Functions and basic derivatives test

IB11

Name:____

Remember to show your work in every exercise.

1. Define:

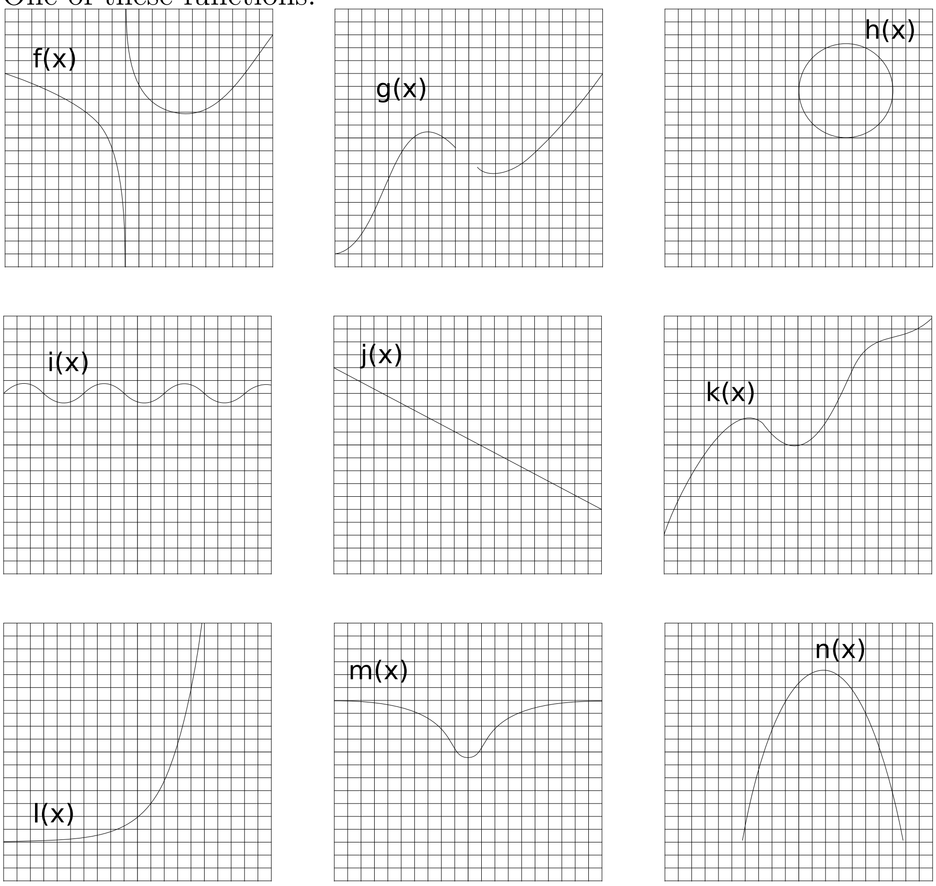
   a) A function is
   (1 0 0)

   b) The domain of a function is
   (1 0 0)

   c) The range of a function is
   (1 0 0)

2. State three causes for a function not to respond, at least two of them with an example.(2 1 0)

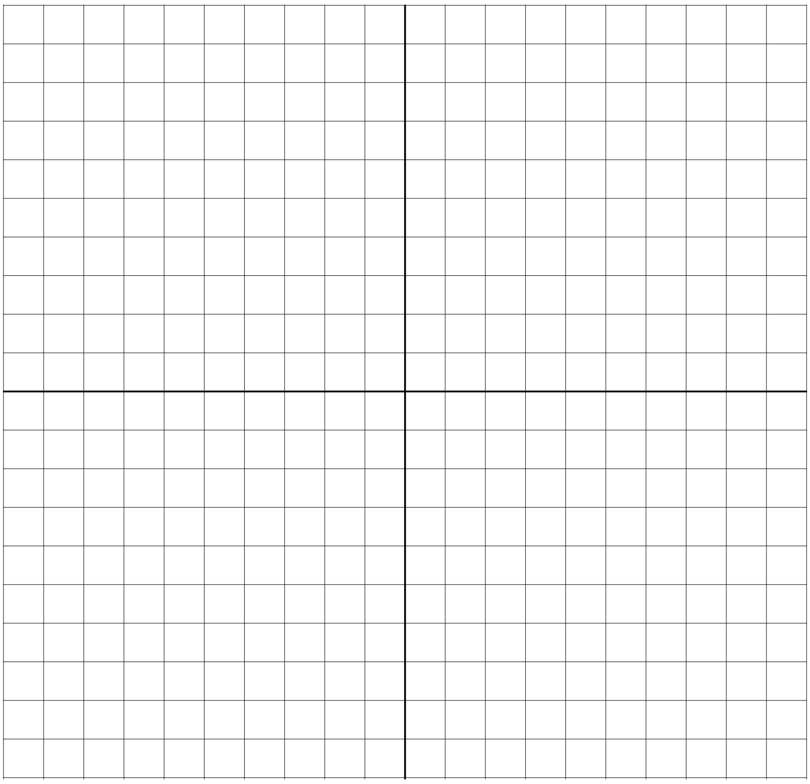
3. One of these functions:



   a) When you give it 6 it gives back 8 (0.5 0 0)
   b) Is linear (0.5 0 0)
   c) Has a horizontal asymptote. (0 0.5 0)
   d) Has a vertical asymptote (0 0.5 0)
   e) Is a parabole (0 0.5 0)
   f) Never gives back the value 0 (0 0.5 0)
   g) Is exponential (explain how you can tell) (0 1 0)

4. Calculate the equation of the linear function that passes through the points (-3,1) and (6,4). (0 1 0)

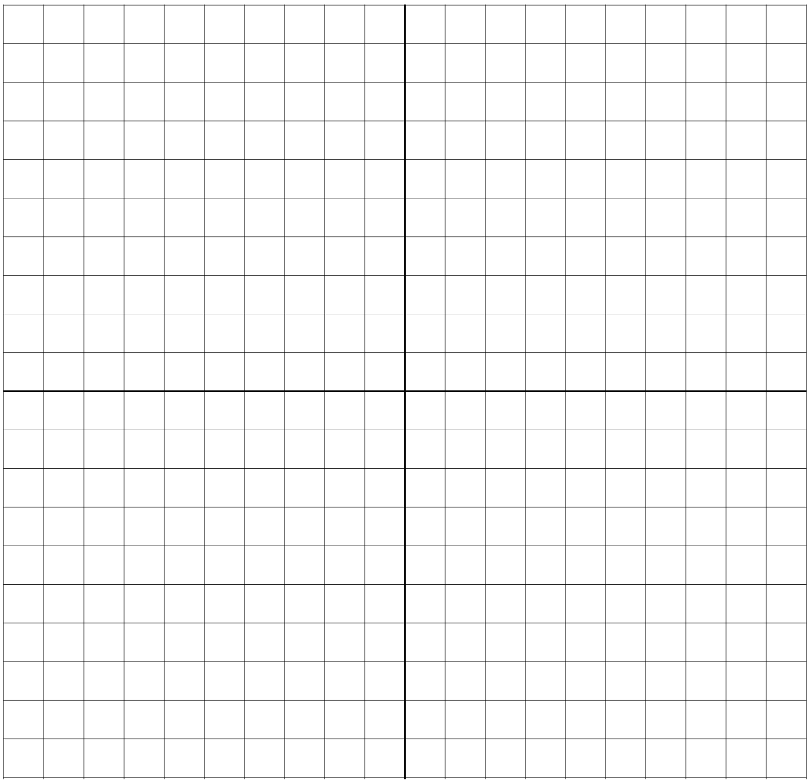
5. Sketch the parabole $o(x) = x^2 + 2x - 3$ by finding its apex and intersections with the axis (0 2 0)



6. For the parabole in exercise 5, find the equation of the line that is tangent to it at x=2. (0 2 0)

7. A large steel beam is being cast from molten steel. The initial temperature of the beam is 1400 degrees Celsius and the air of the foundry is at 31 degrees Celsius.

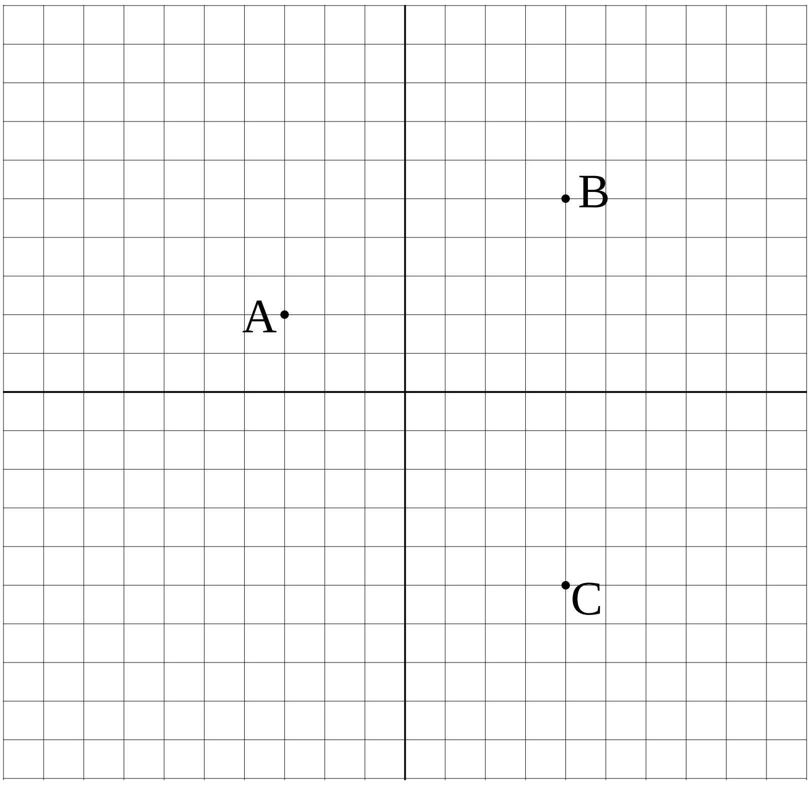
   a) Sketch in a graph how the temperature of the beam will progress in time. (0 1 0)

   b) What kind of function models the progress of the temperature of the beam in time? Why is this so? (0 2 0)

c) In order to fulfill the DIN standard for this type of beam, the cooling down must be done in a more uniform manner. In order to achieve this, the chemical engineer devises a water cooler that cools the beam so that its temperature is lowered constantly 40 degrees per minute. As a result of this, the temperature of the water increases at 2.5 degrees per minute.
If the initial temperature of the water is 9 degrees Celsius, write the functions that model the progress of the temperatures of the beam and the water.(0 2 0)
What is the lowest temperature the beam will reach? How long will that take?(1 2 1)

8. The engineer from exercise 7 is stranded in the desert, don't ask me why.
Her current location is point C, and points A and B are two villages where she could find water and shelter.



a) Make a simple Voronoi diagram to find whether she is on the side of the points closer to A, on the other side or right on the dividing line. (1 2 0)

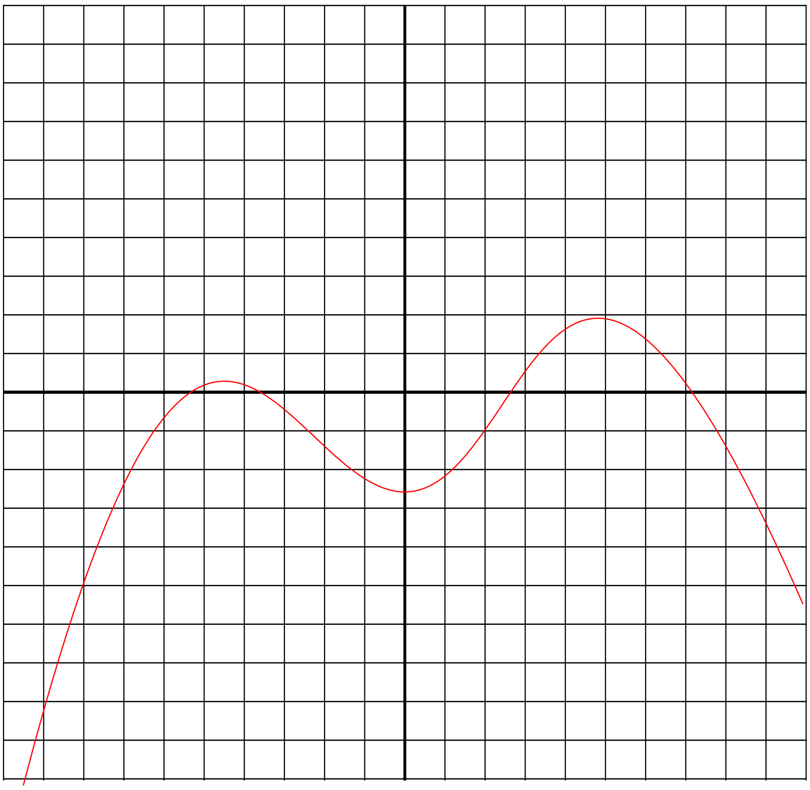
b) Is a Voronoi diagram the best way to solve this problem? Why? (0 0 1)

9. Set your calculator to radians and find (0 2 0)

$$\lim_{x \to 0} \frac{\sin x}{x}$$

What does this result mean? (0 1 1)

10. In this function



- Find a point where the function is positive and its derivative positive too (mark as a)(0 0.5 0)
- Find a point where the function is positive and its derivative is negative (mark as b)(0 0.5 0)
- Find a point where the function is negative and its derivative negative (mark as c)(0 0.5 0)
- Find a point where the function is negative and its derivative more positive than in point a (mark it as d)(0 0.5 0)
- Bonus: Find a point where the function is negative and the derivative is negative too but increasing (mark it as e)(0 1 0)

11. Solve, using the calculator, the equation $-e^{(-x)} + \frac{2}{x^2-8x+16} + \frac{x}{2} + 2 = 3$ (0 1 0)

12. Differentiate; (1 0 0) each

$a)$

$$p(x) = x^2 + 6x - 4$$

$b)$

$$q(x) = x^4 - 8x^3 + 6x^2 + 4x - 11$$

$c)$

$$r(x) = 2x + 7$$

$d)$

$$s(x) = x^{45}$$

$e)$

$$t(x) = 45$$

13. Bonus question: find the equation of the line tangent to q(x) when x=-2. (0 1 1)