

# Paper 1

Time allowed: 2 hours

Answer all the questions.

All numerical answers must be given exactly or correct to three significant figures, unless otherwise stated in the question.

Answers should be supported by working and/or explanations. Where an answer is incorrect, some marks may be awarded for a correct method, provided this is shown clearly.

You need a graphic display calculator for this paper.

## Short questions

1 A window-screen wiper blade on a car is of length 0.5 m. In its motion, it moves through an angle of  $\frac{3\pi}{2}$  and back again.

a Find the area of the window-screen that is wiped by the blade. [2 marks]

b There is an insect stuck on the blade at the far end. Find the distance that the insect travels as the blade moves through the angle of  $\frac{3\pi}{2}$ . [2 marks]

[Total 4 marks]

2 An explorer travels 5 km due South. He then travels 6 km on a bearing of  $100^\circ$ . He then wishes to go straight back to where he started from.

a Calculate the distance he will have to travel. [2 marks]

b Find the bearing that he will have to travel on to return back to his starting point. [4 marks]

[Total 6 marks]

3 [Maximum marks: 6]

Consider the relation  $C = C(A)$  between the circumference of a circle  $C$  and its area  $A$  defined for  $A \geq 0$ .

a Justify that  
i  $C = C(A)$  is a function  
ii  $C$  has an inverse function  $C^{-1}$ . [2 marks]

b Sketch the graphs of  $C(A)$  and  $C^{-1}(A)$  on the same axes. [2 marks]

c Hence, find the area of a circle with circumference 25. [2 marks]

4 [Maximum marks: 6]

A cup of hot chocolate is left on a counter for several hours. Initially its temperature was  $86^\circ\text{C}$ . After 30 minutes the temperature had already dropped to  $28^\circ\text{C}$ .

The temperature,  $T^\circ\text{C}$ , of the hot chocolate is modelled by the function  $T(t) = 22 + a2^{bt}$ , where  $t$  is the number of hours that have elapsed since the hot chocolate was first left to stand.

a Find the value of  
i  $a$   
ii  $b$  [4 marks]

b Write down the equation of the horizontal asymptote of the graph of  $T$ . [1 mark]

c State the meaning of the asymptote found in (b). [1 mark]

5 [Maximum marks: 5]

Kathy is collecting information for a statistics project. She asks a group of students that have pet dogs about the number of times that usually they walk the dog per day.

The data collected is shown in the following table.

Number of walks	1	2	3	4	5
Number of students	4	8	10	5	1

a State, with a reason, whether 'number of walks' is a discrete or continuous variable. [2 marks]

b For the students that Kathy surveyed, find  
i the modal number of dog walks per day [1 mark]

ii the mean number of dog walks per day. [2 marks]

6 [Maximum marks: 6]

Jasmine plays a computer game. In the game, she collects tokens of different colours. Each colour token gives the player a different number of points.

Jasmine records the relative frequencies of obtaining tokens of each colour. She uses this to estimate the probability of obtaining a token of a certain colour.

Colour	Red	Blue	Green	Yellow	Pink	Orange
Points	2	3	5	1	3	4
Estimate of probability	$\frac{1}{9}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{2}{9}$	$\frac{1}{6}$	$p$

a Find the value of  $p$ . [2 marks]

b Find the expected score if Jasmine plays the game once. [2 marks]

Jasmine plays the game 2 times and adds the points together.

c Find the probability that Jasmine scores a total of 4 points. [2 marks]

7 [Maximum marks: 5]

Jeanny has 6 coloured pencils and 8 coloured pens in her colouring box. Every afternoon from Monday through to Friday, Jeanny returns from Kindergarten and picks a pen or pencil at random from this box. She then begins to draw with the object she picked.

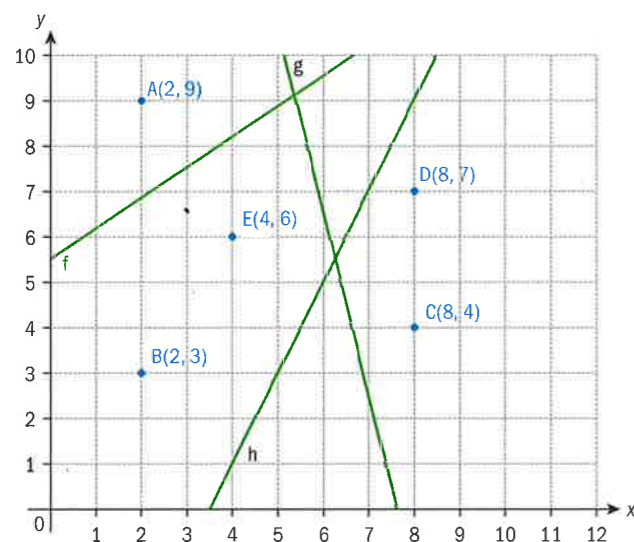
- a State the probability that, on any one day, Jeanny picks a pen. [1 mark]
- b Find the probability that, in one week (Monday through to Friday), Jeanny picks a pen on
  - i exactly two days. [2 marks]
  - ii at least two days [2 marks]

8 [Maximum marks: 8]

Points A(2, 9), B(2, 3), C(8, 4), D(8, 7) and E(4, 6) represent wells in the Savannah National Park. These wells are shown in the coordinate axes below.

Horizontal scale: 1 unit represents 1 km.

Vertical scale: 1 unit represents 1 km.



- a Calculate the distance between the wells A and E. [2 marks]

Pablo, the Park Ranger draws three lines,  $f$ ,  $g$  and  $h$ , around point E and obtains an incomplete Voronoi diagram around the point E.

- b Find the equation of the line which would complete the Voronoi cell around point E. Give your answer in the form  $ax + by + d = 0$  where  $a, b, d \in \mathbb{Z}$ . [5 marks]
- c In the context of the question, explain the significance of the cell around point E. [1 mark]

9 In this question give all monetary answers to 2 decimal places.

Sarah borrows £10 000 from a bank that charges 4% compound interest per year.

- a If Sarah makes no repayments, calculate how much she will owe at the end of 5 complete years. [2 marks]

In the year in which the debt becomes more than £15 000, Sarah will pay the whole loan off with one repayment at the end of that complete year.

- b Calculate how many complete years it will take before Sarah pays the loan off.
- c Calculate how much Sarah has to pay back to the bank. [3 marks]

[Total 5 marks]

10 The random variable,  $X$ , represents the number of emails that Katherine receives during a day.  $X$  satisfies a Poisson distribution with a mean of 30.

- a Find the probability that, on a particular day, she will receive less than 20 emails. [2 marks]
- b Find the probability that, on a particular day, she will receive more than 35 emails. [2 marks]
- c Find the probability that she will receive 200 emails or less in a 7-day week. [2 marks]

The number,  $R$ , of emails that Jane receives during a day satisfies a Poisson distribution with a mean of 20.

- d Find the probability that, on a particular day, the total number of emails received by both Katherine and Jane together is 50 or less. [2 marks]

[Total 8 marks]

11 The height of Gerry the giraffe is given by the equation  $h(t) = \frac{6}{1 + 2e^{-0.4t}}$ ,

where  $h$  is measured in metres and  $t$  is the time in years since his birth.

- a Find Gerry's height when he was born. [2 marks]
- b State the height that Gerry approaches as he becomes incredibly old. [2 marks]
- c Find Gerry's height on his 4th birthday. [2 marks]
- d On Gerry's  $n$ th birthday, he is taller than 5 m for the first time on a birthday. Find the value of  $n$ . [2 marks]

[Total 8 marks]

12 The number of bacteria,  $n$ , in a jug of fruit juice can be modelled

by  $n = 10e^{\frac{t}{2}}$ , where  $t$  is time measured in hours.

- a Write down the number of bacteria present initially. [1 mark]
- b Calculate the number of bacteria present after 6 hours. [2 marks]

If the number of bacteria exceeds 1000 the juice is no longer safe to drink.

- c Calculate how long it takes before the juice is no longer safe to drink. Give your answer in hours and minutes correct to the nearest minute. [3 marks]
- d Describe two limitations to this model. [2 marks]

[Total 8 marks]

13 The height above the ground,  $h$  cm, of the valve on a mountain bike wheel as the bike is being cycled is given by  $h(t) = 35\sin(2\pi t) + 40$ , where  $t$  is time measured in seconds.

- a Write down the initial height of the valve. [1 mark]
- b Find how long it takes for the wheel to make one complete revolution. [2 marks]
- c State
  - i the maximum height that the valve reaches
  - ii the minimum height that the valve reaches. [2 marks]
- d Find the height of the valve after 0.3 s. [1 mark]
- e In the first second, calculate how long the valve is more than 60 cm above the ground. [3 marks]

[Total 9 marks]

14 The pressure  $P$  (measured in Pascals) and the volume  $V$  (measured in  $\text{cm}^3$ ) of a gas satisfy the differential equation  $\frac{dP}{dV} = -\frac{P}{V}$ .

- a Solve the differential equation, simplifying the answer, given that if the pressure is 1000 Pascals then the volume is  $200 \text{ cm}^3$ . [6 marks]
- b Find the pressure required to compress the gas to a volume of  $150 \text{ cm}^3$ . [2 marks]

[Total 8 marks]

15 Melchester Rovers football club have such an attacking style of play that they either win a match or lose it, but never draw. If they win a match, their confidence increases and the probability that they win their next match is 0.9. If they lose a match then their confidence decreases and the probability that they win their next match is only 0.3. Let  $v_n = \begin{pmatrix} p_n \\ 1 - p_n \end{pmatrix}$  where  $p_n$  is the probability that they win their  $n$ th match.

- a Find  $M$ , where  $M$  is the transition matrix defined by  $v_{n+1} = Mv_n$ . [2 marks]
- b Sketch a labelled directed graph that represents the situation. [2 marks]
- c As  $n$  tends to infinity  $v_n$  tends to a steady state of  $V = \begin{pmatrix} r \\ 1 - r \end{pmatrix}$ . [4 marks]

[Total 8 marks]

16 It is known that the IQs of "Grandmaster" chess players are normally distributed, with population mean of  $\mu$ .

A sample of the IQs of ten Grandmasters were tested at a tournament. The sample mean  $\bar{x}$  was 115 and the sample standard deviation  $s_n$  was 4.

- a Find the 90% confidence interval for  $\mu$ , giving your answer to one decimal place. [3 marks]
- b State, with a reason, which distribution should have been used. [2 marks]

[Total 5 marks]

17 The mass,  $m$  kg, of a swan is proportional to  $w^3$ , where  $w$  is its wingspan in metres.

- a State which of the following graphs would generate a straight line.
  - i  $m$  plotted against  $w$
  - ii  $\log m$  plotted against  $\log w$
  - iii  $\log m$  plotted against  $w$
  - iv  $m$  plotted against  $\log w$ . [2 marks]
- b Justify your answer in (a), and state the gradient of whichever graphs generate a straight line. [3 marks]

[Total 5 marks]

**Time allowed: 2 hours**

**Answer all the questions.**

All numerical answers must be given exactly or correct to three significant figures, unless otherwise stated in the question.

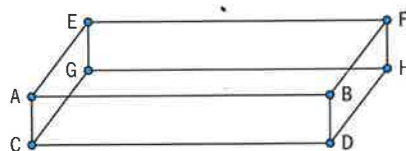
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**You need a graphic display calculator for this paper.**

**1** [Maximum mark: 19]

A company manufactures pencils. Before being sharpened, each pencil can be modelled by a cylinder of length 19.5 cm and diameter 6 mm.

The pencils are packaged in small, closed rectangular boxes. Each box is 19.7 cm long, 6.2 cm wide and 3.2 cm high. One pencil box is shown in the diagram below.



- a Calculate
  - i the surface area of the box in  $\text{cm}^2$
  - ii the volume of the box in  $\text{cm}^3$ . [3 marks]
- b Determine the maximum number  $N$  of pencils that will fit in a single box. You must clearly justify your answer. [2 marks]
- c Hence, calculate total volume occupied by the  $N$  pencils, and express it as a percentage of the volume of the box. [4 marks]

Each month, the company sells  $x$  thousand boxes of pencils.

It is known that  $\frac{dP}{dx} = -x + 1200$ , where  $P$  is the weekly profit (in euros)

from the sale of  $x$  thousand boxes

- d Find the number of boxes that should be sold each week to maximize the profit. [2 marks]

The profit from the sale of 3000 boxes is €1500.

- e Find  $P(x)$ . [5 marks]
- f Find the least number of boxes which must be sold each month in order to make a profit. [3 marks]

[Total marks 19]

- 2 The time,  $T$ , that competitors take to complete a 10 000 m race is normally distributed with mean  $\mu = 50$  minutes and standard deviation  $\sigma = 5$  minutes.
  - a Sketch a diagram to represent this information with the numbers 50 and 5 indicated on it. [3 marks]
  - b Find the probability that a random competitor takes between 45 and 55 minutes. [2 marks]
  - c Find the probability that a random competitor takes less than 40 minutes to complete the race. [2 marks]
  - d The fastest 75% of competitors receive a medal. Find the time that the race has to be completed under, in order for a competitor to receive a medal. [2 marks]
  - e Given that a competitor received a medal, find the probability that they finished the race in less than 40 minutes. [3 marks]
  - f If 10 000 competitors ran the race, estimate (to the nearest minute) how many competitors completed the race in less than 33 minutes. [2 marks]

[Total marks 14]

- 3 Eleven students,  $A-K$ , revised for and then took a maths exam. Let  $h$  represent the number of hours that each student spent revising, and let  $s$  represent the score (out of 100) that they gained. The data for each student is shown in the following table.

	A	B	C	D	E	F	G	H	I	J	K
$h$	10	9.5	9	8.5	8	7.5	7	6.5	6	5	0
$s$	100	91	93	90	80	85	79	70	69	60	65

- a Identify any outliers in the values of  $h$ . Justify your answers. [4 marks]
- b Calculate the Pearson product moment correlation coefficient,  $r$ , for this bivariate data. [2 marks]
- c Write down the equation of the line of best fit of  $s$  on  $h$ . [2 marks]
- d Hence estimate the score, to the nearest integer, of a twelfth student who spent 5.5 hours revising. [2 marks]
- e Rank the students from 1 – 11 according to the number of hours they spent revising,  $h$ . Rank 1 should represent the most hours spent revising.

In a similar way, rank the students according to the test score,  $s$ , they obtained. Rank 1 should represent the highest test score.

Copy and complete the table below to show the rankings for each student.

	A	B	C	D	E	F	G	H	I	J	K
Hour rank	1	2	3								
Score rank	1	3	2								

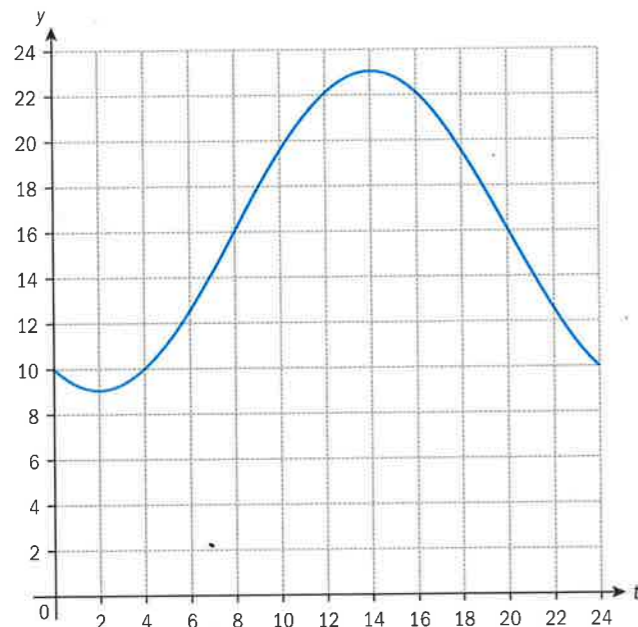
[2 marks]

- f Calculate the Spearman rank correlation coefficient,  $r_s$ , for this data. [2 marks]  
 g Explain why  $r_s > r$ . [1 mark]

[Total marks 15]

4 [Maximum mark: 15]

The following graph shows the temperature in Algarve on a particular day in winter.



The temperature,  $y$  degrees Celsius, can be modelled by the curve  $y = a \sin(b(t - c)) + d$ ,  $a, b, c, d \in \mathbb{R}^+$ , for  $0 \leq t \leq 24$  where  $t$  is the time in hours after midnight. The maximum temperature of  $23^\circ\text{C}$  was recorded at  $t = 14$  hours, and the minimum temperature of  $9^\circ\text{C}$  was recorded at  $t = 2$  hours.

- a Show that  $a = -7$ . [2 marks]  
 b Find the value of  
 i  $b$     ii  $c$     iii  $d$ . [6 marks]

Using the temperature records for the in Algarve in August, it was found that the temperature on a particular day in summer could be modelled by the curve  $y = -6 \sin(0.262(t + 1)) + 25$ , for  $0 \leq t \leq 24$  where  $t$  is the number of hours after midnight.

- c In hours and minutes after midnight (correct to the nearest minute), find  
 i the value of  $t$  when the temperature reaches its minimum  
 ii the value of  $t$  when the temperature first drops below  $21^\circ\text{C}$   
 iii the length of time the temperature is below  $21^\circ\text{C}$ . [7 marks]

5 [Maximum mark: 12]

An drone's position is given by the coordinates  $(x, y, z)$ , where  $x$  and  $y$  are the drone's displacement north and east of its launching point at a deserted beach, and  $z$  is the height of the drone. All displacements are given in metres.

The drone travels with constant velocity  $\begin{pmatrix} -5 \\ -10 \\ -1 \end{pmatrix} \text{ m s}^{-1}$  and, at 1p.m.,

it is detected at a position 150 m north and 100 m east of the launching point, and at a height of 50 m. Let  $t$  be the time, in hours, after 1p.m.

- a Write down the equation for the position of the drone, relative to the launching point, at time  $t$ . [2 marks]  
 b Verify that if the drone continues to fly with constant velocity it will pass directly over a dog located at a point P, 100 m due north from the launching point. State the height of the drone as it passes over point P, and the time at which it occurs. [4 marks]

At this moment, the drone's pilot adjusts the angle of descent so that the drone will travel in a straight line at a constant velocity and land at the point  $(0,0,0)$  in 5 seconds.

- c i Find the distance the drone has to travel from P its landing point.  
 ii Determine the magnitude of the new velocity vector.  
 iii Hence write down an equation for the new velocity vector of the drone. [6 marks]

6 [Maximum mark: 19]

Scientists have been collecting data about the migration habits of a particular species of birds. Annual censuses were conducted in two different regions, A and B, which are inhabited by these birds.

A steady pattern of of change has been observed in their movements: each year 40% of the birds move from location A to location B and 20% of the birds move from location B to location A.

Assume that there are no birds going to, or arriving from, any other locations.

- a Write down a transition matrix  $\mathbf{T}$  representing the movements of the birds between the two regions in a particular year. [2 marks]  
 b Find  
 i the eigenvalues for the transition matrix  $\mathbf{T}$   
 ii the eigenvectors for the transition matrix  $\mathbf{T}$ . [6 marks]  
 c Hence find matrices  $\mathbf{M}$  and  $\mathbf{D}$  such that  $\mathbf{T} = \mathbf{MDM}^{-1}$ . [4 marks]

Initially location A had 4200 and location B had 6000 of these birds.

- d Find an expression for the number of birds at  
 i location A  
 ii location B after  $n$  years. [4 marks]

- e Hence write down the long-term number of birds that each location is expected to have. [1 mark]
- f State two limitations of the model. [2 marks]

7 [Maximum mark: 16]

A law of cooling states that  $\frac{dB}{dt} = -k(B^4 - R^4)$  where  $B$  is the temperature of a body and  $R$  the temperature of the room. In the model,  $B$  and  $R$  are measured in  $^{\circ}\text{C}$ ,  $t$  is the time in hours, and  $k$  is a constant.

Assume that the room temperature  $R$  is constant.

- a Show that  $\ln \frac{B+R}{B-R} + 2 \arctan \frac{B}{R} - 4R^3kt = c$  is an implicit solution to the differential equation. [6 marks]

Consider the solution to the differential equation  $\frac{dB}{dt} = -k(B^4 - R^4)$

that contains the point  $(0, 35)$  when  $k = 0.00005$  and  $R = 20$ .

- b i Use Euler's method with step  $h = 0.1$  to approximate  $B(0.5)$ .
- ii Find an implicit solution to the differential equation. [10 marks]

# Paper 3

Time allowed: 1 hour

Answer all the questions.

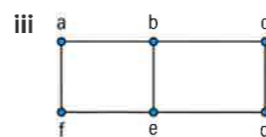
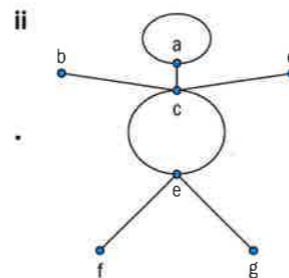
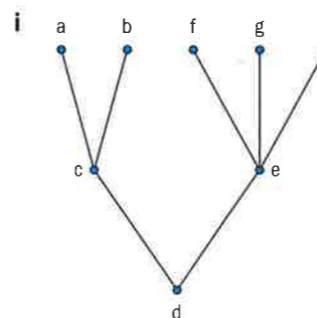
All numerical answers must be given exactly or correct to three significant figures, unless otherwise stated in the question.

Answers should be supported by working and/or explanations. Where an answer is incorrect, some marks may be awarded for a correct method, provided this is shown clearly.

You need a graphic display calculator for this paper.

- 1 a For the following graphs, state the number of edges and the degree of each vertex. For (i), (ii) and (iii) give the degree of each vertex in a table of the form

vertex	a	b
Degree		



- iv The complete graph  $K_n$ . [8 marks]

- b** The Handshaking Lemma states that for any graph the sum of the degrees of the vertices is equal to twice the number of edges.
- Verify that this rule is true for each of the four examples in (a).
  - Explain why this rule must be true for any graph.
- c** A graph is called *planar* if it can be drawn on paper in 2-dimensions without any of the edges crossing (the edges can only touch at a vertex). Graphs (a) (i), (ii) and (iii) are all planar.
- Draw  $K_3$ , the complete graph with 3 vertices, to show that it is planar.
  - Draw  $K_4$ , the complete graph with 4 vertices, to show that it is planar.

[5 marks]

The complete graph with 4 vertices models 4 houses with a cycle track from each house to every other house, where the cycle tracks do not cross (to avoid collisions). We will now consider the same problem with 5 houses. We want to find out if  $K_5$  is planar.

A result has been proved that states that if a graph is planar, simple, and connected, then  $e \leq 3v - 6$ , for  $v \geq 3$ , where  $e$  is the number of edges and  $v$  is the number of vertices.

- Use this result to prove, by contradiction, that  $K_5$  is not planar.
- d** Two newly built houses have to be connected to each of the utility services Gas, Water and Electricity. The houses are represented by vertices A and B, and the utility services by G, W and E. The connecting pipes will be represented by edges. The construction firm do not want any of the pipes to cross over each other.
- Draw a graph of this situation that shows that the graph can be drawn in a planar fashion.

[8 marks]

We will now consider the same situation but with three houses represented by A, B and C, each of which must be connected to Gas, Water and Electricity.

- Draw a graph of this situation, but do not attempt to draw it in a planar fashion.
- State how many vertices and edges this graph has.

Another result has been proved that states that if a graph is planar, simple and connected, and also contains no triangles (i.e. does not have  $K_3$  as a subgraph), then  $e \leq 2v - 4$ , for  $v \geq 3$ , where  $e$  is the number of edges and  $v$  is the number of vertices.

- Use this result to prove, by contradiction, that the graph in (ii) cannot be planar.

[9 marks]

[Total 30 marks]

- 2** In this question, you will perform various tests to determine what type of distribution certain random variables best fit.

100 observations of a discrete random variable  $X$  are taken and are displayed in the table below.

$x$	$\leq 1$	2	3	4	5	6	$\geq 7$
Observed frequency	8	16	18	20	17	12	8

- a** We will first test if this data fits the binomial  $B(40, 0.1)$  distribution.
- Write down the mean of the  $B(40, 0.1)$  distribution.
- 100 values of a variable which fits a  $B(40, 0.1)$  distribution are measured.
- Copy and complete the table below to show the expected frequencies when 100 values of a variable which fits the  $B(40, 0.1)$  distribution are measured.

$x$	$\leq 1$	2	3	4	5	6	$\geq 7$
Expected frequency							

- Perform a  $\chi^2$  goodness of fit test at the 10% level to test the null hypothesis  $H_0$ : This data fits the  $B(40, 0.1)$  distribution.

State the number of degrees of freedom, the  $p$ -value and the conclusion of the test.

[8 marks]

- b** We will now test if this data fits the Poisson  $Po(4)$  distribution.
- Construct a table similar to that in part a (ii) to show the expected frequencies when 100 values of a variable which fits the  $Po(4)$  distribution are measured.
  - Perform a  $\chi^2$  goodness of fit test at the 10% level to test the null hypothesis  $H_0$ : This data fits the  $Po(4)$  distribution.

State the number of degrees of freedom, the  $p$ -value and the conclusion of the test.

[7 marks]

- c** Copy and complete the following conjecture by filling in the gaps.
- For  $n$  large and  $p$  \_\_\_\_\_, the binomial distribution can be approximated by the \_\_\_\_\_ distribution with the same \_\_\_\_\_.

[1 mark]

Another 100 observations of a different discrete random variable  $Y$  are measured and displayed in the table below.

$y$	$y < 15.5$	$15.5 < y < 18.5$	$18.5 < y < 21.5$	$21.5 < y < 24.5$	$24.5 < y$
Observed frequency	8	23	34	28	7

- d** We will first test if this data fits the binomial  $B(40, 0.5)$  distribution.
- Write down the mean and the variance of the  $B(40, 0.5)$  distribution.
  - If  $Y \sim B(40, 0.5)$ , calculate  $P(Y = 22, 23 \text{ or } 24)$ .
  - Copy and complete the table below to show the expected frequencies when 100 values of a variable which fits the binomial distribution  $B(40, 0.5)$  are measured.

$y$	$y < 15.5$	$15.5 < y < 18.5$	$18.5 < y < 21.5$	$21.5 < y < 24.5$	$24.5 < y$
Expected frequency	7.69	24.1	36.4		

- iv Perform a  $\chi^2$  goodness of fit test at the 10% level to test the null hypothesis  $H_0$ : This new data fits the  $B(40, 0.5)$  distribution.

State the number of degrees of freedom, the  $p$ -value and the conclusion of the test. [7 marks]

- e You will now test if this data fits the normal distribution  $N(20, 10)$ .

- i If  $Y \sim N(20, 10)$ , calculate  $P(21.5 < Y < 24.5)$ .  
 ii Hence, Copy and complete the table below to show the expected frequencies when 100 values of a variable which fits the normal distribution  $N(20, 10)$  are measured.

$y$	$y < 15.5$	$15.5 < y < 18.5$	$18.5 < y < 21.5$	$21.5 < y < 24.5$	$24.5 < y$
Expected frequency	7.73	24.1	36.5		

- iii Perform a  $\chi^2$  goodness of fit test at the 10% level to test the null hypothesis  $H_0$ : This new data fits the  $N(20, 10)$  distribution.

State the number of degrees of freedom, the  $p$ -value and the conclusion of the test. [6 marks]

- f Copy and complete the following conjecture by filling in the gaps.

For  $n$  \_\_\_\_\_ and  $p$  not close to 1 or 0, the binomial distribution can be approximated by the \_\_\_\_\_ distribution with the same \_\_\_\_\_ and \_\_\_\_\_, even though the binomial is a discrete distribution and the \_\_\_\_\_ is a \_\_\_\_\_ distribution. [1 mark]

[Total 30 marks]

# Answers

## Chapter 1

### Skills check

- 1 a i 0.69 ii 28.71  
 iii 77.98  
 b i 0.694 ii 28.7  
 iii 78.0  
 2 a  $2^{-3} = \frac{1}{8}$  b  $27^{\frac{1}{3}} = 3$   
 3 a  $x^2 = 9^2 + 13^2 = 250, x = 5\sqrt{10}$   
 b  $7^2 = x^2 + 5^2, x^2 = 24, x = 2\sqrt{6}$

### Exercise 1A

- 1 i 7.3 m (accuracy of the least accurate measurement)  
 ii 7.27 m  
 2 79 cm (to 2 s.f.)

### Exercise 1B

- 1 a 23.5–24.5 mm  
 b 3.25–3.25 m  
 c 1.745–1.755 kg  
 d 1.395–1.405 g  
 2 a 0.09%  
 b 8847.5–8848.5 m  
 3 a 0.44 (2 s.f.)  
 b 2.7% (2 s.f.)  
 c Uncertainty much larger for the measurements done by group 1.  
 4  $66 \times 10^9$  km (2 s.f.)  
 5 Max 0.35 min, 0.30  
 6 a Actual 10°C, approx 9°C  
 b 10%  
 7 a  $7.20 \text{ m} \leq r < 7.21 \text{ m}$  (3 s.f.)  
 b 0.005 m

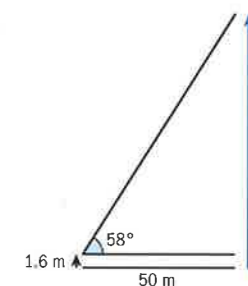
### Exercise 1C

- 1 a i  $9 \times 10^4$  ii  $9.936 \times 10$   
 b i  $10^{11}$  ii  $5.068 \times 10^{11}$   
 2 a  $9.4 \times 10^{-5}$   
 b  $8.35 \times 10^3$   
 c  $5.24 \times 10^{-19}$   
 d  $3.87 \times 10^{-7}$   
 3 a i  $\frac{15}{x^2}$  ii  $15 \cdot x^{\frac{1}{2}}$   
 b i  $7^1$  ii  $1 \cdot 7^1$   
 c i  $\frac{1}{2^{3+3t}}$   
 ii  $2^{-3-3t}$   
 d i  $\frac{25}{3^{2x}}$  ii  $25 \cdot 3^{-2x}$   
 4 a 240, 339, 480  
 b Rate of growth is increasing. When  $t = \frac{3}{2}$ , we have the number of bacteria after one and a half hours.

### Exercise 1D

- 5 a  $\frac{1600}{2^8}$  b 1100  
 6 0.24 mm  
 7 18.1  
 8  $2 \times 10^{-3}$   
 1 a  $\theta = 28.8^\circ, v = 8 \text{ cm}, w = 4 \text{ cm}$   
 b  $\theta = 56^\circ, y = 18.2 \text{ cm}, x = 6.88 \text{ cm}$   
 c  $z = 5.7 \text{ cm}, \alpha = 56^\circ, \beta = 34^\circ$   
 2 i 7.36 m  
 ii 4.25 m  
 iii 6.96 m

### 3 a



- b 81.6 m  
 4 a   
 b 138 m  
 5 1.62 m  
 6 1300 m  
 7 45 km  
 8 a 1.20 m b 2.1%

### Exercise 1E

- 1 a i   
 ii 1 possibility as we have all 3 angles and 1 side  
 iii  $\alpha = 47^\circ, AB = 14.8 \text{ cm}, BC = 10.7 \text{ cm}$   
 b i   
 ii 2 possibilities, ambiguous case of sine rule  
 iii  $\alpha = 62.6^\circ \beta = 95.4^\circ$   
 $DE = 16.5 \text{ cm}$  or  $\alpha = 153^\circ$   
 $\beta = 5^\circ DE = 1.44 \text{ cm}$