Name and date

Maths A & A

Probability exam
Non-calculator section. (13 of 72)

Remember:
- If nothing else is stated, provide your answers with 3 sf.
- Use sigma for the standard deviation

1. Using the example of a perfect die of six:
    a) Define random experiment [1]

    b) Define the probability of getting six [1]

2. In the game StarCraft, three species can be picked to play: Terran, Zerg and Protoss.
Some players can play with only one of them, some can play with two and some players master all three species.
In a tournament there are 47 players who master the Terran, 39 who master the Zerg, 41 who master the Protoss, 14 who master Protoss and Zerg, 19 who master Protoss and Terran, 13 who master Zerg and Terran and 4 who master all three species.

a) Show this information in one or several Venn diagrams. [2]

b) How many players master only the Terran? [1]

c) How many players master the Zerg and **only one** other species? [2]

d) How many players are there in the tournament? [2]

3. A very lazy maths teacher is building a probability exam. Generally, when an item of the syllabus must be covered in an exam, the teacher looks for suitable exercises in previous exams given in the course. The probability of finding one is 2/3. If there is no suitable exercise in past exams, then the teacher looks up in the past official exams. Unfortunately, the official exams tend to mix up the items and the probability that the exercises found are suitable is only 1/3.
If no suitable exercises are found, then the teacher will have to build an exercise expressly.

a) Represent this information in a probability tree [1]

b) Calculate the probability that the teacher needs to build an exercise [1]

c) If she builds the exercise, there is a 1/4 probability that she makes a minor mistake.
An exercise is found with no mistakes. Calculate the probability that she built it expressly [2]

Probability exam
Calculator section.

4. In a large city, a survey is conducted and 160 people are surveyed, of which 60 are children and the rest adults.
Each person in the survey was asked if they preferred Lisbon lemons or one of the other varieties.
It was found that 48 of the children preferred Lisbon lemons. The information is shown in this table:

	Lisbon	Other	Total
Children	48	p	60
Adults	x	y	q

and no one seems to be surprised about the implication that every single person surveyed had an opinion about their preferred variety of lemon

a) Find the values of p and q [1 each]

b) Three people are chosen at random. Find the probability that all three are adults. [2]

c) Given that p(Adult | Lisbon) = 1/3
i) Calculate x [2]

ii) Assuming that the survey sample is perfectly representative of the population of the city, explain whether being and adult is independent of preferring Lisbon lemons. [2]

d) A person is chosen at random from the city. Calculate the probability that it's an adult who prefers Lisbon lemons. [2]

e) Ten people are chosen randomly from the city. Calculate the probability that at least five of them prefer Other varieties of lemons. [4]

5. The number of hairs per square centimetre of scalp (hair density) follows a normal distribution with mean = 480 and standard deviation = 60

a) What percentage of people have a hair density of between 420 and 540? [2]

b) What percentage of people have a hair density that is NOT between 360 and 600? [3]

c) A company is looking for subjects with incipient alopecia for a study on a hair-strengthening shampoo. The subject is eligible if he/she has a hair density below 340. If 5000 individuals are picked at random, how many of them are expected to have a low enough hair density to be eligible for the study? [3]

d) A young girl has been hired to do a shampoo commercial because, according to the company's specialist, she has such a high hair density that "only one person in a thousand would have a higher hair density than her". Calculate her hair density. [4]

6. [IB past exam 2023] A bag contains n citrus fruits. It is known that ten of the fruits are lemons, and the rest of them are oranges. Fruits are drawn from the bag, one after the other, without replacement.

a) Find, in terms of n , the probability that

(i)the first fruit drawn is a lemon [1]

(ii)the first two fruits are lemons. [2]

For the following parts of this question, let n = 25 .

b) Show that the probability that the first two fruits are oranges is 0.35 .[2]

c) Find the probability that the first three fruits are all oranges.[2]

d) Find the probability that at least one of the first three fruits is a lemon.[2]

A game is played where four fruits are drawn, one after the other, from the bag of 25 fruits, without replacement.

A player earns points based on when the first lemon is drawn. At the end of each game, the four fruits are put back in the bag.

A player earns zero points if no lemon is picked, or if the first lemon is picked on the first or second draw.

A player earns 10 points if the first lemon is picked on the third draw and earns 50 points if the first lemon is picked on the fourth draw.

Millie plays this game k times. She finds her score by adding together her points from each game.

e) Find the least value of k such that Millie’s expected score is greater than 100 [6]

7. A conjuror has four special dice, three of 6 and one of 3, and he has figured out a way for them to be indistinguishable at sight or touch. He places all of them in a bag and asks a member of the public to pick one die at random and cast it.
Needles to say, if the die shows 4, 5 or 6, it's clear that the die picked is one of the dice of 6, however the die is cast and the outcome is 2.

a) Formulate the null and alternative hypotheses. [1]

b) Calculate the probability of the observed result if the null hypothesis is true and also if the alternative hypothesis is true. [1]

c) Calculate the probability that it's the die of 3 (and therefore the probability that it's one of the dice of 6) [2]

d) The die is cast two more times, the outcomes being 3 and 2 again. Calculate the probability that the die is the die of 3. [2]

8. Juan de Ribera, captain of the San Juanillo, a XVI century galleon, has had the idea to pre-pack scurvy-preventing kits for the crew. Said kits consist of two lemons, one orange and one strip of beef jerky (which won't help much against scurvy but is traditional).
The weight of lemons follow a normal distribution N(150, 900), the oranges follow N(190, 1600) and the beef jerky follows N(50,64).

a) Calculate the mean and standard deviation of the weight of the packages. [3]

b) In order to save space, the ship's quartermaster Iñaki Gurrea has the idea to pre-peel the oranges and lemons (probably unwise, since the peel is essential for conservation and also contains a large amount of nutrients that can save one in a scurvy-level famine).
20% of the lemons are peel and 30% of the oranges.
Calculate the mean and standard deviation of the anti-scurvy kits.[4]

9. [IB exam 2023 p1] Consider an arithmetic sequence with $u_1=0.6$ and $u_4=0.15$.

a) find d [2]

b) The follfollowing table shows the probability distribution of a discrete random variable X such that $p(x=n)=\frac{u_n}{k}$ where n is integer between 1 and 4 and k is positive.

n	1	2	3	4
p(x=n)	$\frac{0.6}{k}$	$\frac{u_2}{k}$	$\frac{u_3}{k}$	$\frac{0.15}{k}$

Find k [4]

Bonus question:
A very large survey is conducted among male humans to record the average length of their hands. The survey is conducted worldwide and with so many participants that it can be considered very representative of the population.
The data reveals that the average length of hand is 18 cm with a standard deviation of 2.3 cm.
An individual wants to know if his hand can be considered "large" (that's humans for you) and, using a simple normal distribution calculation, sees that, given that his hand is 21 cm long, it is above average and since the probability of finding another human with a larger hand is 0.0961, only one in ten humans would have a larger hand than him.

Discuss these conclusions [2]