How to use this book

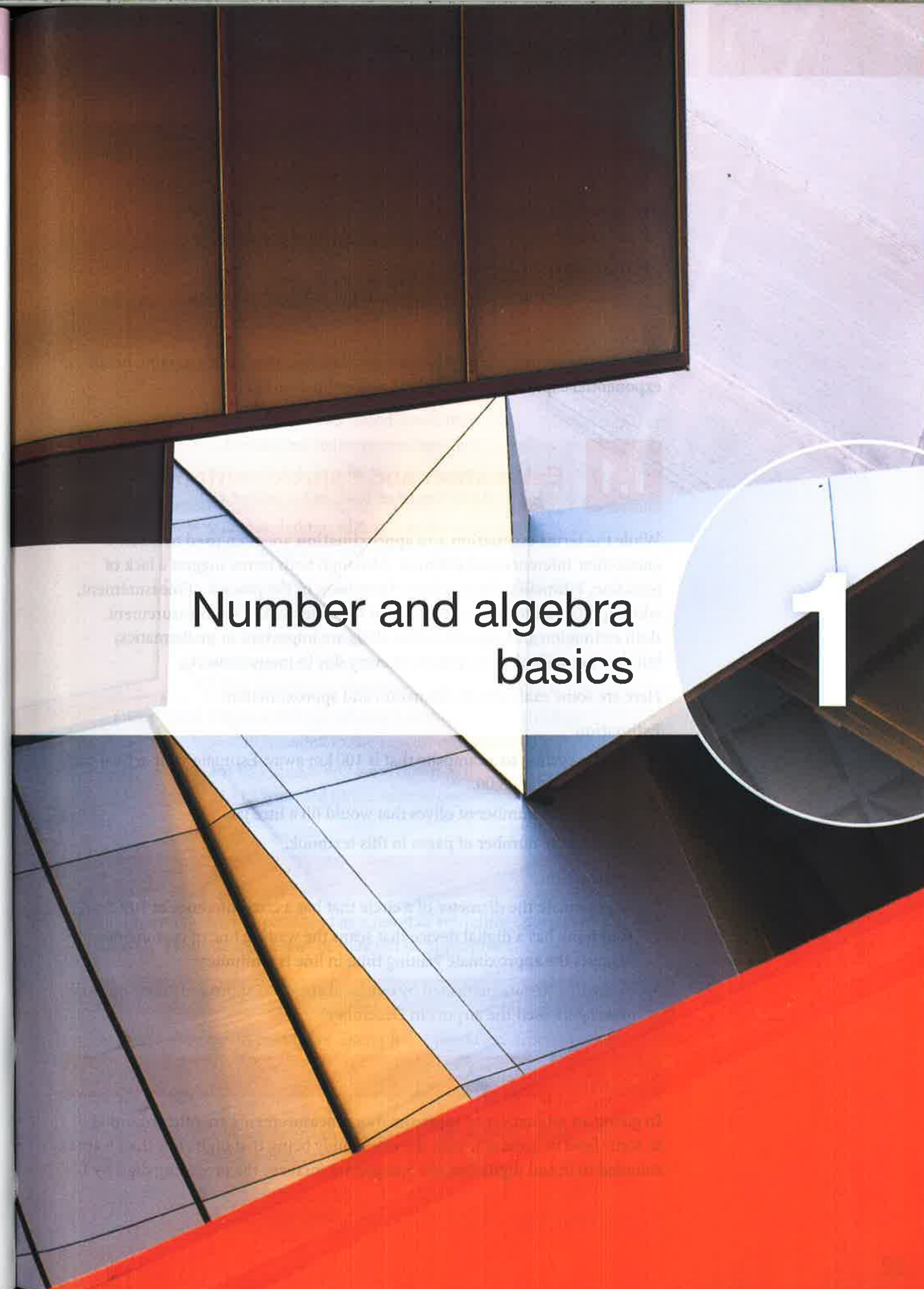
This book is designed to be read by you – the student. It is very important that you read this book carefully. We have strived to write a readable book – and we hope that your teacher will routinely give you reading assignments from this textbook, thus giving you valuable time for productive explanations and discussions in the classroom. Developing your ability to read and understand mathematical explanations will prove to be valuable to your long-term intellectual development, while also helping you to comprehend mathematical ideas and acquire vital skills to be successful in the *Applications and Interpretation* HL course. Your goal should be understanding, not just remembering. You should always read a chapter section thoroughly before attempting any of the exercises at the end of the section.

Our aim is to support genuine inquiry into mathematical concepts while maintaining a coherent and engaging approach. We have included material to help you gain insight into appropriate and wise use of your GDC and an appreciation of the importance of proof as an essential skill in mathematics. We endeavoured to write clear and thorough explanations supported by suitable worked examples, with the overall goal of presenting sound mathematics with sufficient rigour and detail at a level appropriate for a student of HL mathematics.

Our thanks go to Jim Nakamoto, Kevin Frederick and Stephen Lumb who joined our team for this edition, helping us to add richness and variety to the series.

For over 10 years, we have been writing successful textbooks for IB mathematics courses. During that time, we have received many useful comments from both teachers and students. If you have suggestions for improving this textbook, please feel free to write to us at globalschools@pearson.com. We wish you all the best in your mathematical endeavours.

Ibrahim Wazir and Tim Garry



1 Number and algebra basics

Learning objectives

By the end of this chapter, you should be familiar with…

- making reasonable estimations and better approximations
- demonstrating an understanding of the rules of exponents
- using correct scientific notation
- demonstrating an understanding of the rules of logarithms.

This chapter revises and consolidates previous knowledge of scientific notation, exponential expressions, logarithms and estimation skills.

1.1 Estimation and approximation

While the terms **estimation** and **approximation** are often used to mean a guess, their inferences are different. Although both terms suggest a lack of precision, estimation infers a lack of precision in the process of measurement, while approximation lacks precision in the statement of the measurement. Both estimation and approximation skills are important in mathematics, but they are skills that are practiced every day in many contexts.

Here are some examples of estimation and approximation.

Estimation

1. You are cycling to a campsite that is 100 km away. Estimate your arrival time if you depart at 08:00.
2. Estimate the number of olives that would fill a litre jar.
3. Estimate the number of pages in this textbook.

Approximation

1. Approximate the diameter of a circle that has a circumference of 109.2 cm.
2. Your bank has a digital device that scans the waiting line of customers and suggests the approximate waiting time in line is 6 minutes.
3. According to data published by our local airport, "Approximately 2 million passengers used the airport in December".

Rounding answers

In giving an estimation or approximation, measurements are often rounded to some level of accuracy, with the rule simply being that digits less than 5 are rounded to 0; and digits that are 5 or greater increase the preceding digit by 1.

Example 1.1

(a) Round each value to the nearest unit.

(i) 256.4 (ii) 1.49 (iii) 63.5 (v) 700.9

(b) Round each value to the nearest one-hundredth.

(i) 1.006 (ii) 7.295 (iii) 67.085 (iv) 34.113

Solution

(a) (i) 256.4 — 4 is less than 5, so round down to 0
256.4 rounded to the nearest unit is 256

(ii) 1.49 — 4 is less than 5, so round down to 0
1.49 rounded to the nearest unit is 1

(iii) 63.5 — 5 is rounded up, adding 1 to the **units** digit
63.5 rounded to the nearest unit is 64

(iv) 700.9 — 9 is rounded up, adding 1 to the **units** digit
700.9 rounded to the nearest unit is 701

(b) (i) 1.006 — 6 is rounded up, adding 1 to the **hundredths** digit
1.006 rounded to the nearest one-hundredth is 1.01

(ii) 7.295 — 5 is rounded up, adding 1 to the **hundredths** digit, which in turn adds 1 to the tenths digits in this case
7.295 rounded to the nearest one-hundredth is 7.30

(iii) 67.085 — 5 is rounded up, adding 1 to the **hundredths** digit
67.085 rounded to the nearest one-hundredth is 67.09

(iv) 34.113 — 3 is less than 5, so round down to 0
34.113 rounded to the nearest one-hundredth is 34.11

Percentage error

The approximate answer produced as a result of rounding depends on the digit to which it is rounded, and may or may not be appropriate.

Example 1.2

The rounded values in part (a) of Example 1.1 produced differences of

(i) $256.4 - 256 = 0.4$ (ii) $1.49 - 1 = 0.49$

(iii) $63.5 - 64 = -0.5$ (iv) $700.9 - 701 = -0.1$

What are the percentage errors in rounding if the original values are assumed to be precise measurements?

Solution

Dividing the differences by the original values we obtain:

(i) $\frac{0.4}{256.4} \approx 0.156\%$ (ii) $\frac{0.49}{1.49} \approx 32.9\%$

(iii) $\frac{-0.5}{63.5} \approx -0.787\%$ (iv) $\frac{-0.1}{700.9} \approx -0.0143\%$

The errors are all quite small except for the second one. (Note that this percentage is rounded, too!) Choosing an arbitrary decimal place to which a measurement is rounded produces inaccuracies that may not be acceptable.

In IB mathematics, where an exact final answer is not required, an approximate answer, to the required accuracy, is important. To achieve this, a thorough understanding of the notion of **significant figures (s.f.)** is critical. We will revisit percentage errors after studying significant figures.

Significant figures (s.f.)

Rule	Example
All non-zero digits are significant	74 818 226 has 8 s.f. 123.45 has 5 s.f.
All zeros between non-zero digits are significant	103.05 has 5 s.f. 780 002 has 6 s.f.
Zeros to the left of an implied decimal point are **not** significant, whereas zeros to the right of an explicit decimal **are** significant.	23 000 has 2 s.f., while 23 000.0 has 6 s.f.
To the right of a decimal point, all leading zeros are **not** significant, whereas all zeros that follow non-zero digits **are** significant.	0.0043 has 2 s.f., while 0.0043000 has 5 s.f.

Table 1.1 Significant figures rules and examples

Example 1.3

Indicate the number of significant figures in each value.

(a) 30 020 (b) 30 020.0 (c) 0.008 (d) 1000.0

(e) 1.09 (f) 7.00101 (g) 0.02 (h) 0.020

Solution

(a) 4: The non-zero digits and the zeros in between are significant.

(b) 6: All digits between the leading non-zero digit and the decimal point are significant. The zero after the decimal point is also significant.

(c) 1: Only the '8' is significant.

(d) 5: All digits between the leading non-zero digit and the decimal point are significant. The zero after the decimal point is also significant.

(e) 3: The non-zero digits and the zero in between are significant.

(f) 6: The non-zero digits and the zeros in between are significant.

(g) 1: Only the '2' is significant.

(h) 2: The '2' and the trailing zero are significant.

Percentage error revisited

In Example 1.1, we looked at the following values rounded to the nearest unit:

256.4 1.49 63.5 700.9

Now, instead of rounding to the nearest unit, consider these values given to 3 significant figures:

256 1.49 63.5 701

The differences of these values from the original values would be:

$256.4 - 256 = 0.4$ $1.49 - 1.49 = 0$

$63.5 - 63.5 = 0$ $700.9 - 701 = -0.1$

Hence, their percentage errors would be:

$\frac{0.4}{256.4} \approx 0.156\%$ $\frac{0}{1.49} = 0\%$

$\frac{0}{63.5} = 0\%$ $\frac{-0.1}{700.9} \approx -0.0143\%$

Exercise 1.1

1. Round each value to the nearest unit.
 (a) 25.8 (b) 0.61 (c) 1200.7 (d) 83.47
2. Round each value to the nearest one-hundredth.
 (a) 27.047 (b) 800.008 (c) 3.14159 (d) 0.0009
3. Give an example that would justify the use of measurements given to the following levels of precision.
 (a) 1.72 m (b) 0.014 s (c) 250 km (d) 2.43 mB
 (e) 1.27 cm (f) 1200 g (g) 23°C (h) 0.2 A
4. If the original values in questions 1 and 2 were precise measurements, find the percentage errors in their rounded values, to two decimal places.
5. Determine the number of significant figures in each value.
 (a) 3910 (b) 3901 (c) 8200 (d) 8200.0
 (e) 100.3 (f) 100.03 (g) 0.002 (h) 0.0020
6. Give each value correct to 3 significant figures.
 (a) 5627 (b) 3098 (c) 4 762 311 (d) 3.14159
 (e) 0.0002070 (f) 100.03 (g) 0.02013 (h) 0.020003

1.2 Rules of exponents

Exponents and the rules for their use will be required throughout this course.

The rules of exponents for real values a, m and n are:

1. $a^m \cdot a^n = a^{m+n}$
2. $\dfrac{a^m}{a^n} = a^{m-n}$, provided $a \neq 0$
3. $(a^m)^n = a^{mn}$
4. $a^0 = 1$, provided $a \neq 0$
5. $a^{-m} = \dfrac{1}{a^m}$
6. $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$, provided $a > 0$

Example 1.4

Use the rules of exponents to write each of the following in the form 2^n, where $n \in \mathbb{Z}$.

(a) $2^4 \cdot 2^2$ (b) $2^8 \cdot 2^{-2}$ (c) $4^2 \cdot 2^3$ (d) $2^{-4} \cdot 4^{\frac{1}{2}}$

(e) $\dfrac{2^8}{2^2}$ (f) $\dfrac{2^{-2}}{2^{-6}}$ (g) $(2^4)^2$ (h) $(2^4)^0$

Solution

(a) 2^6 The bases are the same. Add the exponents.

(b) 2^6 The bases are the same. Add the exponents.

(c) 2^7 The bases are different. First convert 4^2 to 2^4, then add the exponents.

(d) 2^{-3} First convert $4^{\frac{1}{2}}$ to 2^1, then add the exponents.

(e) 2^6 The bases are the same. Subtract the exponents.

(f) 2^4 The bases are the same. Subtract the exponents.

(g) 2^8 Multiply the exponents.

(h) 2^0 Multiply the exponents.

Exercise 1.2

Use the rules of exponents to write each of the following in the form 3^n, where $n \in \mathbb{Z}$.

1. $3^4 \cdot 3^2$ **2.** $3^4 \cdot 3^{-2}$ **3.** $\dfrac{3^6}{3^2}$ **4.** $\dfrac{3^{-6}}{3^2}$

5. $(3^4)^2$ **6.** $(3^{-1})^{-3}$ **7.** $9^4 \cdot 3^2$ **8.** $9^4 \cdot 81^2$

9. $9^4 \cdot 81^{-2}$ **10.** $\sqrt{27}$ **11.** $\sqrt{27} \cdot 9\sqrt{3}$ **12.** $\sqrt[3]{9} \cdot 3\sqrt[3]{9}$

13. $\sqrt{3} \cdot \sqrt[3]{3}$ **14.** $\sqrt{3} \cdot \sqrt[4]{9}$ **15.** $9^{-2} \cdot (3\sqrt{27})^2$ **16.** 1

17. $9^2 \cdot 3^{\frac{1}{2}}$ **18.** $\sqrt{3} \cdot \sqrt[3]{81}$ **19.** $3^{\frac{1}{2}} \cdot \sqrt[3]{9}$ **20.** $\sqrt{27} \cdot 3^{\frac{3}{2}}$

21. $\dfrac{3^6}{\sqrt{3}}$ **22.** $\dfrac{3^2}{\sqrt[3]{9}}$ **23.** $\dfrac{3^{\frac{2}{3}}}{\sqrt[3]{9}}$ **24.** $\dfrac{9^{-\frac{1}{2}}}{\sqrt[3]{3}}$

1.3 Scientific notation

Scientific notation is used to represent very large and very small measurements without having to count decimal places. For example, the approximate distance from the Earth to the sun is 149 600 000 km. Using scientific notation this would be written as 1.496×10^8 km.

The ångström (Å) is a unit of length equal to one ten-billionth of a metre. In scientific notation, it is written as 1×10^{-10} m. It is very useful to have a notation that immediately shows the magnitude of this number that would otherwise be written as 0.0000000001 m.

Provided that measurements with comparable units are used, addition and subtraction is straightforward. If the units are not comparable, they need to be converted to a common unit.

The least number of significant figures in any measurement determines the number of significant figures in the answer.

Example 1.5

Find the perimeter of a rectangle with length $l = 2.3 \times 10^{-1}$ m and width $w = 9.5 \times 10^{-2}$ m.

Solution

There is a difference in the order of magnitude between the length and width, so a conversion is required.

$l = 2.3 \times 10^{-1}\,\text{m} = 23 \times 10^{-2}\,\text{m}$

$w = 9.5 \times 10^{-2}\,\text{m}$

The perimeter, p, is given by:

$$\begin{aligned} p &= 2(l + w) \\ &= 2(23 \times 10^{-2} + 9.5 \times 10^{-2}) \\ &= 2(32.5 \times 10^{-2}) \\ &= 65 \times 10^{-2}\,\text{m} \\ \text{or} \quad &= 6.5 \times 10^{-1}\,\text{m} \end{aligned}$$

Example 1.6

Find the area of the rectangle in Example 1.5

Solution

Area $A = l \times w = (2.3 \times 10^{-1}) \times (9.5 \times 10^{-2})$

$= (2.3 \times 9.5) \times (10^{-1} \times 10^{-2})$

$= 21.85 \times 10^{-3}$

$= 2.185 \times 10^{-2}$

But the given measurements were given to 2 s.f. so the answer should use the same degree of accuracy.

Area $A = 2.2 \times 10^{-2}\ \text{m}^2$ (2 s.f.)

Don't forget to include units.

Exercise 1.3

1. Express the following in scientific notation.

(a) 1203 **(b)** 7 billion **(c)** 0.000301 **(d)** 20.01
(e) 2000 **(f)** 0.00070 **(g)** 12.03 **(h)** 10006
(i) 10.001 **(j)** 1 googol

Consider the number of significant figures in each value.

2. Work out each calculation and give your answer in scientific notation.

(a) 210×8000 **(b)** 200×0.00018

(c) $(2.3 \times 10^9) \times (8 \times 10^2)$ **(d)** $(2.3 \times 10^{-3}) \times (8 \times 10^3)$

3. Find each value to 3 significant figures and express the answer in scientific notation.

(a) 2^{30} **(b)** $2^{31} - 1$ **(c)** e^{π} **(d)** π^{e}

1.4 Exponents and logarithms

$y = a^x \Rightarrow x = \log_a y$

Exponents and logarithms are inverses of each other. For instance, should a relation be expressed as $y = a^x$, then x is the exponent of the base a which yields the quantity y, written simply as $x = \log_a y$

When the base is 10, we typically do not write it. We can simply write **log *y***. A logarithm with base 10 is called a **common logarithm**.

When the base is **e**, we write **ln *y***. A logarithm with base **e** is called a **natural logarithm**.

Base	Expression
a	$\log_a m$
10	$\log m$
e	$\ln m$

Example 1.7

The squares of a chessboard are numbered consecutively from 1 to 64. If one grain of rice is placed on the first square, two grains on the second square, four grains on the third square, doubling the number of grains with each square, on which square would there first be more than a million (10^6) grains of rice? Theoretically, of course!

Solution

The number of grains of rice follows an exponential pattern.

Square (x)	Grains of rice (y)	In exponential form
1	1	2^0
2	2	2^1
3	4	2^2
4	8	2^3
Noting the number of the square and the exponent…		
n	$\geq 10^6$	2^{n-1}

So, we need to find a **solution** to $y = 2^{x-1}$ when $y = 10^6$. Stating this as a logarithm problem, we must solve the equation $x - 1 = \log_2 10^6$ or $x = \log_2(10^6) + 1$

We can use our GDC to solve the equation.

Since the number of the square must be a positive integer, the 21st square will be the first to hold in excess of one million grains of rice.

We can do the calculation using the Solver feature of a GDC (see Figure 1.1).

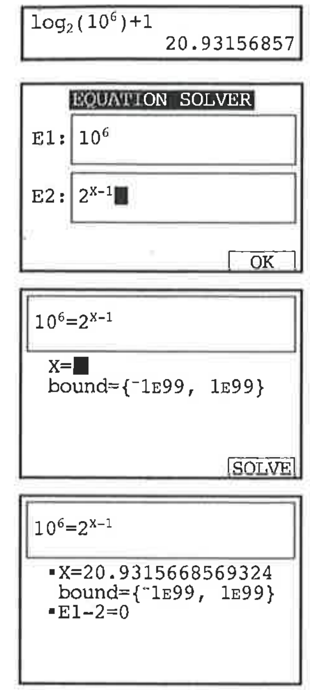


Figure 1.1 GDC screens for the solution to Example 1.7

Example 1.8

Earthquake magnitudes (R) are measured on the Richter scale which is a base-10 logarithmic scale, and relative comparisons are useful. For example, an earthquake of magnitude $R = 4$ is ten times as strong as an earthquake of magnitude $R = 3$. What is the magnitude of an earthquake R_1 that is twice as strong as another of magnitude $R = 3$?

Solution

Comparing the relative magnitudes, the equation to be solved for R_1 is:

$$2 = \frac{10^{R_1}}{10^3}$$

$$\Rightarrow 2 \cdot 10^3 = 10^{R_1}$$

$$\Rightarrow R_1 = \log(2 \cdot 10^3) \text{ and using a GDC gives}$$

$$\Rightarrow R_1 \approx 3.301$$

Exercise 1.4

1. Write each equation in logarithmic form.

(a) $1000 = 10^3$ (b) $64 = 4^3$ (c) $100^{\frac{3}{2}} = 1000$

(d) $9^{\frac{1}{2}} = 3$ (e) $2\sqrt{2} = 8^{\frac{1}{2}}$ (f) $10^0 = 1$

(g) $e^0 = 1$ (h) $6^{-2} = \frac{1}{36}$ (i) $(\sqrt{2})^{-2} = \frac{1}{2}$

(j) $3^{-\frac{1}{2}} = \frac{1}{\sqrt{3}}$ (k) $\left(\frac{1}{2}\right)^{-3} = 8$ (l) $8^{-\frac{1}{2}} = \frac{\sqrt{2}}{4}$

(m) $(-2)^3 = -8$ (n) $(0.01)^{-1} = 100$ (o) $\left(\frac{\sqrt{2}}{2}\right)^3 = \frac{\sqrt{2}}{4}$

2. Express each equation in the form $x = \ldots$

(a) $y = 2^x$ (b) $y = 10^x$ (c) $y = e^x$

(d) $y = 2^{3x}$ (e) $y = 3 \cdot 2^x$ (f) $y = 5 - 2^x$

(g) $y = 3^{2x}$ (h) $y = 3^{\frac{x}{2}}$ (i) $y = e^{2x}$

(j) $y = 2^{x-3}$ (k) $y = e^{\frac{x}{2}}$ (l) $y = \frac{1}{2}e^{2x}$

3. Consider Example 1.7 about the grains of rice on a chess board. Would any one of the 64 squares ever hold more than exactly 1 billion grains of rice?

4. Using the earthquake context of Example 1.8, find the magnitude of an earthquake that is:

(a) ten times as powerful as one of magnitude $R = 5.2$

(b) twice as powerful as one of magnitude $R = 5.2$

1.5 Rules of logarithms

Logarithms may seem to be just mirror images of exponents, and rules regarding logarithms may appear to be similar to those for exponents. However, logarithms are defined only when the base b and its arguments are positive.

1. $\log_b(m \cdot n) = \log_b m + \log_b n$
2. $\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$
3. $\log_b(m^n) = n \cdot \log_b m$
4. $\log_b 1 = 0$
5. $\log_b m = \frac{1}{\log_m b}$
6. $\log_b m = \frac{\log_a m}{\log_a b}$, for any $a > 0$

Example 1.9

Let's revisit Example 1.8, but this time we'll use the first rule of logarithms. What is the magnitude of an earthquake R_1 that is twice as strong as another of magnitude $R = 3$?

Solution

Comparing the relative magnitudes, the equation to be solved for R_1 is:

$$2 = \frac{10^{R_1}}{10^3}$$

$$\Rightarrow 2 \cdot 10^3 = 10^{R_1}$$

$\Rightarrow R_1 = \log(2 \cdot 10^3)$ and this time, by using the first rule of logarithms:

$$\Rightarrow R_1 = \log 2 + \log 10^3$$

$\Rightarrow R_1 = \log 2 + 3$ and since $\log 2 \approx 0.301$

$$\Rightarrow R_1 \approx 3.301$$

It is worth noting that regardless of any other magnitude to which one compares an earthquake, an earthquake that is twice as strong has a magnitude that is greater by $R \approx 0.301$.

Exercise 1.5

1. Determine the value of each of the following.

(a) $\log_2 16$ (b) $\log_{16} 2$ (c) $\log_{\sqrt{2}} 16$

(d) $\log_2 \sqrt{2}$ (e) $\log_2(-16)$ (f) $\log_2 2\sqrt{2}$

(g) $\log_{\sqrt{2}} 2\sqrt{2}$ (h) $\log_{2\sqrt{2}} 2$ (i) $\log 4 + \log 25$

(j) $\log 30 - \log 300$ (k) $\ln\left(\frac{1}{e^2}\right)$ (l) $\ln\sqrt{e}$

2. Simplify each expression.

(a) $\log_a a^3$ (b) $\log_a \sqrt{a}$ (c) $\log_{\sqrt{a}} a\sqrt{a}$

(d) $\log_{\sqrt{a}} \sqrt[3]{a}$ (e) $\log_{a^2} a^3$ (f) $\log_{a^2} \sqrt{a}$

(g) $\log_{a^2} \sqrt[3]{a}$ (h) $\log_{a^2} a\sqrt{a}$ (i) $\log_a a^{-3} + \log_a a^4$

(j) $\log_{a^2} a^{-3} + \log_{a^2} a^4$ (k) $\log_a a^3 - \log_a a^2$ (l) $\log_a a^3 - \log_a \sqrt{a}$

3. Solve each equation for x.

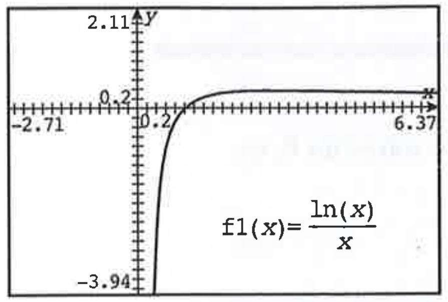
(a) $\log_3 x = -2$ (b) $\log_2(x - 3) = 5$

(c) $\log_x 3 = -2$ (d) $\log_3(x^2 + 2x + 1) = 0$

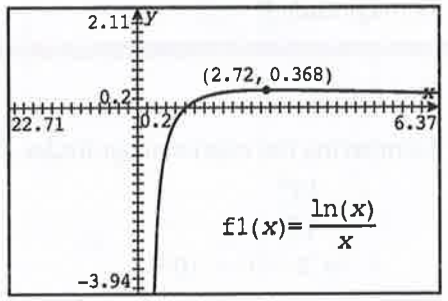
Optional

Which is larger, e^{π} or π^{e}?

The numerical values of e^{π} and π^{e} were calculated in Exercise 1.3. Now, consider an analysis using the graph of $f(x) = \frac{\ln x}{x}$



with its maximum at



Note that the maximum is found at $x = e$ (which can also be found using calculus later), and π is further to its right, below the maximum.

Hence, $f(e) > f(\pi) \Rightarrow \frac{\ln e}{e} > \frac{\ln \pi}{\pi}$

Now, $\ln e = 1 \Rightarrow \frac{1}{e} > \frac{\ln \pi}{\pi}$

$\pi > e\ln \pi$

$\pi > \ln \pi^{e}$

Then $e^{\pi} > e^{\ln \pi^{e}}$

But $e^{\ln \pi^{e}} = \pi^{e}$

And so $e^{\pi} > \pi^{e}$

Chapter 1 practice questions

1. Determine the number of significant figures in each of the following.

(a) 2308 **(b)** 2300 **(c)** 1000 **(d)** 4000.9

(e) 570.03 **(f)** 0.0003 **(g)** 0.00320

2. Give each value correct to 3 significant figures.

(a) 58 261 **(b)** 6107 **(c)** 123 807

(d) 1.618 **(e)** 0.003051 **(f)** 400.01

3. Use the rules of exponents to write each of the following in the form 2^n, where $n \in \mathbb{Q}$.

(a) $2^6 \cdot 2^2$ **(b)** $2^8 \cdot 2^{-2}$ **(c)** $\frac{2^9}{2^3}$ **(d)** $\frac{2^{-6}}{2^3}$

(e) $(2^3)^2$ **(f)** $(2^{-3})^{-2}$ **(g)** $4^4 \cdot 2^2$ **(h)** $4^4 \cdot 16^4$

(i) $8^6 \cdot 16^{-3}$ **(j)** $\sqrt{8}$ **(k)** $\sqrt{8} \cdot 4\sqrt{2}$ **(l)** $\sqrt[3]{4} \cdot 2\sqrt[3]{16}$

(m) $\sqrt{2} \cdot \sqrt[3]{2}$ **(n)** $\sqrt{2} \cdot \sqrt[4]{8}$ **(o)** $16^{-2} \cdot (4\sqrt{8})^2$ **(p)** $\left(\frac{\sqrt{2}}{2}\right)^{-3}$

(q) $4^2 \cdot 2^{\frac{1}{2}}$ **(r)** $\sqrt[3]{16} \cdot \sqrt[4]{4}$ **(s)** $\sqrt{8} \cdot 2^{\frac{3}{2}}$ **(t)** $2^{\frac{1}{3}} \cdot \sqrt[3]{4}$

(u) $\frac{2}{\sqrt{2}}$ **(v)** $\frac{2^{\frac{1}{3}}}{\sqrt[3]{64}}$ **(w)** $\frac{2^{\frac{2}{3}}}{\sqrt[3]{4}}$ **(x)** $\frac{4^{-\frac{1}{3}}}{\sqrt[3]{2}}$

4. Express each number in scientific notation.

(a) 52 270 **(b)** 13.1401 **(c)** 0.0000604

(d) 0.0009 **(e)** 0.0090 **(f)** 32.001

(g) 500 003 **(h)** 100.00 **(i)** 1 μm

5. State each value in scientific notation, correct to 3 significant figures.

(a) 10^{18} **(b)** e^e

(c) one nanosecond **(d)** $\frac{1 + \sqrt{5}}{2}$

6. Write each equation in logarithmic form.

(a) $243 = 3^5$ **(b)** $256 = 2^8$

(c) $100^{\frac{1}{2}} = 10$ **(d)** $64^{\frac{1}{6}} = 2$

(e) $9\sqrt{3} = 3^{\frac{5}{2}}$ **(f)** $10^{-3} = 0.001$

(g) $e^0 = 1$ **(h)** $5^{-3} = \frac{1}{125}$

(i) $(3\sqrt{3})^{-2} = \frac{1}{27}$ **(j)** $8^{-\frac{1}{2}} = \frac{1}{2\sqrt{2}}$

(k) $\left(\frac{1}{4}\right)^{-3} = 64$ **(l)** $27^{-\frac{1}{2}} = \frac{\sqrt{3}}{9}$

(m) $(-2)^{-3} = -\frac{1}{8}$ **(n)** $(0.1)^{-2} = 100$

(o) $\left(\frac{\sqrt{3}}{3}\right)^3 = \frac{\sqrt{3}}{9}$ **(p)** $\left(\frac{1}{\sqrt{2}}\right)^{-3} = 2\sqrt{2}$

7. Express each equation with x as the subject.

(a) $y = 5^x$ **(b)** $y = 10^x$

(c) $y = e^x$ **(d)** $y = 2^{2x}$

(e) $y = 3 \cdot 3^x$ **(f)** $y = 7 + 3^x$

(g) $y = 2^{-2x}$ **(h)** $y = 2^{\frac{x}{3}}$

(i) $y = e^{\frac{x}{2}}$ **(j)** $y = 5^{x+3}$

(k) $y = e^{x-1}$ **(l)** $y = \frac{1}{e^{2x}}$

8. Determine the value of each of the following.

(a) $\log_3 243$ **(b)** $\log_{243} 3$

(c) $\log_{\frac{1}{2}} 16$ **(d)** $\log_3 3\sqrt{3}$

(e) $\log(-16)$ **(f)** $\log_4 2\sqrt{2}$

(g) $\log 50 + \log 20$ **(h)** $\log 4000 - \log 4$

(i) $\ln e^{-2}$ **(j)** $\ln\left(\frac{1}{\sqrt{e}}\right)$

9. Write down the value of:

(a) $2\log_{64} 8$ **(b)** $\log_8 \sqrt{8}$

(c) $\log_{\sqrt{3}} 3\sqrt{3}$ **(d)** $\log_{\sqrt{16}} \sqrt[3]{16}$

(e) $\log_{\sqrt{8}} 8^3$ **(f)** $\log_{5^2} \sqrt{5}$

(g) $\log_9 9^{-3} + \log_3 9^4$ **(h)** $\log_{\sqrt{8}} 2^{-3} + \log_8 4^4$

(i) $\log_{2\sqrt{2}} 4^3 - \log_{\sqrt{2}} 2$ **(j)** $\log_{\frac{1}{3}} 3 - \log_3 \sqrt{3}$

10. Solve each equation for x.

(a) $\log_5 x = -3$

(b) $\log_x \frac{1}{4} = -2$

(c) $\log_3 (x^2 - 2x - 5) = 1$

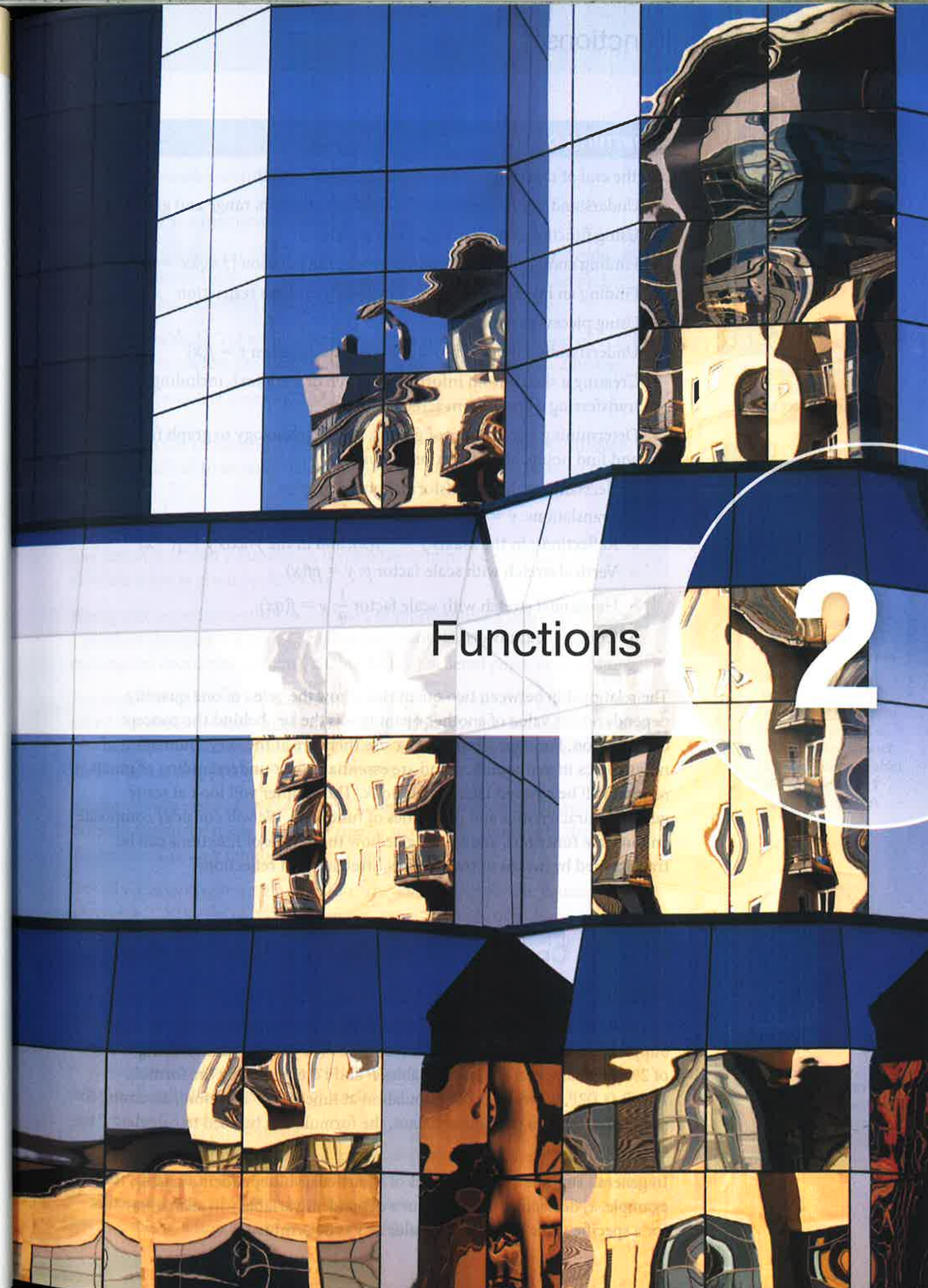
11. Given that $2^m = 8$ and $2^n = 16$,

(a) write down the value of m and of n

(b) hence or otherwise solve $8^{2x+1} = 16^{2x-3}$

12. Consider $a = \log_2 3 \times \log_3 4 \times \log_4 5 \times \ldots \times \log_{31} 32$
Given that $a \in \mathbb{Z}$, find the value of a.

13. Given that $\log_x y = 4\log_y x$, find all the possible expressions of y as a function of x.



Functions 2