

# Functions test

Name and date:

Show your work in every exercise.

All results must be given exact or with three significant figures, unless otherwise stated.

1. Define function (2 0 0)

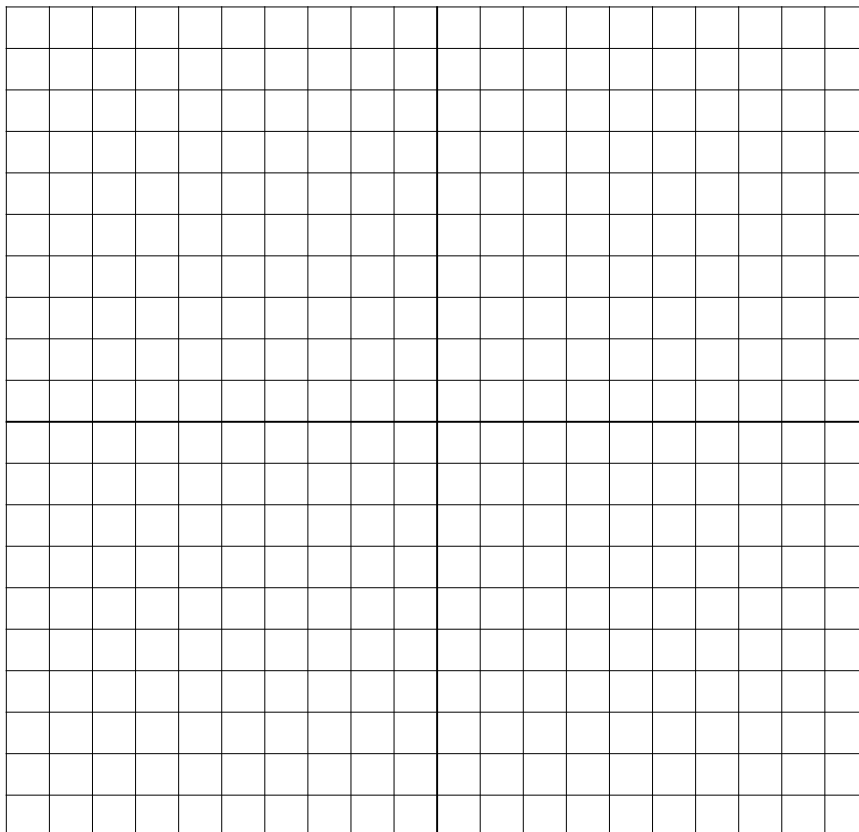
2. Find the domain of the function  $a(x) = \frac{1}{x^2 - 4}$  (1 1 0)

3. Explain why the domains of these two functions are not the same  $b(x) = \sqrt{x - 6}$   
 $g(x) = \log(x - 6)$  (1 1 0)

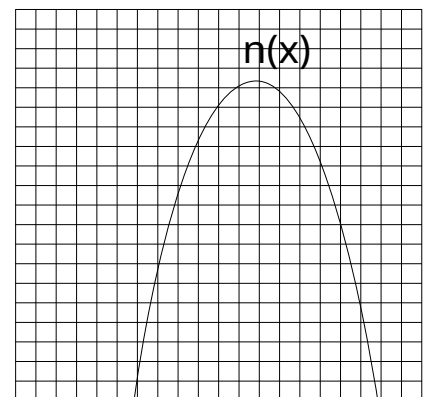
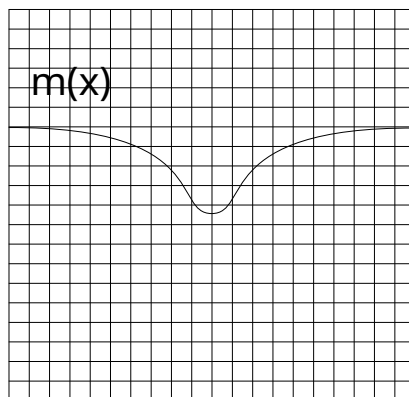
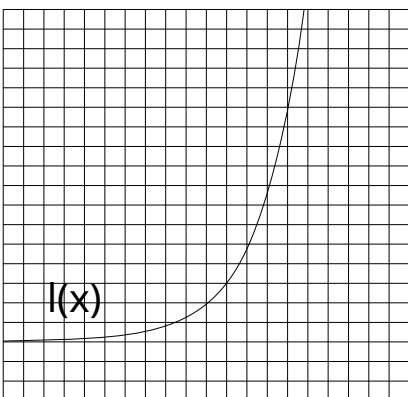
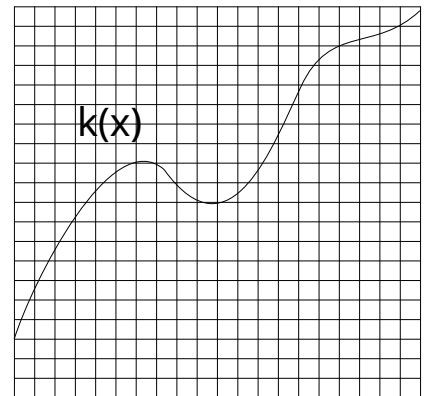
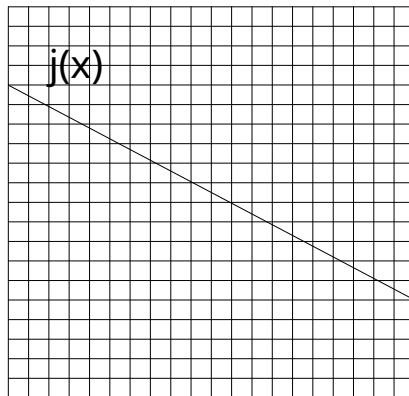
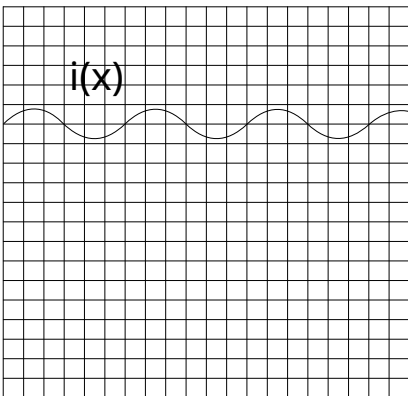
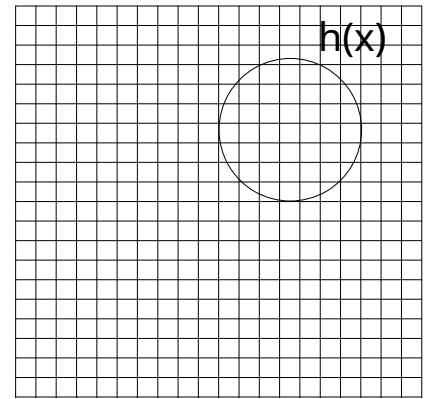
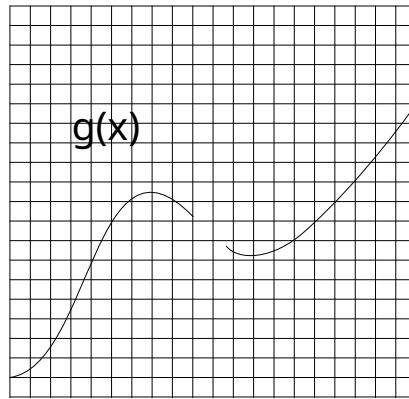
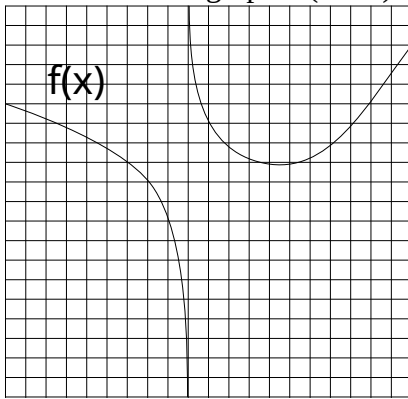
4. Does the function  $c(x) = x^3$  have inverse? If it does, write it. If it doesn't, state why. (0 2 0)

5. a) Calculate the equation of the linear function that passes through the points (1,0) and (-5,3). [1]  
 b) Find the equation of the line that is perpendicular to it and passes through the point (-2,2) [2]

6. Sketch the function  $d(x) = x^2 + x - 12$ . Show relevant points. (0 2 0)



7. In these graphs: (0 5 0)



- i) One could be a fourth degree polynomial.
- ii) Only horizontal asymptotes are found.
- iii) One of them is not a function.
- iv) Two are functions with restricted domain.
- v) Only one of them has a range of all real numbers.

- a) Only i and ii are true
- b) Only i and iii are true
- c) Only i and iv are true
- d) Only i and v are true
- e) Only iii and iv are true
- f) Only v is true
- g) Only iv is true
- h) Only i and iv are true

8. Assa and Åsa are bringing fruit to their grandmother. Assa brings strawberries and Åsa brings blueberries, but as they enter the Lagoon Road they notice that there are more berries growing in the bushes by the road, so they decide to pick some more along the way.

Assa picks four strawberries per meter and puts them in her basket, where she had 388 strawberries to begin with. Åsa picks 7 blueberries per meter and puts them in her bag, where she had 280 blueberries to begin with.

a) Establish two linear models (functions) for the number of Assa's strawberries and Åsa's blueberries depending on the distance they have walked. (0 1 0)

b) If the Lagoon Road is 50m long, how many berries will each of them have? (0 1 0)

c) At what point will they have the same number of berries? (0 2 0)

9. An engineer builds a model glider with rocket propulsion to get it airborne.

The rocket follows the trajectory of the function  $f(x) = -0.0005x^2 + 5x$

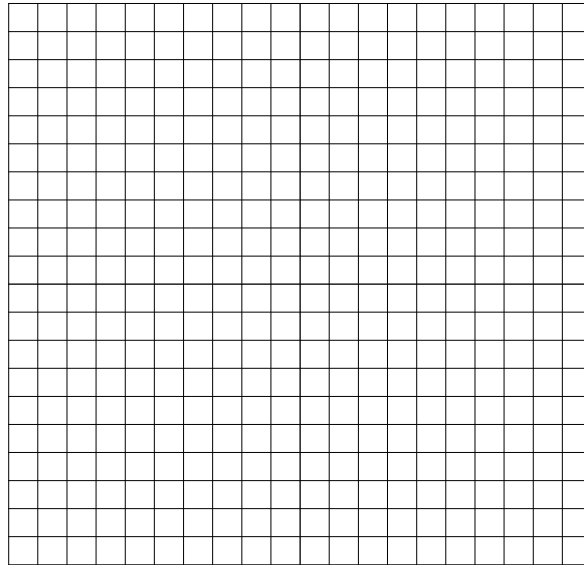
At a given point in time, the engineer will press a button in the remote control and the glider will detach from the rocket and start flying.

a) What is the best (highest) position to detach the glider? (1 2 0)

b) Where will the glider land if it glides from the point of detachment in a straight trajectory with a slope of  $-1/5$ ? (1 2 0)

10. The rocket from exercise 9 falls to the ground and is recovered by the engineer. When the rocket reaches the ground, it is still at  $240^{\circ}\text{C}$ , and it cools down on the Saharan sand and an air temperature of  $56^{\circ}\text{C}$ .

a) Sketch a graph for the temperature of the rocket over time showing the relevant points. (1 1 0)



b) If after 15 min the temperature of the rocket is  $160^{\circ}\text{C}$ , establish a model for the temperature showing the known parameters and explain why they have these values. (1 2 0)

c) The engineer can pick up the rocket with gloves, but only if the temperature is  $125^{\circ}\text{C}$  or below. How long time must pass before she can pick it up? (0 2 1)

11. The engineer from exercise 9 and 10 is spending the night in the desert, where the temperature follows a sinusoidal model with maximum at 4pm and minimum at 4am.

The maximum temperature is  $57^{\circ}\text{C}$  and the minimum temperature is  $9^{\circ}\text{C}$ .

a) Establish a sinusoidal model and calculate the parameters  $a$ ,  $b$  and  $d$  (Hint: start counting time at the time of the day that is most convenient for you) [4]

b) The engineer has a small heater but it must be activated manually. She doesn't want to waste fuel so she wants to turn it on when the temperature gets uncomfortably low for her, which is any temperature below  $15^{\circ}\text{C}$ .

At what time will she have to turn it on and for how long will it have to be on? [3]

Bonus question 1: explain why the domain of the function  $n(x) = \sqrt{-x^2 + 2x - 2}$  is what it is. [2 marks]

Bonus question 2: The exponential model  $i(x) = 45 \cdot 1.3^x + 100$  uses a number other than  $e$  as base of the exponential part. This will make things complicated later when we have to differentiate, so express this model using  $e$  as base. [2 marks]

Bonus question 3: The sinusoidal model  $j(x) = 20 \cos(18x) + 6$  can be turned into a sine function by making a modification. 2 marks for doing said modification.