

16 Exploration

All IB Diploma subjects have an Internal Assessment (IA). The IA in Mathematics is called an exploration. The exploration will be assessed internally by your teacher and externally moderated by the IB and counts for 20% of your final grade.

This chapter gives you advice on planning your exploration, as well as hints and tips to help you to achieve a good grade by making sure that your exploration satisfies the assessment criteria. There are also suggestions on choosing a topic and how to get started on your exploration.

About the exploration

The exploration is an opportunity for you to show that you can apply mathematics to an area that interests you. It is a piece of written work investigating an area of mathematics.

There are 30 hours in the syllabus for developing your mathematical toolkit and your exploration. The “toolkit” is the inquiry, investigative, problem solving and modelling skills you need to write a good exploration. You can build these skills throughout this book—in particular, in the Investigations, Developing Inquiry Skills and Modelling activities in each chapter.

You should expect to spend around 10–15 hours of class time on your exploration and up to 10 hours of your own time.

During **class time** you will:

- go through the assessment criteria with your teacher
- brainstorm to come up with suitable topics/titles
- look at previous explorations and the grading
- meet with your teacher to discuss your choice of topic and your progress.

During **your own time** you will:

- research the topic you have chosen, to make sure that it is appropriate for an exploration (if not, you will have to conduct further research to help you select a suitable topic)
- collect and organize your information/data and decide which mathematical processes to apply
- write your exploration
- submit a draft exploration to your teacher (your teacher will set a deadline for this)
- present your draft exploration to some of your peers, for their feedback.
- submit the final exploration (your teacher will set a deadline for this). If you do not submit an exploration then you receive a grade of “N” and will not receive your IB Diploma.

How the exploration is marked

After you have submitted the final version of your exploration your teacher will mark it. This is “internal assessment” (in school). Your teacher submits these marks to the IB, from which a random sample of explorations is selected automatically. Your teacher uploads these sample explorations to be marked by an external moderator. This external moderation of IA ensures that all teachers in all schools are marking students’ work to the same standards.

To begin with, the external moderator will mark three of your school’s explorations. If the moderator’s mark is within 2 marks of your teacher’s mark, then all your teacher’s marks stay the same.

If the moderator’s mark is more than 2 marks higher or lower than your teacher’s mark, the external moderator will mark the remaining explorations in the sample. This may increase the mark if the teacher marked too harshly or decrease the mark if the teacher marked too leniently. The moderator sends a report to the school to explain the reason for any change in the marks.

Internal assessment criteria

Your exploration will be assessed by your teacher, against the criteria given below. The IB external moderator will use the same assessment criteria.

The final mark for each exploration is the sum of the scores for each criterion. The maximum possible final mark is 20. This is 20% of your final mark for Mathematics: applications and interpretation Higher level.

The criteria cover five areas, A to E

| | |
|-------------|----------------------------|
| Criterion A | Presentation |
| Criterion B | Mathematical communication |
| Criterion C | Personal engagement |
| Criterion D | Reflection |
| Criterion E | Use of mathematics |

Criterion A: Presentation

This criterion assesses the organization, coherence and conciseness of your exploration.

| Achievement level | Descriptor |
|-------------------|---|
| 0 | The exploration does not reach the standard described by the descriptors below. |
| 1 | The exploration has some coherence or some organization. |
| 2 | The exploration has some coherence and shows some organization. |
| 3 | The exploration is coherent and well organized. |
| 4 | The exploration is coherent, well organized and concise. |

IA tip

When coloured graphs are uploaded in black and white, they are very difficult to follow, as information such as colour-coded keys is lost. Make sure your exploration is uploaded in colour if it contains colour diagrams.

IA tip

Make sure that all the pages are uploaded. It is almost impossible to mark an exploration with pages missing.

IA tip

Make sure you understand these criteria. Check your exploration against the criteria frequently as you write it.

To get a good mark for Criterion A: Presentation

- A **well organized** exploration has:
 - a **rationale** which includes an explanation of why you chose this topic
 - an **introduction** in which you discuss the context of the exploration
 - a statement of the **aim** of the exploration, which should be clearly identifiable
 - a **conclusion**.
- A **coherent** exploration:
 - is logically developed and easy to follow
 - should “read well” and express ideas clearly
 - includes any graphs, tables and diagrams where they are needed—not attached as appendices to the document.
- A **concise** exploration:
 - focuses on the aim and avoids irrelevancies
 - achieves the aim you stated at the beginning
 - explains all stages in the exploration clearly and concisely.
- References must be cited where appropriate. Failure to do so could be considered academic malpractice.

Criterion B: Mathematical communication

This criterion assesses how you:

- use appropriate mathematical language (notation, symbols, terminology)
- define key terms, where required
- use multiple forms of mathematical representation, such as formulas, diagrams, tables, charts, graphs and models, where appropriate.

| Achievement level | Descriptor |
|-------------------|--|
| 0 | The exploration does not reach the standard described by the descriptors below. |
| 1 | There is some relevant mathematical communication which is partially appropriate. |
| 2 | The exploration contains some relevant, appropriate mathematical communication. |
| 3 | The mathematical communication is relevant, appropriate and is mostly consistent. |
| 4 | The mathematical communication is relevant, appropriate and consistent throughout. |

To get a good mark for Criterion B: Mathematical communication

- Use appropriate mathematical language and representation when communicating mathematical ideas, reasoning and findings.
- Choose and use appropriate mathematical and ICT tools such as graphic display calculators, screenshots, mathematical software, spreadsheets, databases, drawing and word-processing software, as appropriate, to enhance mathematical communication.

IA tip

For more on citing references, academic honesty and malpractice, see pages 741 – 742.

IA tip

Only include forms of representation that are relevant to the topic, for example, don't draw a bar chart and pie chart for the same data. If you include a mathematical process or diagram without using or commenting on it, then it is irrelevant.

IA tip

Use technology to enhance the development of the exploration—for example, by reducing laborious and repetitive calculations.



IA tip

Students often copy their GDC display, which makes it unlikely they will reach the higher levels in this criterion. You need to express results in proper mathematical notation, for example, use 2^x and not 2^x use \times not $*$ use 0.028 and not 2.8E-2

IA tip

Just showing personal interest in a topic is not enough to gain the top marks in this criterion. You need to write in your own voice and demonstrate your own experience with the mathematics in the topic.

- Define key terms that you use.
- Express results to an appropriate degree of accuracy.
- Label scales and axes clearly in graphs.
- Set out proofs clearly and logically.
- Define variables.
- Do not use calculator or computer notation.

Criterion C: Personal engagement

This criterion assesses how you engage with the exploration and make it your own.

| Achievement level | Descriptor |
|-------------------|---|
| 0 | The exploration does not reach the standard described by the descriptors below. |
| 1 | There is evidence of some personal engagement. |
| 2 | There is evidence of significant personal engagement. |
| 3 | There is evidence of outstanding personal engagement. |

To get a good mark for Criterion C: Personal engagement

- Choose a topic for your exploration that you are interested in, as this makes it easier to display personal engagement
- Find a topic that interests you and ask yourself “What if...?”
- Demonstrate personal engagement by using some of these skills and practices from the mathematician’s toolkit.
 - Creating mathematical models for real-life situations.
 - Designing and implementing surveys.
 - Running experiments to collect data.
 - Running simulations.
 - Thinking and working independently.
 - Thinking creatively.
 - Addressing your personal interests.
 - Presenting mathematical ideas in your own way.
 - Asking questions, making conjectures and investigating mathematical ideas.
 - Considering historical and global perspectives.
 - Exploring unfamiliar mathematics.

Criterion D: Reflection

This criterion assesses how you review, analyse and evaluate your exploration.

| Achievement level | Descriptor |
|-------------------|---|
| 0 | The exploration does not reach the standard described by the descriptors below. |
| 1 | There is evidence of limited reflection. |
| 2 | There is evidence of meaningful reflection. |
| 3 | There is substantial evidence of critical reflection. |

To get a good mark for Criterion D: Reflection

- Include reflection in the conclusion to the exploration, but also throughout the exploration. Ask yourself “What next?”
- Show reflection in your exploration by:
 - discussing the implications of your results
 - considering the significance of your findings and results
 - stating possible limitations and/or extensions to your results
 - making links to different fields and/or areas of mathematics
 - considering the limitations of the methods you have used
 - explaining why you chose this method rather than another.

Criterion E: Use of mathematics

This criterion assesses how you use mathematics in your exploration.

| Achievement level | Descriptor |
|-------------------|--|
| 0 | The exploration does not reach the standard described by the descriptors below. |
| 1 | Some relevant mathematics is used. |
| 2 | Some relevant mathematics is used. The mathematics explored is partially correct. Some knowledge and understanding are demonstrated. |
| 3 | Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Good knowledge and understanding are demonstrated. |
| 4 | Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Good knowledge and understanding are demonstrated. |
| 5 | Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct and demonstrates sophistication or rigour. Thorough knowledge and understanding are demonstrated. |
| 6 | Relevant mathematics commensurate with the level of the course is used. The mathematics explored is precise and demonstrates sophistication and rigour. Thorough knowledge and understanding are demonstrated. |

To get a good mark for Criterion E: Use of mathematics

- Produce work that is commensurate with the level of the course you are studying. The mathematics you explore should either be part of the syllabus, at a similar level or beyond.
- If the level of mathematics is not commensurate with the level of the course you can only get a maximum of two marks for this criterion.
- Only use mathematics relevant to the topic of your exploration. Do not just do mathematics for the sake of it.
- Demonstrate that you fully understand the mathematics used in your exploration.

IA tip

Discussing your results without analysing them is not meaningful or critical reflection. You need to do more than just describe your results. Do they lead to further exploration?

IA tip

Make sure the mathematics in your exploration is not only based on the prior learning for the syllabus. When you are deciding on a topic, consider what mathematics will be involved and whether it is commensurate with the level of the course.

- Justify **why** you are using a particular mathematical technique (do not just use it).
- Generalize and justify conclusions.
- Apply mathematics in different contexts where appropriate.
- Apply problem-solving techniques where appropriate.
- Recognize and explain patterns where appropriate.

- **Precise** mathematics is error-free and uses an appropriate level of accuracy at all times.
- Demonstrate **sophistication** of mathematics in your exploration.
 - Show that you understand and can use challenging mathematical concepts
 - Show that you can extend the applications of mathematics beyond those you learned in the classroom
 - look at a problem from different mathematical perspectives
 - identify underlying structures to link different areas of mathematics.
- Demonstrate **rigour** by using clear logic and language in your mathematical arguments and calculations.

IA tip

Each step in the mathematical development of the exploration needs to be clearly explained so that a peer can follow your working without stopping to think.

Academic honesty

This is very important in all your work. Your school will have an Academic Honesty Policy which you should be given to discuss in class, to make sure that you understand what malpractice is and the consequences of committing malpractice.

According to the IB Learner Profile for Integrity:

“We act with integrity and honesty, with a strong sense of fairness and justice, and with respect for the dignity and rights of people everywhere. We take responsibility for our actions and their consequences.”

Academic Honesty means:

- that your work is authentic
- that your work is your own intellectual property
- that you conduct yourself properly during examinations
- that any work taken from another source is properly cited.

Authentic work:

- is work based on your own original ideas
- can draw on the work of others, but this must be fully acknowledged in footnotes and bibliography
- must use your own language and expression
- must acknowledge all sources fully and appropriately in a bibliography.

IA tip

Reference any photographs you use in your exploration, including to decorate the front page.

Malpractice

The IB Organization defines malpractice as “behaviour that results in, or may result in, the candidate or any other candidate gaining an unfair advantage in one or more assessment components.”

Malpractice includes:

- plagiarism—copying from others’ work, published or otherwise, whether intentional or not, without the proper acknowledgement
- collusion—working together with at least one other person in order to gain an undue advantage (this includes having someone else write your exploration)
- duplication of work—presenting the same work for different assessment components
- any other behaviour that gains an unfair advantage such as taking unauthorized materials into an examination room, stealing examination materials, disruptive behaviour during examinations, falsifying CAS records or impersonation.

Collaboration and collusion

It is important to understand the distinction between collaboration (which is allowed) and collusion (which is not).

Collaboration

In several subjects, including mathematics, you will be expected to participate in group work. It is important in everyday life that you are able to work well in a group situation. Working in a group entails talking to others and sharing ideas. Every member of the group is expected to participate equally and it is expected that all members of the group will benefit from this collaboration. However, the end result must be your own work, even if it is based on the same data as the rest of your group.

Collusion

This is when two or more people work together to intentionally deceive others. Collusion is a type of plagiarism. This could be working with someone else and presenting the work as your own or allowing a friend to copy your work.

References and acknowledging sources

The IB does not tell you which style of referencing you should use—this is left to your school.

¹Words & Ideas. The Turnitin Blog. Top 15 misconceptions about Turnitin. Misconception 11: matched text is likely to be completely coincidental or common knowledge (posted by Katie P., March 09, 2010).

IA tip

Plagiarism detection software identifies text copied from online sources. The probability that a 16-word phrase match is “just a coincidence” is $\frac{1}{10^{12}}$.¹

IA tip

Discussing individual exploration proposals with your peers or in class before submission is collaboration.

Individually collecting data and then pooling it to create a large data set is collaboration. If you use this data for your own calculations and write your own exploration, that is collaboration. If you write the exploration as a group, that is collusion.

IA tip

Be consistent and use the same style of referencing throughout your exploration.

The main reasons for citing references are:

- to acknowledge the work of others
- to allow your teacher and moderator to check your sources.

To refer to someone else’s work:

- include a brief reference to the source in the main body of your exploration—either as part of the exploration or as a footnote on the appropriate page
- include a full reference in your bibliography.

The bibliography should include a list with full details of **all** the sources that you have used.

Choosing a topic

You need to choose a topic that interests you, as then you will enjoy working on the exploration and you will be able to demonstrate personal engagement by using your own voice and demonstrating your own experience.

Discuss the topic you choose with your teacher and your peers before you put too much time and effort into developing the exploration. Remember that the work does not need to go beyond the level of the course which you are taking, but you can choose a topic that is outside the syllabus and is at a commensurate level. You should avoid choosing topics that are too ambitious, or below the level of your course.

These questions may help you to find a topic for your exploration:

- What areas of the syllabus are you enjoying most?
- What areas of the syllabus are you performing best in?
- Would you prefer to work on purely analytical work or on modelling problems?
- Have you discovered, through reading or talking to peers on other mathematics courses, areas of mathematics that might be interesting to look into?
- What mathematics is important for the career that you eventually hope to follow?
- What are your special interests or hobbies? Where can mathematics be applied in this area?

One way of choosing a topic is to start with a general area of interest and create a mind map. This can lead to some interesting ideas on applications of mathematics to explore. The mind map on the following pages shows how the broad topic “Transport” can lead to suggestions for explorations into such diverse topics as baby carriage design, depletion of fossil fuels and queuing theory.

On page 744 there is an incomplete mind map for you to continue, either on your own or by working with other mathematics students.

IA tip

Cite references to others’ work even if you paraphrase or rewrite the original text.

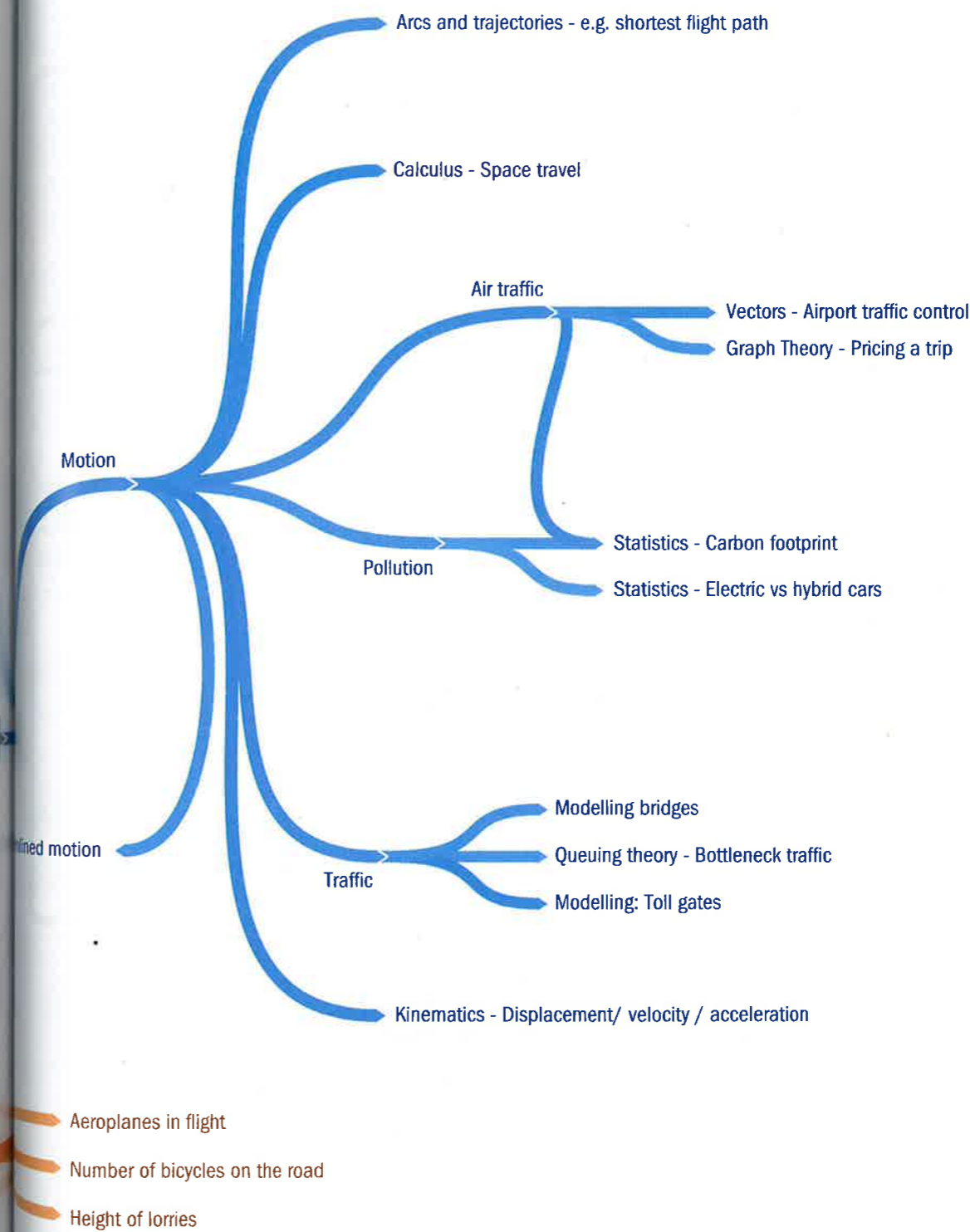
You do not need to cite references to formulas taken from mathematics textbooks.

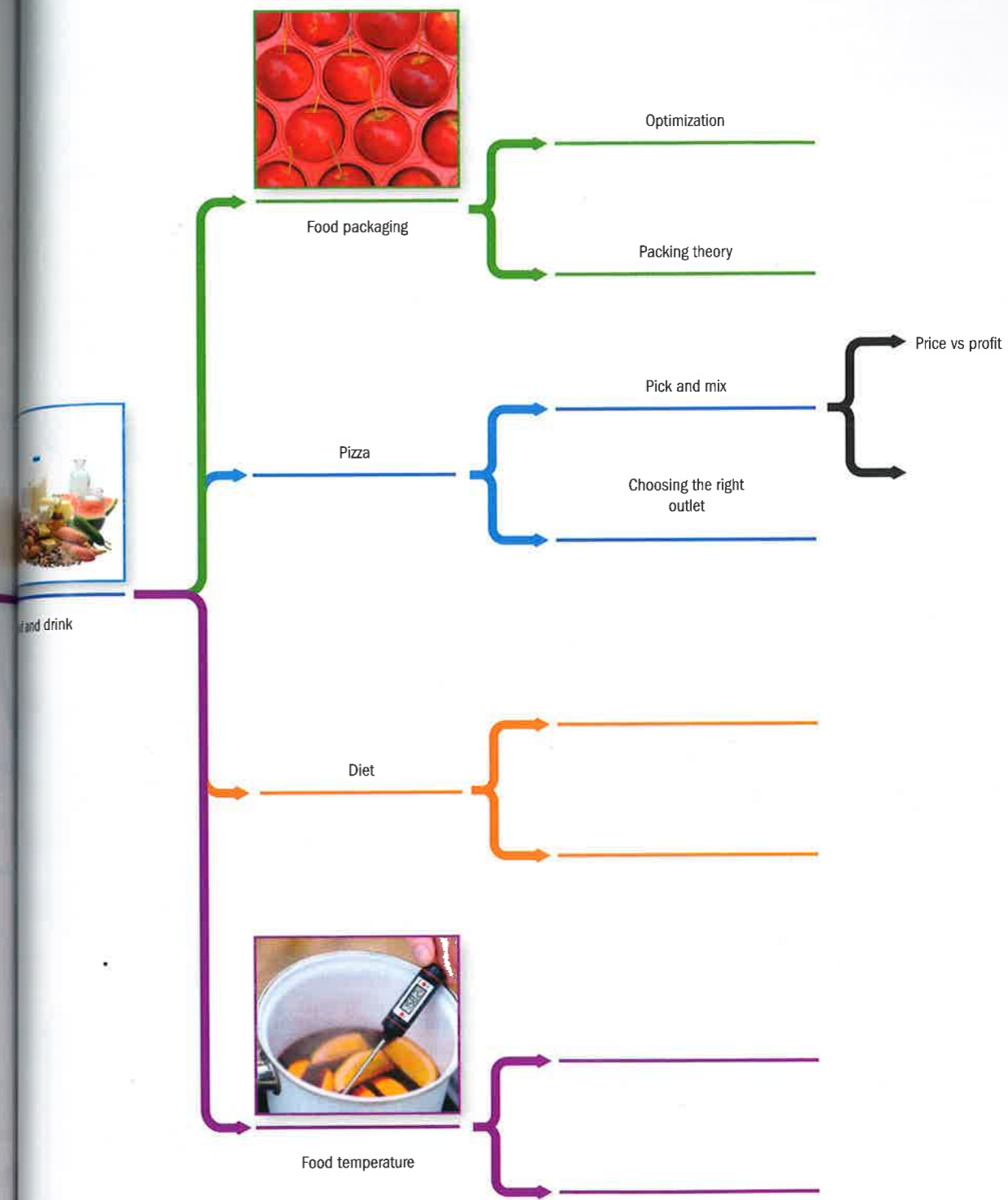
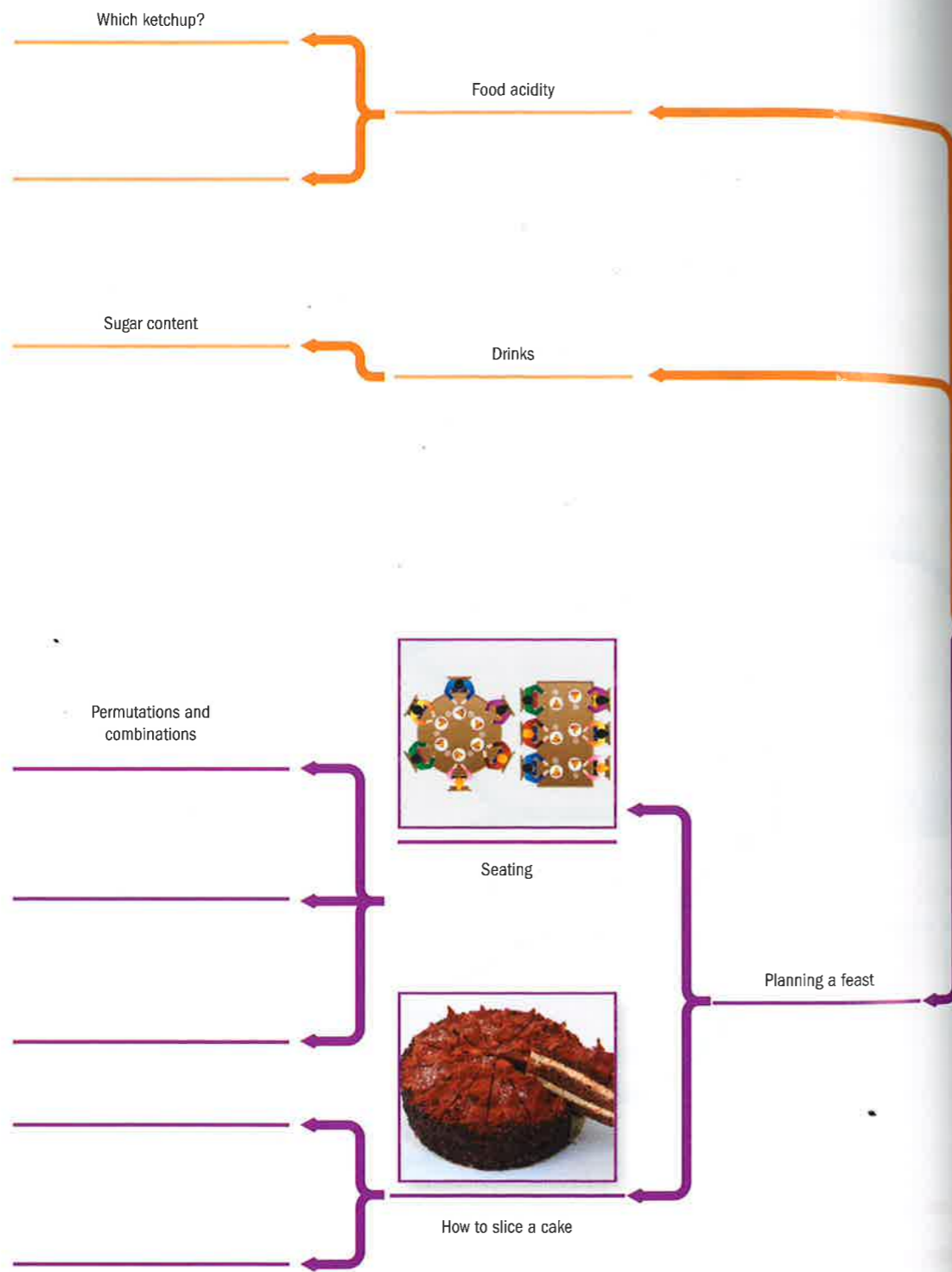
IA tip

You must include a brief reference in the exploration as well as in the bibliography. It is not sufficient just to include a reference in the bibliography.

IA tip

Your exploration should contain a substantial amount of mathematics at the level of your course, and should not just be descriptive. Although the history of mathematics can be very interesting it is not a good exploration topic.





Research

Once you have chosen a topic, you will need to do some research. The purpose of this research is to help you determine how suitable your topic is.

- Don't rely on the Internet for all your research—you should also make use of books and academic publications.
- Plan your time wisely—make sure that you are organized.
- Don't put it off—start your research in good time.
- For Internet research: refine your topic so you know exactly what information you are looking for, and use multiple-word searches. It is very easy to spend hours on the Internet without finding any relevant information.
- Make sure that you keep a record of all the websites you use—this saves so much time afterwards. You will need to cite them as sources, and to include them in your bibliography.
- Make sure that the sources are reliable—who wrote the article? Are they qualified? Is the information accurate? Check the information against another source.
- Research in your own language if you find this easier.

These questions will help you to decide whether the topic you have chosen is suitable.

- What areas of mathematics are contained in the topic?
- Which of these areas are contained in the syllabus that you are following?
- Which of these areas are not in the syllabus that you are following but are contained in the other IB mathematics course?
- Which of these areas are in none of the IB mathematics courses? How accessible is this mathematics to you?
- Would you be able to understand the mathematics and write an exploration in such a way that a peer is able to understand it all?
- How can you demonstrate personal engagement in your topic?
- Will you manage to complete an exploration on this topic and meet all the top criterion descriptors within the recommended length of 12 to 20 pages (double spaced and font size 12)?

Writing an outline

Once you think you have a workable topic, write a brief outline including:

- Why you chose this topic.
- How your topic relates to mathematics.
- The mathematical areas in your topic, for example, algebra, geometry, calculus, etc.
- The key mathematical concepts covered in your topic, for example, modelling data, areas of irregular shapes, analysing data, etc.
- The mathematical skills you will use in the exploration, for example, integration by parts, working with complex numbers, using polar coordinates, etc.
- Any mathematics outside the syllabus that you need to learn.
- Technology you could use to develop your exploration.
- New key terms that you will need to define or explain.
- How you are going to demonstrate personal engagement.
- A list of any resources you have/ will use in the development of your exploration. If this list includes websites you should include the URL and the date when this was accessed.

Share this outline with your teacher and with your peers. They may ask questions that lead you to improve your outline.

IA tip

Try to avoid writing a research report in which you merely explain a well-known result that can easily be found online or in textbooks. Such explorations have little scope for meaningful and critical reflection and it may be difficult to demonstrate personal engagement

IA tip

Learning new mathematics is not enough to reach the top levels in Criterion C: Personal engagement.

IA tip

Popular topics such as the Monty Hall problem, the Birthday paradox, and so on are not likely to score well on all the criteria.

This template may help you write the outline for the exploration when presenting a formal proposal to your teacher.

Mathematics exploration outline

| |
|----------------------|
| Topic: |
| Exploration title: |
| Exploration aim: |
| Exploration outline: |
| Resources used: |
| Personal engagement: |

Writing your exploration

Now you should be ready to start writing your exploration in detail.

You could ask one of your classmates to read the exploration and give you feedback before you submit the draft to your teacher. If your exploration is related to another discipline, for example, Economics, it would be better if the peer reading your exploration is someone who does not study economics.

Mathematical exploration checklist

Work through this checklist to confirm that you have done everything that you can to make your exploration successful.

- | | |
|---|---|
| <ul style="list-style-type: none"> <input type="checkbox"/> Does your exploration have a title? <input type="checkbox"/> Have you given a rationale for your choice of exploration? <input type="checkbox"/> Have you ensured your exploration does not include any identifying features—for example, your name, candidate number, school name? <input type="checkbox"/> Does your exploration start with an introduction? <input type="checkbox"/> Have you clearly stated your aim? <input type="checkbox"/> Does your exploration answer the stated aim? <input type="checkbox"/> Have you used double line spacing and 12-point font? <input type="checkbox"/> Is your exploration 12–20 pages long? <input type="checkbox"/> Have you cut out anything that is irrelevant to your exploration? <input type="checkbox"/> Have you checked that you have not repeated lots of calculations? <input type="checkbox"/> Have you checked that tables only contain relevant information and are not too long? <input type="checkbox"/> Is your exploration easy for a peer to read and understand? <input type="checkbox"/> Is your exploration logically organized? <input type="checkbox"/> Are all your graphs, tables and diagrams correctly labelled and titled? <input type="checkbox"/> Are any graphs, tables and diagrams placed appropriately and not all attached at the end? <input type="checkbox"/> Have you used appropriate mathematical language and representation (not computer notation, eg *, ^, etc.)? | <ul style="list-style-type: none"> <input type="checkbox"/> Have you used notation consistently through your exploration? <input type="checkbox"/> Have you defined key terms (mathematical and subject specific) where necessary? <input type="checkbox"/> Have you used appropriate technology? <input type="checkbox"/> Have you used an appropriate degree of accuracy for your topic/exploration? <input type="checkbox"/> Have you shown interest in the topic? <input type="checkbox"/> Have you used original analysis for your exploration (eg simulation, modelling, surveys, experiments)? <input type="checkbox"/> Have you expressed the mathematical ideas in your exploration in your own way (not just copy-and-pasted someone else's)? <input type="checkbox"/> Does your exploration have a conclusion that refers back to the introduction and the aim? <input type="checkbox"/> Do you discuss the implications and significance of your results? <input type="checkbox"/> Do you state any possible limitations and/or extensions? <input type="checkbox"/> Do you critically reflect on the processes you have used? <input type="checkbox"/> Have you explored mathematics that is commensurate with the level of the course? <input type="checkbox"/> Have you checked that your results are correct? <input type="checkbox"/> Have you clearly demonstrated understanding of why you have used the mathematical processes you have used? <input type="checkbox"/> Have you acknowledged direct quotes appropriately? <input type="checkbox"/> Have you cited all references in a bibliography? <input type="checkbox"/> Do you have an appendix if one is needed? |
|---|---|

IA tip

As you write your exploration, remember to refer to the criteria below.

IA tip

Remember that your peers should be able to read and understand your work.