Derivative use test

Maths 3c

Non-calculator section

The questions marked as "bonus questions" are optional and will count if they are correct but not if they are wrong or unanswered.

Name:_____

1. For a given function f, state whether these sentences are true or false and explain why.

   a) If I want to know for what values of x the function is increasing or decreasing I do sign analysis of f, and when it's positive then the function is increasing and when it's negative then it's decreasing. (1/0/0)

   b) If I want to know for what falues of x the curvature of the function is positive or negative I do sign analysis of the second derivative of f, and when it's positive the curvature is positive and when it's negative then the curvature is negative.(1/0/0)

2. Briefly state the difference between:

   a) Absolute maximum and local maximum. (1/0/0)

b) Inflection point and terrace point.(1/0/0)

c) Bonus question: suggest a function that has a terrace point.(1/0/0)

3. For these functions, find all critical points and identify them.

a) $g(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x$ (0/1/0)

b) $h(x) = x^3 - x^2 - 4x + 4$ (0/2/0)

4. Find absolute maximum and absolute minimum of these functions.

   a) $j(x) = \frac{1}{5}x + \frac{1}{3}$ within the interval between x=-3 and x=6. (0/1/0)

   b) $k(x) = 2x - \frac{6(x+4)-10}{3}$ within the interval between x=220 and x=255 (0/0/1)

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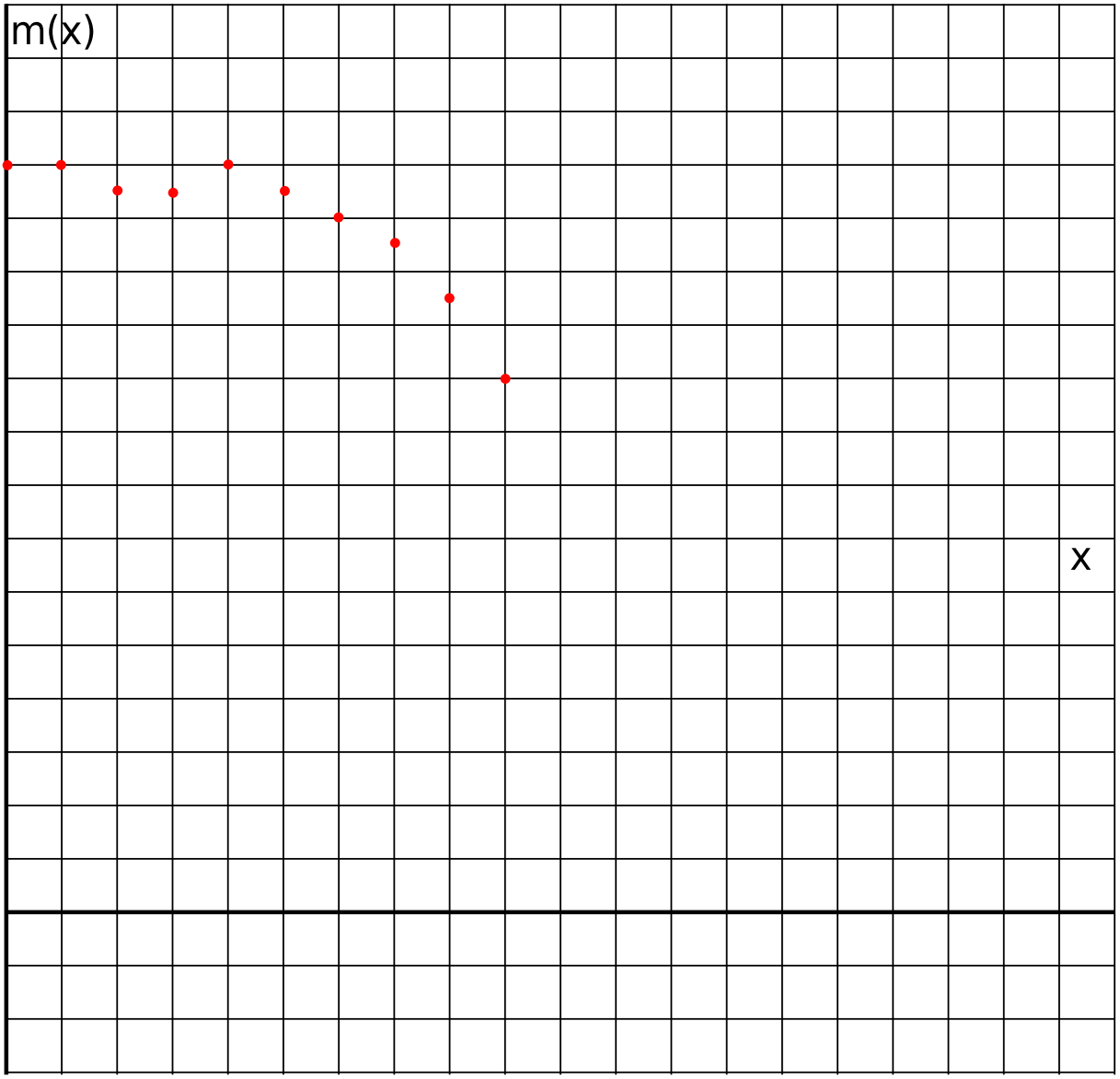
5. Here is a graph of the function m(x).



a) Use the rate of change between the first and last points to extrapolate for what value of x the function will reach 0. (0/1/0)

b) Do the same thing using the last two points. (0/1/0)

6. Given this function:
   $i(x) = \frac{1}{4}x^2 - 2x + 6$

   a) Find the tangent line in x=6 (1/0/0)

   b) Find the tangent line in x=8 (1/0/0)

   c) According to the way the function is behaving when x=6, for what value of x will it reach 20? (0/1/0)

d) According to the way the function is behaving when x=8, for what value of x will it reach 20? (0/1/0)

e) Bonus question: are these estimations earlier or later than the value for which the function will reach 20? (0/1/0)

7. The temperature of the air depends on the time of the day according to the following function:

$$T(t) = 0{,}02t^3 - 0{,}3t^2 + t + 6$$

Althought this function is only valid between 0 and 12h.

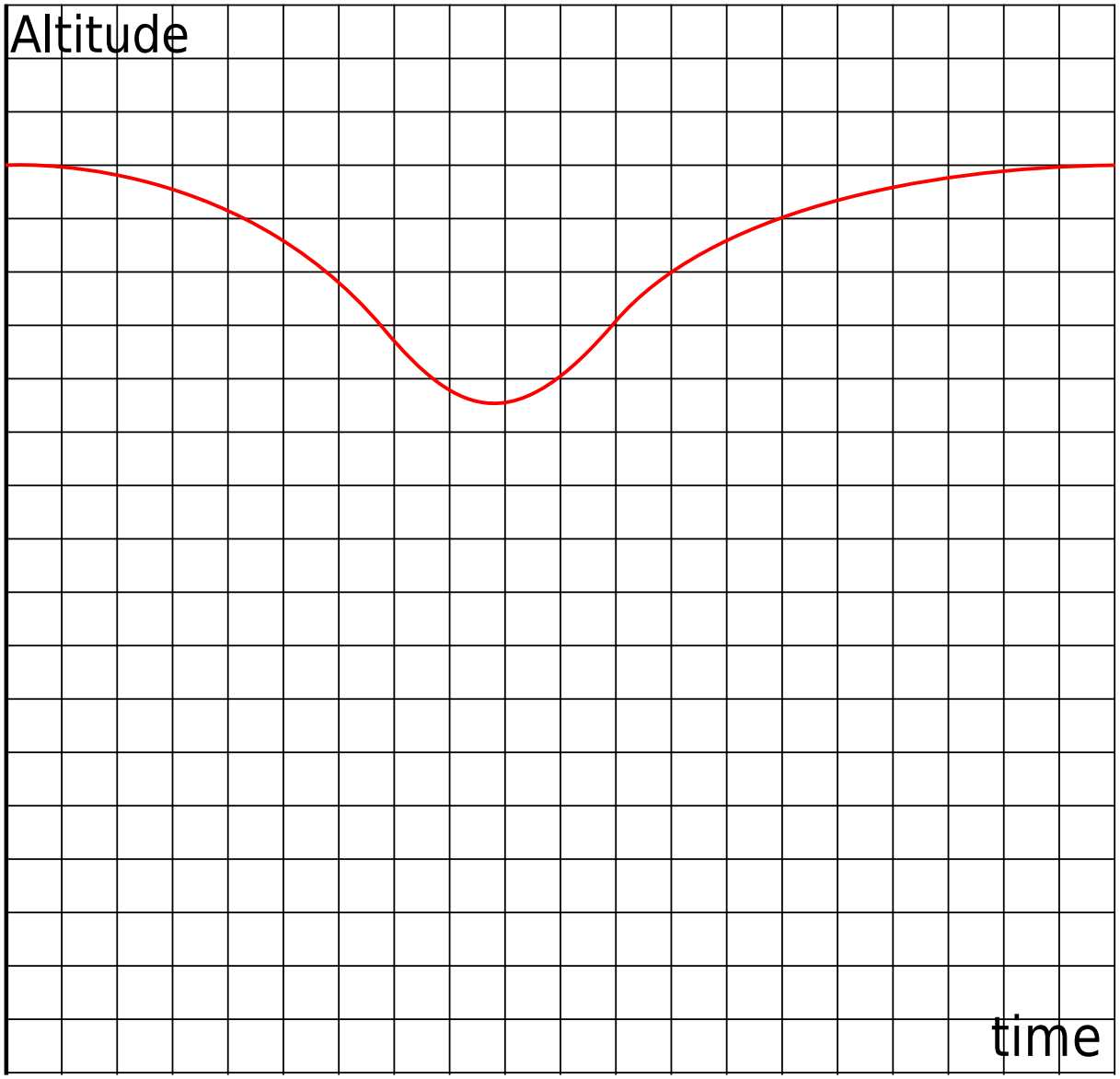
a) Find the maximum value of T during that interval. (0/1/0)

b) Find the minimum value of T during that interval. (0/1/0)

8. We build a metal box out of a single sheet of metal that has an area of 550cm^2. The box must have a square base and no lid (open top). Find the dimensions of the box that allow for maximum volume and find said volume too. (0/1/1)

9. For a given item that is sold in a store, the number of items sold per month depend on the item's price. If the price is 100kr, then it's too expensive and no one will buy it, but the sales would increase if the price is lower: 50 items will be sold for each Krona below 100 that the price is set. Manufacturing the item costs 20Kr per item plus 30000Kr as fixed cost. Find the price that makes the maximum profit and find the maximum profit. (hint: they are natural numbers) (0/1/2)

10. The spaceship Nicki Minaj is orbiting around the planet [insert humorous name here] and the helmsman notices that the altitude is decreasing, therefore the ship is slowly falling onto the planet.
Being an experienced helmsman, he decides to activate the vertical thrusters (that apply a vertical force on the ship) to pull the ship up and back to orbit.
When the first officer wakes up he checks the altitude and sees a graph like this:



Can you mark the time when the thrusters were activated? Explain what makes you think that is the correct moment. (0/0/1)