

# Paper 1

**Time allowed: 1 hour 30 minutes**

Answer all the questions.

All numerical answers must be given exactly or correct to three significant figures, unless otherwise stated in the question.

Answers should be supported by working and/or explanations. Where an answer is incorrect, some marks may be awarded for a correct method, provided this is shown clearly.

**You need a graphic display calculator for this paper.**

- 1** Diego receives COP 1 000 000 from his grandfather. He invests some of his money into a saving account which pays annual interest at a rate of 6%, which is compounded monthly.
- a** Find the least amount of money that Diego must initially deposit into the account if he is to have COP1 500 000 in this account after 10 years. Give your answer to two decimal places. [2 marks]
- With the remainder of the money that Diego does not invest, he buys a car. The value of the car decreases at an average of 10% every year.
- b** Find the value of the car after 10 years. [3 marks]
- 2** Alessandra starts a three-week fitness programme that requires her to walk at a fast pace over a distance which increases each day. The first day, a Monday, she walks a distance of 200 m. On Friday, she walks a distance of 360 m. The distance she walks increases by a constant amount  $d$  m every day.
- a** Write down the value of  $d$ . [1 mark]
- b** Calculate the distance Alessandra will walk on the second Sunday after the start of her programme. [2 marks]
- c** Find the total distance Alessandra will have walked at the end of the three-week fitness programme. [3 marks]
- 3** Consider the relation  $C = C(A)$  between the circumference of a circle  $C$  and its area  $A$  defined for  $A \geq 0$ .
- a** Justify that:
- i**  $C = C(A)$  is a function. [2 marks]
- ii**  $C$  has an inverse function  $C^{-1}$ . [2 marks]
- b** Sketch the graphs of  $C(A)$  and  $C^{-1}(A)$  on the same axes. [2 marks]
- c** Hence, find the area of a circle with circumference 25. [2 marks]

- 4** A cup of hot chocolate is left on a counter for several hours. Initially its temperature was  $86^\circ\text{C}$ . After 30 minutes the temperature had already dropped to  $28^\circ\text{C}$ . The temperature,  $T^\circ\text{C}$ , of the hot chocolate is modelled by the function  $T(t) = 22 + a2^{bt}$ , where  $t$  is the number of hours that have elapsed since the hot chocolate was first left to stand.
- a** Find the value of:
- i**  $a$       **ii**  $b$  [4 marks]
- b** Write down the equation of the horizontal asymptote of the graph of  $T$ . [1 mark]
- c** State the meaning of the asymptote found in part **b**. [1 mark]
- 5** Kathy is collecting information for a statistics project. She asks a group of students who have pet dogs about the number of times that they usually walk the dog per day.

The data collected is shown in the following table.

Number of walks	1	2	3	4	5
Number of students	4	8	10	5	1

- a** State, with a reason, whether "number of walks" is a discrete or continuous variable. [2 marks]
- b** For the students that Kathy surveyed, find:
- i** the modal number of dog walks per day [1 mark]
- ii** the mean number of dog walks per day. [2 marks]
- 6** Jasmine plays a computer game. In the game, she collects tokens of different colours. Each colour token gives the player a different number of points. Jasmine records the relative frequencies of obtaining tokens of each colour. She uses this to estimate the probability of obtaining a token of a certain colour.
- | Colour                  | Red           | Blue          | Green          | Yellow        | Pink          | Orange |
|-------------------------|---------------|---------------|----------------|---------------|---------------|--------|
| Points                  | 2             | 3             | 5              | 1             | 3             | 4      |
| Estimate of probability | $\frac{1}{9}$ | $\frac{1}{6}$ | $\frac{1}{12}$ | $\frac{2}{9}$ | $\frac{1}{6}$ | $p$    |
- a** Find the value of  $p$ . [2 marks]
- b** Find the expected score if Jasmine plays the game once. [2 marks]
- 7** Jeanny has six coloured pencils and eight coloured pens in her colouring box. Every afternoon from Monday through to Friday, Jeanny returns from kindergarten and picks a pen or pencil at random from this box. She then begins to draw with the object she picked.
- a** State the probability that, on any one day, Jeanny picks a pen. [1 mark]
- b** Find the probability that, in one week (Monday through to Friday), Jeanny picks a pen on exactly two days. [3 marks]

8 A farmer fed two groups of cows according to different feeding programmes. The first group was fed according to the "Intensive" programme, and the second group was fed according to the "Extensive" programme.

After two years, the size of the fat layer on the belly of each cow was measured. The data, in centimetres, is given below.

Intensive programme: 20.5, 20, 19, 24, 24.5, 25, 26

Extensive programme: 19, 19, 21, 20, 20.5, 18.5

a On the same axis, construct two box-and-whisker diagrams to represent the results of each set of cows. [3 marks]

b Describe what your box-and-whisker diagrams tell you about the size and spread of the fat layers on the cows from each of the feeding programmes. [2 marks]

9 The loudness of a sound,  $L$  decibels (dB), is a function of the sound intensity,  $S$  watts per square metre ( $\text{W/m}^2$ ).

Loudness is given by the formula  $L = 10 \cdot \log_{10}(S \times 10^{12})$ .

Humans can hear sounds between 0 and 140 decibels.

a Determine whether a human can hear a sound with intensity of  $0.000001 \text{ W/m}^2$ . [3 marks]

In some countries, workers must wear hearing protection when the loudness of sounds reaches 80 decibels or more.

b In these countries, determine the maximum intensity a sound can be before a worker must wear hearing protection. [2 marks]

10 As part of a study into colour preference, Alicia surveyed 100 students at her school. She recorded each student's gender, and their favourite colour from blue, red, brown or black. Alicia's results are shown in the table below.

	Colour			
	Blue	Red	Brown	Black
Female	16	15	10	13
Male	23	12	9	2

Alicia conducted a  $\chi^2$  test for independence at a 5% level of significance.

a State the null hypothesis. [1 mark]

b Write down the associated  $p$ -value. [2 marks]

c State, giving a reason, whether the null hypothesis should be accepted. [2 marks]

11 Consider the function  $f(x) = x(x - 2)(x - 4)$ .

a Sketch the graph of  $f$  and shade the two regions bounded by the graph of  $f$  and the  $x$ -axis. Label the two regions A and B. [3 marks]

b Find:  
 i the area of region A  
 ii the area of region B  
 iii the **total** shaded area. [4 marks]

12 Lisa measures the arm spans (the length from finger tips on one hand to finger tips on the other hand, when arms are held horizontally outstretched) of students in two mathematics classes.

She is interested to see whether the mean arm span,  $\mu_1$ , of class A is the same as the mean arm span,  $\mu_2$ , of class B. Lisa's data is shown in the table below.

Class A	161	178	194	204	173	162	177	196	194	160		
Class B	165	146	159	160	170	173	199	140	178	200	155	194

In this question, you will use a  $t$ -test to compare the means of the two groups at the 10% level of significance.

You may assume the data is normally distributed and the standard deviations are equal between the two groups.

a i State the null hypothesis. [2 marks]  
 ii State the alternative hypothesis. [2 marks]

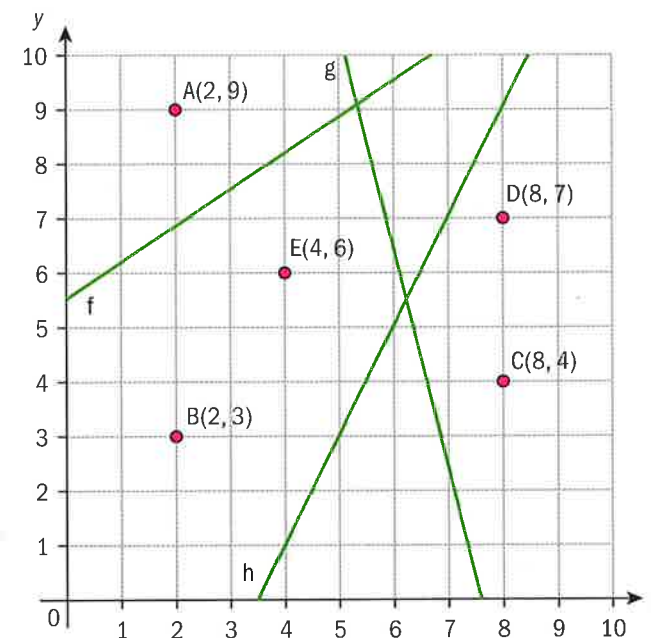
b Find the associated  $p$ -value for this test. [2 marks]

c State, giving a reason, whether Lisa should accept the null hypothesis. [2 marks]

13 Points A(2, 9), B(2, 3), C(8, 4), D(8, 7) and E(4, 6) represent wells in the Savannah National Park. These wells are shown in the coordinate axes below.

Horizontal scale: 1 unit represents 1 km.

Vertical scale: 1 unit represents 1 km.



a Calculate the distance between the wells A and E. [2 marks]

Pablo, the Park Ranger draws three lines,  $f$ ,  $g$  and  $h$ , around point E and obtains an incomplete Voronoi diagram around the point E.

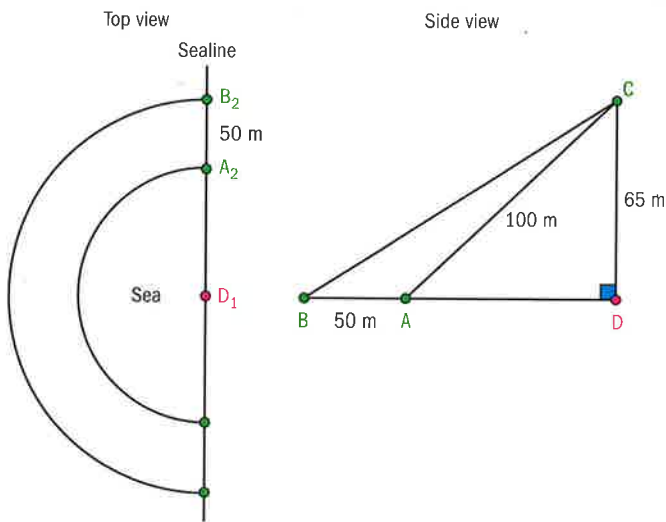
- b Find the equation of the line which would complete the Voronoi cell around point E. Give your answer in the form  $ax + by + d = 0$  where  $a, b, d \in \mathbb{Z}$ . [5 marks]
- c In the context of the question, explain the significance of the cell around point E. [1 mark]

14 A lighthouse is situated at point D. Light is emitted from point C, 65 m above sea level.

The beam of light from the lighthouse traces a semicircular arc which, on the surface of the water, is 50 m wide.

The edges of the beam pass through the two sets of points A and B. The distance from any point A to C is 100 m.

Not to scale



- a Find the area of the surface of the sea swept by the light beam, correct to the nearest square metre. [4 marks]
- b Find  $\widehat{CAD}$ . [2 marks]
- c Hence find the length BC. [3 marks]

# Paper 2

Time allowed: 1 hour 30 minutes

Answer all the questions.

All numerical answers must be given exactly or correct to three significant figures, unless otherwise stated in the question.

Answers should be supported by working and/or explanations. Where an answer is incorrect, some marks may be awarded for a correct method, provided this is shown clearly.

You need a graphic display calculator for this paper.

- 1 A surveyor is marking out an ornamental garden in the shape of a triangle. She places a marker in the ground at point A and then travels 80 m due North to a point B, where she places a second marker. She then travels 70 m on a bearing of  $125^\circ$  and places a third marker at point C. Finally she walks in a straight line back to the first marker at A.
- a Sketch this information on a diagram. [2 marks]
- b Calculate the length of the side AC. [3 marks]
- c Calculate the area of the triangular garden. [2 marks]
- d Find the bearing that the surveyor was walking on when she travelled from C to A. [5 marks]
- 2 The time,  $T$ , that competitors take to complete a 10 000 m race is normally distributed with mean  $\mu = 50$  minutes and standard deviation  $\sigma = 5$  minutes.
- a Sketch a diagram to represent this information with the numbers 50 and 5 indicated on it. [3 marks]
- b Find the probability that a random competitor takes between 45 and 55 minutes. [2 marks]
- c Find the probability that a random competitor takes less than 40 minutes to complete the race. [2 marks]
- d The fastest 75% of competitors receive a medal. Find the time that the race has to be completed under, in order for a competitor to receive a medal. [2 marks]
- e Given that a competitor received a medal, find the probability that they finished the race in less than 40 minutes. [3 marks]
- f If 10 000 competitors ran the race, estimate (to the nearest minute) how many competitors completed the race in less than 33 minutes. [2 marks]

- 3 On a particular day, the depth of water ( $d$  m) at a harbour entrance can be modelled by the equation  $d = 11 + 5\sin 30t^\circ$ . Here,  $t$  is the time (in hours) that have elapsed since midnight.
- a State the depth of water at midnight. [1 mark]
- b State the period of this function. [2 marks]
- c i State the maximum value of the depth and the times during the day when this will occur.
- ii State the minimum value of the depth and the times during the day when this will occur. [4 marks]
- d Sketch the graph of  $d$  as a function of  $t$ , for  $0 \leq t \leq 24$ . [3 marks]
- e A large boat can only come into the harbour when the depth of water is greater than 14 m. Find the morning time interval, in hours and minutes, during which the boat can enter. [3 marks]
- 4 a When Mich takes a free throw in a basketball game, the probability that he scores is always constant at  $\frac{2}{3}$ .  
Mich takes nine free throws. Let  $X$  be the number of times he scores.
- i State the distribution that  $X$  satisfies, and state the parameters.
- ii Write down the mean of this distribution.
- iii Calculate the probability that Mich scores exactly seven times.
- iv Calculate the probability that he scores four or more times. [8 marks]
- b When Ken takes a free throw at basketball the probability that he scores is always constant at  $\frac{1}{3}$ . Find the minimum number of free throws that Ken would have to take to be at least 99% certain of scoring at least once. [5 marks]
- 5 Eleven students,  $A - K$ , revised for and then took a maths exam. Let  $h$  represent the number of hours that each student spent revising, and let  $s$  represent the score (out of 100) that they gained. The data for each student is shown in the following table.

	A	B	C	D	E	F	G	H	I	J	K
$h$	10	9.5	9	8.5	8	7.5	7	6.5	6	5	0
$s$	100	91	93	90	80	85	79	70	69	60	65

- a Identify any outliers in the values of  $h$ . Justify your answers. [4 marks]
- b Calculate the Pearson's product moment correlation coefficient,  $r$ , for this bivariate data. [2 marks]
- c Write down the equation of the line of best fit of  $s$  on  $h$ . [2 marks]
- d Hence estimate the score, to the nearest integer, of a twelfth student who spent 5.5 hours revising. [2 marks]

- e Rank the students from 1 to 11 according to the number of hours they spent revising,  $h$ . Rank 1 should represent the most hours spent revising.

In a similar way, rank the students according to the test score,  $s$ , they obtained. Rank 1 should represent the highest test score.

Copy and complete the table below to show the rankings for each student.

	A	B	C	D	E	F	G	H	I	J	K
Hour rank	1	2	3								
Score rank	1	3	2								

[2 marks]

- f Calculate the Spearman's rank correlation coefficient,  $r_s$ , for this data. [2 marks]
- g Explain why  $r_s > r$ . [1 mark]

- 6 Alison rides a rollercoaster. The height of the rollercoaster over the first part of the ride can be modelled by the equation

$$h(x) = \frac{x^3}{3} - \frac{5x^2}{2} + 4x + 5 \text{ for } 0 \leq x \leq 7, \text{ where } x \text{ m is the horizontal}$$

distance from the start, and  $h$  m is the height.

- a Find  $h'(x)$ . [2 marks]
- b Solve  $h'(x) = 0$ . [2 marks]
- c Hence, or otherwise, find the coordinates of the stationary points of  $h(x)$ , and determine the nature of each. [4 marks]
- d Sketch the graph of  $h(x)$  against  $x$ . [2 marks]
- e If Alison is more than 10 m above the ground she suffers from acrophobia. Find the values of  $x$  for which this happens. [1 mark]
- f If the gradient is smaller than  $-1$ , Alison will scream. Find the values of  $x$  for which this happens. [2 marks]

c The curve is concave down in the interval  $0 \leq x \leq 5$ , so each trapezium will be an underestimate. (1 mark)  
Therefore the sum of the trapezia will also be an underestimate. (1 mark)

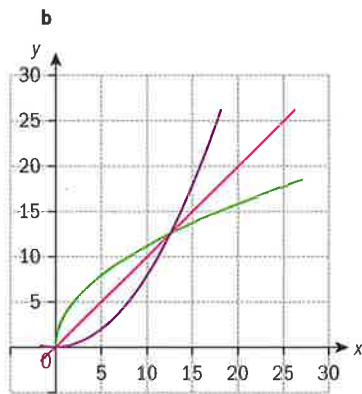
d  $\int_0^5 \left(5 - \frac{x^3}{25}\right) dx = \left[5x - \frac{x^4}{100}\right]_0^5$   
 $= 25 - \frac{625}{100} = 18.75$  units<sup>2</sup> (2 marks)

e Percentage error  
 $= \frac{18.75 - 18.5}{18.5} \times 100 = 1.35\%$  (2 marks)

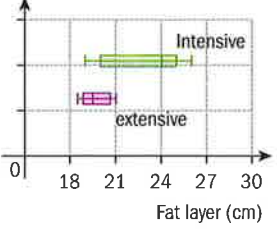
## Paper 1

### Exam-style questions

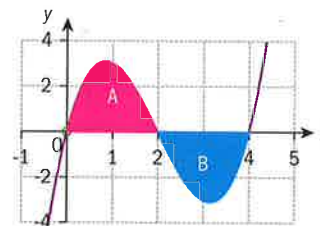
- 1 a  $P\left(1 + \frac{6}{100 \times 12}\right)^{12 \times 10}$   
 $= 1\,500\,000$  (1 mark)  
 $P = 824\,449.10$  (1 mark)  
 Ans: COP 824 449.10 (2 marks)
- b  $V_0 = 1500000 - 824449.10 = 675550.90$  (1 mark)  
 $V_{10} = 675550.90 \times (0.9)^{10} = 235550.03$  (2 marks)
- 2 a  $d = 40$  (metres) (1 mark)
- b EITHER  
 $200 + 40 \times 13 = 720$  (2 marks)
- OR  
 $360 + 40 \times 9 = 720$  (2 marks)
- c  $S_{21} = \frac{2 \times 200 + 40 \times 20}{2} \times 21$   
 $= 12\,600$  (1 mark)



- 3 a i For each given area  $A$ , there is only one possible value for the radius, which is  $\sqrt{\frac{A}{\pi}}$ .  
 Therefore, since  $C = 2\pi r$ ,  $C$  is also unique for that value of  $A$ . (1 mark)
- ii For each given circumference  $C$ , there is only one possible value for the radius, which is  $\frac{C}{2\pi}$ .  
 Therefore,  $A = \pi r^2$ ,  $A$  is unique for that value of  $C$ , so  $C^{-1}$  exists. (1 mark)
- c  $A = C^{-1}(25) \Rightarrow C(A) = 25 \Rightarrow A = 49.7$  (2 marks)
- 4 a i  $T(0) = 86 \Rightarrow 22 + a = 86 \Rightarrow a = 64$  (2 marks)
- ii  $T(0.5) = 28 \Rightarrow 22 + 64 \times 2^{0.5b} = 28$  (1 mark)  
 $b = -6.83$  (1 mark)
- b  $T = 22$  (1 mark)
- c The temperature of the hot chocolate approaches 22 °C as  $t$  gets very large, indicating that the temperature of the room is 22 °C. (1 mark)
- 5 a Discrete (1 mark)  
 Number of walks can be counted (1 mark)

- b i 3 (1 mark)  
 ii 2.68 (3 s.f.) (2 marks)
- 6 a  $\frac{1}{9} + \frac{1}{6} + \frac{1}{12} + \frac{2}{9} + \frac{1}{6} + p = 1$  (2 marks)  
 $p = \frac{1}{4}$  (2 marks)
- b  $\frac{1}{9} \times 2 + \frac{1}{6} \times 3 + \frac{1}{12} \times 5 + \frac{2}{9} \times 1 + \frac{1}{6} \times 3 + \frac{1}{4} \times 4 = \frac{103}{36}$   
 (2.86, 3 sf) (2 marks)
- 7 a  $\frac{8}{14} \left(\frac{4}{7}\right)$  or 0.571 (3 s.f.) (1 mark)
- b  $\left(\frac{5}{2}\right) \left(\frac{4}{7}\right)^2 \left(\frac{3}{7}\right)^3 = 0.257$   
 (3 s.f.) (3 marks)
- 8 a   
 Intensive  
 Scale and labelled axis (1 mark)  
 Correct diagrams (2 marks)  
 extensive
- b The average layer of fat was thicker on the cows from the Intensive programme. (1 mark)  
 The interquartile range and range of fat layer sizes was also much greater for cows from the Intensive programme. (2 marks)
- 9 a  $L = 10 \log_{10}(10^{-6} \times 10^{12}) = 60$  dB (2 marks)  
 Yes as the value is between 0 and 140. (1 mark)
- b  $10 \log_{10}(S \times 10^{12}) = 80$  (1 mark)  
 $S = 0.0001$  Wm<sup>-2</sup> (1 mark)

- 10 a  $H_0$ : Favourite colour is independent of gender. (1 mark)
- b 0.0276 (1 mark)
- c As  $0.0276 < 0.05$  the null hypothesis is rejected at 5% level of significance. (2 marks)
- 11 a Labelled axes (1 mark)  
 Each region (2 marks)



- b Use of definite integration (1 mark)
- i  $\text{area}(A) = \int_0^2 f(x) dx = 4$  (1 mark)
- ii  $\text{area}(B) = \int_2^4 |f(x)| dx = 4$  (1 mark)
- iii  $\text{area}(A) + \text{area}(B) = 8$  (1 mark)
- 12 a i  $H_0: \mu_1 = \mu_2$  (1 mark)  
 ii  $H_1: \mu_1 \neq \mu_2$  (1 mark)
- b  $H_0: p = 0.209$  (3 sf) (2 marks)
- c As  $0.209 > 0.1$  there is no evidence to reject the null hypothesis at the 10% level of significance. (2 marks)

- 13 a  $AE = \sqrt{(2-4)^2 + (9-6)^2} = 3.61$  (3 s.f.) (2 marks)
- b Attempt to find perpendicular bisector of BE. (1 mark)  
 One technique is shown here:  
 $PB = PE \Rightarrow \sqrt{(x-2)^2 + (y-3)^2} = \sqrt{(x-4)^2 + (y-6)^2}$  (1 mark)

Attempt to expand (1 mark)

$x^2 - 4x + 4 + y^2 - 6y + 9 = x^2 - 8x + 16 + y^2 - 12y + 36$  (1 mark)

$4x + 6y - 39 = 0$  (1 mark)

- c The cell corresponds to the region of the park that has  $E$  as the closest well. (1 mark)
- 14 a  $AD^2 = 100^2 - 65^2 = 5775$   
 or  $AD = 76.0$  3 s.f. (1 mark)

Area  
 $= \frac{\pi}{2} ((50 + AD)^2 - AD^2)$   
 $= 15\,864$  m<sup>2</sup> (3 marks)

b  $\hat{C}\hat{A}\hat{D}$   
 $= \sin^{-1}\left(\frac{65}{100}\right) = 40.54\dots$   
 $= 40.5^\circ$  (2 marks)

c  $\hat{B}\hat{A}\hat{C} = 180 - \hat{C}\hat{A}\hat{D} = 139.46$   
 Cosine rule  
 $\Rightarrow (BC)^2 = (AB)^2 + (AC)^2 - 2AB \times AC \times \cos \hat{B}\hat{A}\hat{C}$  (1 mark)

$BC = \sqrt{50^2 + 100^2 - 2 \times 50 \times 100 \cos(139.46\dots)}$

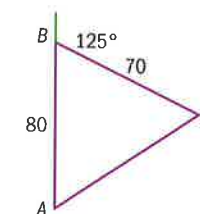
$BC = 142$  m (3 s.f.) (1 mark)

$BC = 142$  m (3 s.f.) (1 mark)

## Paper 2

### Exam-style questions

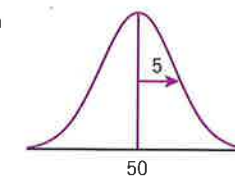
1 a



(2 marks)

- b  $\hat{A}\hat{B}\hat{C} = 180 - 125 = 55$  (1 mark)  
 $AC^2 = 80^2 + 70^2 - 2 \times 70 \times 80 \times \cos 55$  (1 mark)  
 $AC = 69.8279\dots = 69.8$  m (3 s.f.) (1 mark)
- c  $\text{Area} = \frac{1}{2} \times 80 \times 70 \times \sin 55$   
 $= 2293.62\dots$   
 $= 2.29 \times 10^3$  m (3 s.f.) (1 mark)
- d  $\frac{69.8279}{\sin 55} = \frac{80}{\sin C} \Rightarrow C = 69.8$  (2 marks)  
 Bearing is  $360 - 55 - 69.8 = 235^\circ$  (3 s.f.) (1 mark)

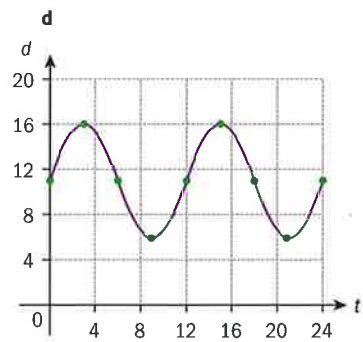
2 a



- b  $P(45 < t < 55) = 0.683$  (3 s.f.) (2 marks)
- c  $P(t < 40) = 0.0228$  (3 s.f.) (2 marks)
- d  $P(t < M) = 0.75 \Rightarrow M = 53.4$  (3 s.f.) mins (2 marks)
- e  $P(t < 40 | \text{gained medal}) = \frac{P(t < 40 \cap \text{gained medal})}{P(\text{gained medal})} = \frac{0.02275\dots}{0.75} = 0.303$  (3 s.f.) (3 marks)

- f  $10\,000 \times P(t < 33) = 3.37\dots$   
 so 3 competitors (2 marks)

- 3 a 11 m (1 mark)
- b  $\frac{360}{30} = 12$  hours (2 marks)
- c since  $-1 \leq \sin \leq +1$
- i 16 m at 03:00 and 15:00 (2 marks)
- ii 6 m at 09:00 and 21:00 (2 marks)



- d** First two intersections between sine curve and  $d = 14$  are 1.23... and 4.77...  
So time period is  $01:14 < t < 04:46$  a.m. (3 marks)
- 4 a i** Binomial;  $X \sim B(9, \frac{2}{3})$  (3 marks)  
**ii**  $\mu = np = 6$  (1 mark)  
**iii**  $P(X = 7) = 0.234$  (3s.f.) (2 marks)  
**iv**  $P(X \geq 4) = 1 - P(X \leq 3) = 0.958$  (3s.f.) (2 marks)

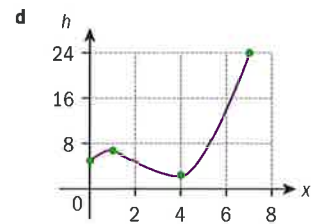
	A	B	C	D	E	F	G	H	I	J	K
Hour rank	1	2	3	4	5	6	7	8	9	10	11
Score rank	1	3	2	4	6	5	7	8	9	11	10

- e** (1 mark)
- f**  $r_s = 0.973$  (3s.f.) (1 mark)
- g** Spearman's is less sensitive to outliers like  $K$ , which distort the data, so Spearman's shows greater correlation. (1 mark)

- d**  $P(\text{scoring at least once}) > 0.99 \Rightarrow P(\text{missing all shots}) \leq 0.01$  (1 mark)  
Let him take  $n$  shots, so require  $(\frac{2}{3})^n \leq 0.01$  (2 marks)  
Solving by GDC  $\Rightarrow n \geq 11.5$  so  $n = 12$  is the minimum (2 marks)

- 5 a**  $Q_1 = 6, Q_3 = 9 \Rightarrow \text{IQR} = 3$  (2 marks)  
 $6 - 1.5 \times 3 = 1.5$  so value of  $h$  for student  $K$  is an outlier (2 marks)  
**b**  $r = 0.816$  (3s.f.) (2 marks)  
**c**  $s = 3.79h + 53.6$  (3s.f.) (2 marks)  
**d**  $3.79 \times 5.5 + 53.6 = 74$  to the nearest integer (2 marks)

- 6 a**  $h'(x) = x^2 - 5x + 4$  (2 marks)  
**b**  $x = 1$  or  $x = 4$  (2 marks)  
**c** max (1,6.83)  
min (4,2.33) (4 marks)



- d** from intersection of  $h = 10$  and graph,  $5.90 < x \leq 7$  (3s.f.) (1 mark)  
**e** from intersection of  $y = x^2 - 5x + 4$  and  $y = -1, 1, 3.8 < x < 3.62$  (3s.f.) (2 marks)

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