

Maths AA HL Functions test

Non-calculator section

Show your work in every exercise.

All results must be given exact or with three significant figures, unless otherwise stated.

1. Define function [1]

2. State the domain of the following functions: [3]

$$f(x) = \sqrt{x^2 - 3x - 4}$$

$$g(x) = \log|x|$$

$$h(x) = \begin{cases} \frac{x-3}{x^2-x-6} & \text{for } x \neq 3 \\ 6 & \text{for } x = 3 \end{cases}$$

3. Does the function $i(x) = (x-3)^3$ have inverse? If it does, write it. If it doesn't, state why. [3]

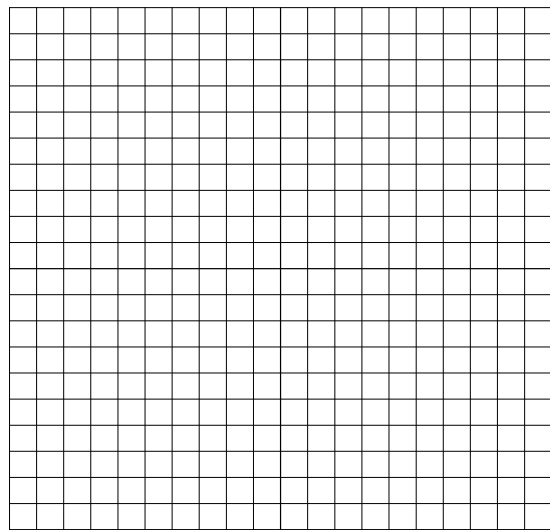
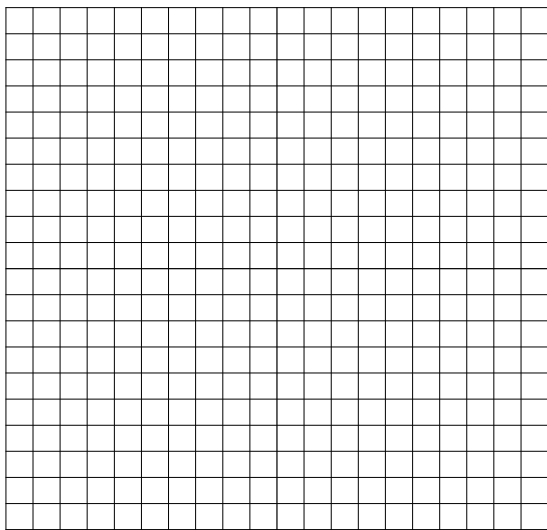
4. A line is in vector form by the function $(x, y) = (4, 3) + \lambda \cdot (3, -6)$

Write the equation of the line in general form and point-gradient form. [2]

5. a) find $(i \circ f)$ [1]

b) find $(f \circ i)$ [2]

6. Sketch the functions $j(x) = |\sin x|$ and $k(x) = \sin|x|$ at least from -360 to 360° [2]



7. Given these limits [3]

I $\lim_{x \rightarrow 3} \frac{x^3 - 9x^2 + 27x - 27}{x^2 - 6x + 9}$

II $\lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt{x}}$

III $\lim_{x \rightarrow \infty} \frac{x^6}{e^{0.1x}}$

IV $\lim_{x \rightarrow \infty} \frac{\cos x}{\log x}$

- a) They are all zero.
- b) They are all zero except I, which is 2.
- c) They are all zero except I, which is infinity.
- d) They are all zero except I, which is infinity and IV, which doesn't exist.
- e) They are all zero except I and IV, which don't exist.
- f) They are all zero except I and IV, which are infinity.
- g) Only II is zero
- h) Only III is zero

Calculator section

Show your work in every exercise.

All results must be given exact or with three significant figures, unless otherwise stated.

8. An engineer builds a model glider with rocket propulsion to get it airborne.

The rocket follows the trajectory of the function $f(x) = -5 \cdot 10^{-4}x^2 + 5x$

At a given point in time, the engineer will press a button in the remote control and the glider will detach from the rocket and start flying.

The glider's trajectory from the point of detachment is slightly downwards by 25% (that means for every 100m forward, it goes down by 25m)

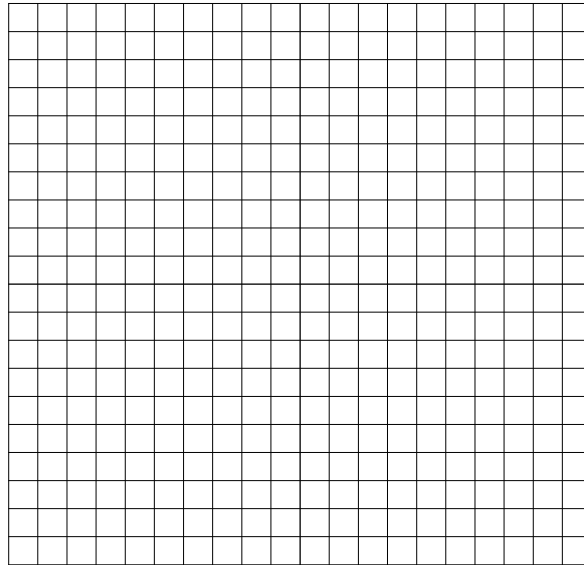
a) What is the highest point the model glider will reach? [2]

b) Find the equation of the line that the glider will follow after detaching at the highest possible point. [2]

c) Where will the glider land? [3]

9. The rocket from exercise 2 falls to the ground and is recovered by the engineer. When the rocket reaches the ground, it is still at 240°C , and it cools down on the Saharan sand and an air temperature of 56°C .

a) Sketch a graph for the temperature of the rocket showing the relevant points. [2]



b) If after 15 min the temperature of the rocket is 160°C , establish a model for the temperature showing the known parameters and explain why they have these values. [2]

c) The engineer can pick up the rocket with gloves, but only if the temperature is 125°C or below. How long time must pass before she can pick it up? [2]

10. The loudness L of a sound is measured in decibels and is related to the intensity I of said sound through the function.

$$L=10 \cdot \log (I \cdot 10^{12})$$

Given two sounds S_1 and S_2

S_1 has a loudness of 60 decibels.

S_2 has an intensity that is twice that of S_1 .

a) Calculate the intensity and the loudness of S_2 [3]

b) The maximum loudness of thunder during a thunderstorm was measured to be 115 decibels. Find the sound intensity of the thunder. [3]

11. A weight suspended on a spring at a height of 12cm above a table. The weight is pulled down and released so that it oscillates vertically.

Its maximum height is 21 cm and its frequency is 1.3 Hz.

a) Find a model for the motion [4]

b) Find the number of times the weight reaches its maximum height in the first five seconds of motion.

c) Find when the weight first reaches a height of 15 cm. [2]

d) A camera is set to take a picture at a random time during the oscillation. Find the probability that when the picture is taken, the height of the weight is 17cm or higher. [4]

Bonus.

In an endless and infinitely slow fall descended countless specks of dust through the air of the dim lit corridor as the champion ran towards the seventh and last test, her footsteps on the ancient flagstones echoing in the gloom and her cape stirring the air for the first time in centuries. She hesitated in front of the door, but resolutely pushed it aside and stepped through, her gaze immediately sweeping the room beyond.

Uncertainty shook her when she identified the dark room for what it was: void. The floor ended in only a few metres into a chasm of unfathomable depths and a darkness that seemed to devour the light even from the soul.

On the other side of the chasm, possibly on an unseen, distant wall but could just as well be hovering on nothingness, a white doorway through which another room could be seen as though illuminated by daylight.

It took only a few seconds before she aspired a round object crossing through the doorway on the other side. She waited for a few seconds and saw the object cross again, this time in the other direction. A pendulum, she concluded, one that may become an obstacle? In an effort to elucidate the way through, she looked at the room on this side of the chasm and a crushing wave of adrenaline flooded her spine when she saw that a hooded figure was sitting on a stool behind the door she had just pushed open, holding a compound bow and an arrow. Had it always been there? Was it a statue? Though immobile, it radiated an indescribable aura of surreality and somehow... time.

She was about to address the figure when it stood up, slowly and solemnly proffered the bow and, in a deep voice that seemed to come from a long way away, spoke thusly:

"The arrow, slike, magic, y'know? Like, it goes straight and snot like it falls or anything... just like, goes. Whatimsaynis like, you don't need to like, aim, y'know, snot an aiming kinda test, slike timing sorathin, ynowsaing?"

The champion looked at the hooded figure quizzically as she took the bow and arrow and replied, clearly pronouncing every syllable "yes, I do know what you are saying".

"Gotta hit the... thing, y'know, it goes like, three metres right'nleft in like, seven seconds, and the other like, thing, the... frame sorathing, slike just... like, one metre wide, y'know? So slike, timing sdathing" the hooded figure continued.

Many questions arose in the mind of the champion, but she made an effort to dismiss them and mentally called upon her experience. Her best judgement was that the pendulum's diameter was 40cm, it was at a distance of 50m and the speed of the arrow was 34m/s. The question therefore was *when*.

She took out her calculator from her waistband and turned it on...

[4]