

Functions exercises set 2

1. Given the function $f(x) = \frac{4x+2}{x-2}, x \neq 2$

a) Sketch its graph. On the sketch, indicate the values of any axis intercepts and label all asymptotes with their equations. (note: this is taken from paper 1, so it ought to be done without calculator)

b) Write down its range.

2. Given the function $g(x) = x^2 + bx + c$ It is known that the function has an axis of symmetry at $x=5$ and one of the x -intercepts is at $x=1/4$.

a) Find the other x -intercept.

b) Find a and b

c) Find the vertex

3. Factorise the polynomial $P(x) = 3x^3 + 5x^2 + x - 1$

4. Consider the function $h(x) = \frac{P(x)}{(x+1)^2(2x+1)}$

a) Find $h(x) = \lim_{x \rightarrow -1} h(x)$

b) Find $h(x) = \lim_{x \rightarrow \infty} h(x)$

5. A sinusoidal model describes an oscillation whose maximum value is 12 and its smallest value is 4. The time between a maximum and a minimum is 3s, and the first maximum takes place one second after the oscillation starts.

a) Find a , b , c and d

b) Calculate what fraction of the time the oscillation has a value of 5 or lower.

6. A cup of hot chocolate cools down from its original 88°C . The room temperature is 16°C and after 3 minutes the chocolate is still at 65°C .

a) Write a model for the temperature as a function of time.

b) How long before it can safely be drunk? (say, 55°C)

7. Given the model for the position of the piston of an engine*

$$y(t) = 64 \cos(900t) + 128 \quad \text{where } t \text{ is in seconds and } y \text{ is in mm.}$$

a) Calculate the maximum height, minimum height and equilibrium position

b) Calculate the period and the frequency.

c) Given that the diameter of the pistons is 48mm and that the engine has six cylinders, calculate the engine displacement.

*Not realistic, since the motion of a piston in a reciprocating engine is not exactly sinusoidal.

Answers:

1. a) Asymptote at $x=2$. Convergence at $y=4$. Intercepts at $(-1/2,0)$ and $(0,-1)$. b) $\mathbb{R} - \{4\}$
2. a) $39/4$. b) $a=1, b=-10$. c) $(5,-22.6525)$
3. $3(x+1)(x+1)(x-1/3)$
4. a) 4. b) $3/2$.
5. a) $a=4, b=60, c=30, d=8$. b) 0.230053456
- 6.a) $y=72e^{-0.1282819t} + 16$. b) 4.77935 min
- 7.a) 192mm, 64mm, 128mm. b) 0.4s, 2.5Hz. c) 1389740.1 mm³ (1.3897 l)