Maths 3c Units 1-3 test

Non-calculator section

1. What is the derivative of a function f(x) at a given point x_0?

2. Solve the equation $|2x+3|+6=15$

3. Differentiate

a) $6x^4-3x^3+11x^2-6x-\dfrac{1}{x}$

b) e^{8x+13}

c) $\sqrt[3]{x^5}$

bonus $\ln x+e^{x^2+2x+3}$

4) Given that $f(x) = 4x^{-2}$

a) Calculate f(2)

b) Calculate the derivative of f at 2. What does this derivative mean?

5. Given the piecewise function

$$g(x) = \begin{cases} x^2 - 2x + 1 \text{ if } x \leqslant 2 \\ ax + b \quad \text{ if } x > 2 \end{cases}$$

a) Find a and b so that the function is continuous

b) Find a and b so that the function is continuous and differentiable at all values of x.

6) Given $g(x) = \dfrac{x}{3} - \dfrac{2}{x\sqrt{x}}$

a) Differentiate it

b) For what values of x is the function non-differentiable?

7) Given this graph of a function

Find points (mark them as a, b, c...) where:
a) The function is positive and the derivative is negative
b) The function is negative and the derivative is positive
c) The function is increasing and the derivative is negative
d) The function is negative and the derivative is negative but increasing
e) The function is positive and the derivative is positive and increasing
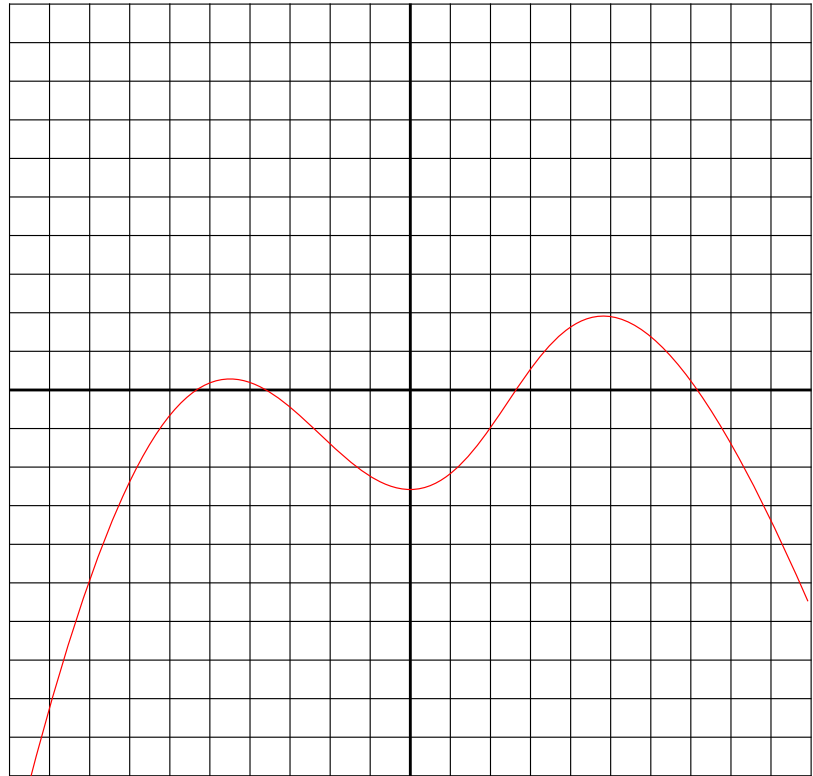
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Calculator section

8. The spaceship Creamy Chocolate fires its thrusters and starts gathering speed. Its speed increases in time according to the function

$$v(t) = -6e^{-0.21t} + 6$$

Where v is the speed in km/s and t is the time in min.

a) Calculate the rate of change between t=0 and t=3. What did we just calculate? (answer in terms of the properties of the motion)

b) What will be the acceleration after 5 minutes?

c) After said time, if it goes on accelerating that fast, when will the speed reach 5 km/s?

d) When will the ship actually reach that speed?

9. Use the definition of the derivative to find the derivative of 2x² "the hard way".

10. The function $i(x)=x^2-2x+1$ has one and only one pair of tangents that intersect at the point (1,-1). Find the equations for those tangents (note: the points awarded may depend on the method used)