

**Mathematics: applications and interpretation**  
**Higher level**  
**Mock paper 1**

Monday 30 May 2022 (morning)

Candidate session number

2 hours

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**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**



Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1.

$$T = \frac{(\tan(2z) + 1)(2\cos(z) - 1)}{y^2 - x^2}$$

Where  $x=9$ ,  $y=41$  and  $z = 30^\circ$

- Calculate the exact value of T [2]
- Give the value of T with 2 sf and with 3 sf [2]
- An engineering student at Umeå University estimates T to be 0.002. Calculate the percentage error in this estimation. [2]

Working.

Answers

a)

b)

c)

2.

These are the grades of a group of IB students:

6 4 5 3 7 3 5 4 2 5

a) Find the mean [2]

b) Find the relative standard deviation [3]

b) Find the interquartile range [2]

Working.

Answers

a)

b)

c)

3.

Assuming that Earth is a sphere with a radius of 6371 Km:

a) Calculate the volume of the Earth in  $\text{Km}^3$ .

Give your answer in the form of  $a \cdot 10^k$ , where  $1 \leq a < 10$  and  $k \in \mathbb{Z}$  [3]

b) The volume of the Moon is  $2.1958 \cdot 10^{10} \text{ Km}^3$ . Calculate how many times greater the volume of the Earth is compared to that of the Moon. Give your answer correct to the nearest integer [3]

Working.

Answers

a)

b)

4.

Pietro arrives in Singapore and, at the airport, changes 800 euros ( EUR ) to Singapore dollars ( SGD ).

The bank rates quoted at the airport for exchanging EUR with SGD are given in the following table. Also given are the rates for exchanging SGD with British pounds ( GBP ) and US dollars ( USD ). There is no commission charged on exchanges.

Bank buys	Bank sells
1 EUR = 1.55 SGD	1 EUR = 1.75 SGD
1 GBP = 1.92 SGD	1 GBP = 2.05 SGD
1 USD = 1.15 SGD	1 USD = 1.28 SGD

a) Calculate the number of SGD Pietro receives. [2]

b) Pietro also has 100 GBP that he wishes to change to USD for a trip to Cambodia. To perform this transaction, the GBP must first be converted to SGD and then to USD . Calculate the number of USD Pietro receives. [4]

Working.

Answers

a)

b)

5.

The second term of an arithmetic sequence is 30 . The fifth term is 90 .

(a) Calculate [3]

- (i) the common difference of the sequence.
- (ii) the first term of the sequence.

The first, second and fifth terms of this arithmetic sequence are the first three terms of a geometric sequence.

(b) Calculate the seventh term of the geometric sequence. [3]

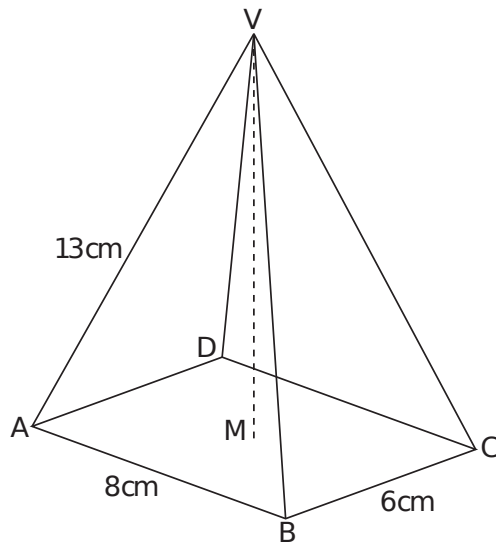
Working.

Answers

a)

b)

6. A right pyramid has apex  $V$  and rectangular base  $ABCD$ , with  $AB = 8\text{ cm}$ ,  $BC = 6\text{ cm}$  and  $VA = 13\text{ cm}$ . The vertical height of the pyramid is  $VM$ .



a) Calculate  $VM$  [4]

b) Calculate the volume of the pyramid [2]

Working.

Answers

a)

b)

7. Pierre invests 5000 euros in a fixed deposit that pays a nominal annual interest rate of 4.5 % , compounded monthly, for seven years.

(a) Calculate the value of Pierre's investment at the end of this time. Give your answer correct to two decimal places. [3]

Carla has 7000 dollars to invest in a fixed deposit which is compounded annually. She aims to double her money after 10 years.

(b) Calculate the minimum annual interest rate needed for Carla to achieve her aim. [3]

Working.

Answers

a)

b)



8. An iron bar is heated. Its length,  $L$ , in millimetres can be modelled by a linear function,  $L = mT + c$ , where  $T$  is the temperature measured in degrees Celsius ( $^{\circ}\text{C}$ ).

At  $150^{\circ}\text{C}$  the length of the iron bar is  $180\text{ mm}$ .

(a) Write down an equation that shows this information. [1]

At  $210^{\circ}\text{C}$  the length of the iron bar is  $181.5\text{ mm}$ .

(b) Write down an equation that shows this second piece of information. [1]

(c) Hence, find the length of the iron bar at  $40^{\circ}\text{C}$ . [4]

Working.

Answers

a)

b)

c)

9. Consider the quadratic function,  $f(x) = px \cdot (q - x)$   
where  $p$  and  $q$  are positive integers.

The graph of  $y = f(x)$  passes through the point  $(6, 0)$ .

(a) Calculate the value of  $q$ . [2]

The vertex of the function is  $(3, 27)$ .

(b) Find the value of  $p$ . [2]

(c) Write down the range of  $f$ . [2]

Working.

Answers

a)

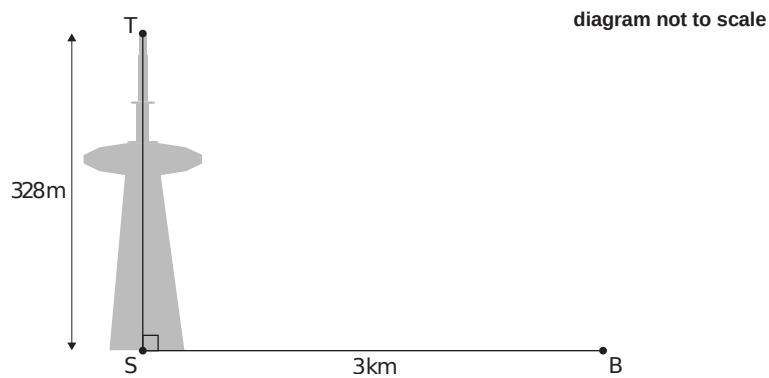
b)

c)

10. Complete the table below placing a tick (✓) to show which of the number sets  $\mathbb{N}$ ,  $\mathbb{Z}$ ,  $\mathbb{Q}$ , and  $\mathbb{R}$  these numbers belong to. The first row has been completed as an example. [6]

	$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}$
0.8			✓	✓
$\sqrt{14}$				
$\sin 30$				
4				
-3				
$4.12 \cdot 10^1$				
$3\frac{1}{3}$				

11. From point B, a vertical tower can be seen. Point B is 3 kilometres, horizontally, from point S at the base of the tower. The height of the tower, TS, is 328 metres.



- Write down the height of the tower, TS, in kilometres. [1]
- On the diagram, label the angle of elevation from B to T with an  $x$ . [1]
- Find the size of the angle of elevation from B to T. [2]
- Find TB, the distance in kilometres from the top of the tower to point B. [2]

Working.

Answers

- 
- 
- 
-

12. At the end of 2008, the population of the city of Nouméa was 97 579 .  
A year later, at the end of 2009 , the population of Nouméa was 99 921 .

(a) Find the percentage change in the population from the end of 2008 to the end of 2009 . [2]  
Assume that the population of Nouméa continues to grow at the same percentage rate.

(b) Use your answer to part (a) to estimate the population at the end of 2016 . [2]

(c) Write your answer to part (b) in the form  $a \times 10^k$  , where  $1 \leq a < 10$  ,  $k \in \mathbb{Z}$  . [2]

Working.

Answers

a)

b)

c)

13. Consider the geometric sequence 2048 , 1536 , 1152 , 864 ...

(a) Find the common ratio,  $r$  . [1]

(b) Write down the next term of the sequence,  $u_5$  . [1]

(c) Find the largest term in the sequence that is not an integer. [2]

(d) Find the sum of the first 20 terms. [2]

Working.

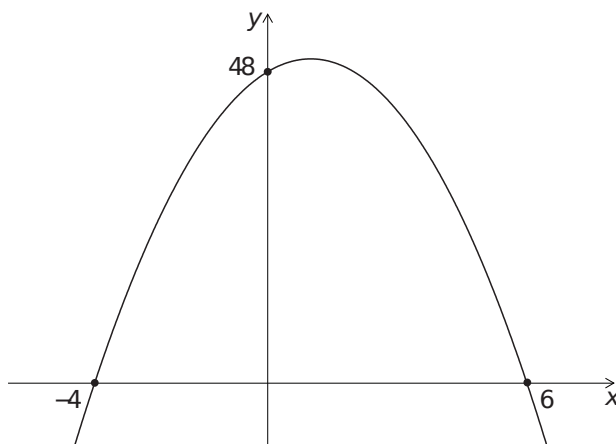
Answers

a)

b)

c)

14. The graph shows the curve of a quadratic function of the form  $f(x) = ax^2 + bx + 48$



(a) Write down the equation for the axis of symmetry of the curve. [2]

(b) Hence, or otherwise, find the value of  $a$  and of  $b$ . [3]

(c) Find the  $y$ -coordinate of the vertex of the curve. [1]

Working.

Answers

a)

b)

c)

15. In an experiment, the number of bacteria,  $N(t)$ , is modelled by the function,  $N(t) = 25 \cdot (8)^t$   
Where  $t$  is the time, in hours, since the start of the experiment, and  $t \geq 0$ .

(a) Write down the number of bacteria at the start of the experiment. [1]

(b) Find the number of bacteria after 1 hour. [1]

(c) Calculate how long it will take for the number of bacteria to exceed five million.  
Give your answer to the nearest hour. [4]

Working.

Answers

a)

b)

c)

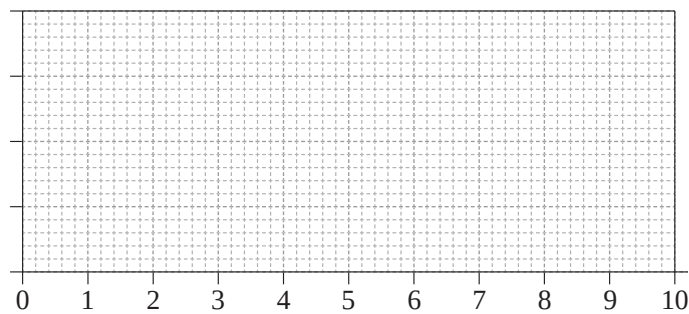
16. Two groups of 40 students were asked how many books they have read in the last two months. The results for the first group are shown in the following table.

Number of books	Frequency
2	5
3	8
4	13
5	7
6	4
7	2
8	1

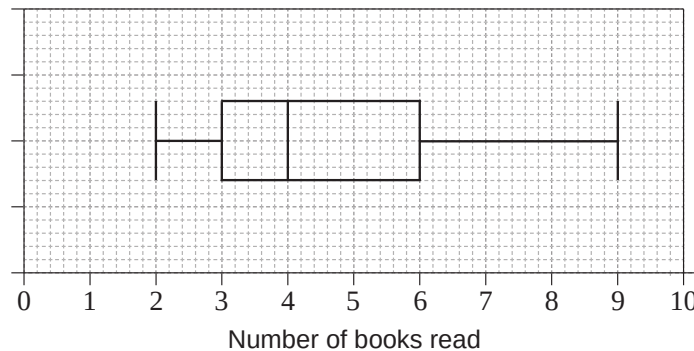
The quartiles for these results are 3 and 5.

(a) Write down the value of the median for these results. [1]

(b) Draw a box-and-whisker diagram for these results on the following grid. [3]



The results for the second group of 40 students are shown in the following box-and-whisker diagram.



(c) Estimate the number of students in the second group who have read at least 6 books. [2]

Working.

Answers

a)

c)



	c)
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**Mathematics: applications and interpretation**  
**Higher level**  
**Mock paper 2**

Monday 30 May (afternoon)

2 hours

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**Instructions to candidates**

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**

1. Engineers at Umeå University are launching an experimental model rocket. The rocket has foldable wings that allow it to glide when a button is pressed. The trajectory of the rocket is initially that of the function

$$f(x) = \frac{-1}{80}x^2 + x$$

a) Find the coordinates of the point of maximum height. [3]

At any point after the maximum height has been reached, the engineer will press the glide button and from that moment on the rocket will go on with a straight trajectory.

b) What will be the equation of the line describing the trajectory if the button is pressed when  $x$  is 60? [3]

c) If the button is pressed when  $x$  is 50 and the rocket is allowed to glide onwards, where will it land? [5]

2. The temperature of a small lake varies along the year. Because of the large mass of water, we can say it is not affected by daily changes, so it oscillates periodically every year. The minimum temperature is 1 °C, and it's reached in day 40 of the year. The maximum temperature is 19 °C. (note: assume that the year has 360 days).

a) Write an equation that models the temperature along the year. [3]

b) An IB student measures the temperature on day 120 and again in day 205. What temperatures will she measure if the model is correct? [2]

c) The student assumes that the temperature behaves linearly rather than periodically, so she makes a linear model. What model will she make? [2]

d) According to this model, how long before the temperature of the lake reaches 37 °C? [3]

3. Antonio and Barbara start work at the same company on the same day. They each earn an annual salary of 8000 euros during the first year of employment. The company gives them a salary increase following the completion of each year of employment. Antonio is paid using plan A and Barbara is paid using plan B.

Plan A: The annual salary increases by 450 euros each year.

Plan B: The annual salary increases by 5 % each year.

a) Calculate [3]

i) Antonio's annual salary during his second year of employment;

ii) Barbara's annual salary during her second year of employment.

b) Write down an expression for [4]

i) Antonio's annual salary during his  $n$ th year of employment;

ii) Barbara's annual salary during her  $n$ th year of employment.

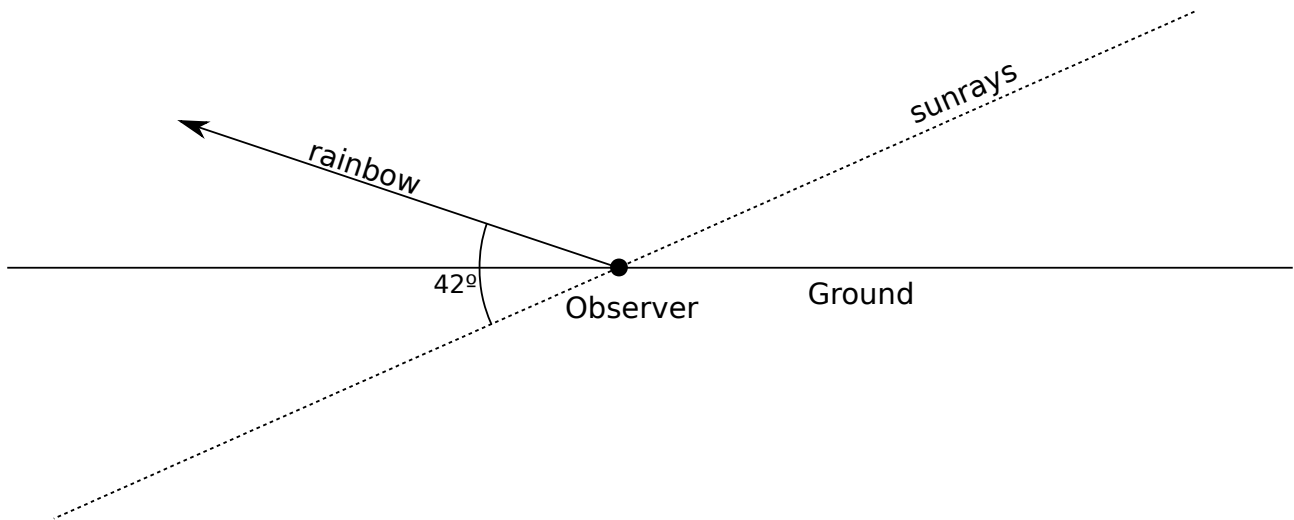
c) Determine the number of years for which Antonio's annual salary is greater than or equal to Barbara's annual salary.[2]

Both Antonio and Barbara plan to work at the company for a total of 15 years.

(d) [7]

- i) Calculate the total amount that Barbara will be paid during these 15 years.
- ii) Determine whether Antonio earns more than Barbara during these 15 years.

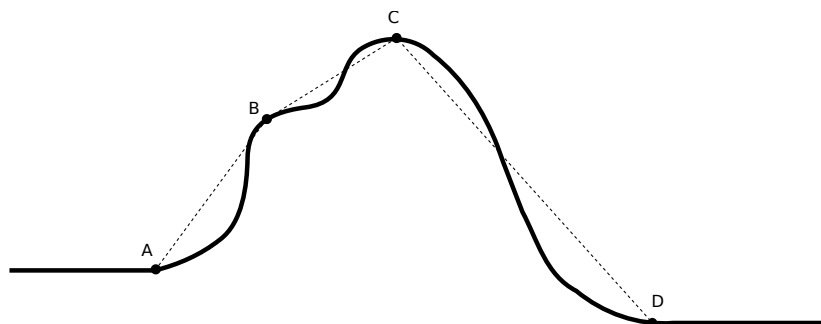
4. The rainbow appears with an angle of  $42^\circ$  above the direction of sunlight, as seen in the picture (the observer is simplified to a point).



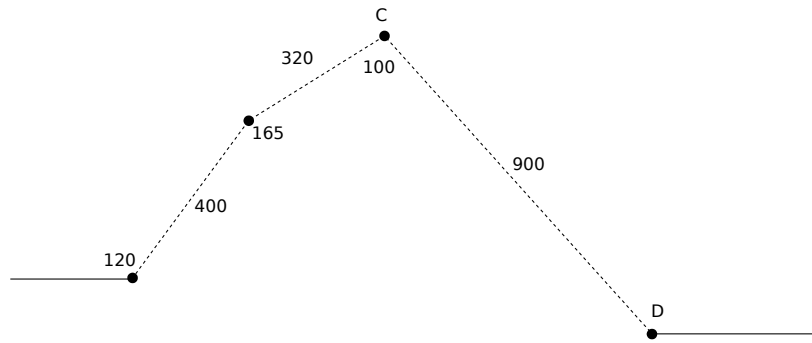
Marco is studying the rainbow phenomenon and has a water curtain to artificially reproduce it. His house is surrounded by trees that are about 22 metres tall, and at a distance of 140 m. When the sun sets behind the trees, the rainbow will disappear.

What is the highest angle of elevation he will be able to observe the rainbow at? [6]

5. Engineers from Umeå University are surveying the land in order to plan for a tunnel. The mountain is part of a range, so they can't go around it, so they decide they'll have to resort to *math*. They set four checkpoints on the mountain as shown in the picture.



The angles and distances are as shown in the following picture.



- a) Calculate the distance from A to D [8]
- b) Calculate the slope of the tunnel if digged directly from A to D. [4]

6. These are the weights (in grams) of garlic cloves from the subspecies A:

- |      |                                    |
|------|------------------------------------|
| 3.81 | Find:                              |
| 3.43 |                                    |
| 3.88 | a) Average weight[1]               |
| 3.51 |                                    |
| 3.85 | b) Median weight[1]                |
| 3.61 |                                    |
| 3.73 | c) Variance [1]                    |
| 3.76 |                                    |
| 3.71 | d) Standard deviation [1]          |
| 3.56 |                                    |
|      | e) Relative standard deviation [1] |

These are the weights of garlic cloves from subspecies B.

- |      |   |
|------|---|
| 3.99 | a) Compare the averages and the ranges of the two garlic subspecies. [2]  |
| 3.91 |   |
| 3.62 |   |
| 3.57 |   |
| 4.01 |   |
| 3.67 |   |
| 3.97 |   |
| 3.52 | b) Discuss whether or not the garlic cloves of subspecies B are bigger or smaller than the ones of subspecies A [3] |
| 3.78 |   |
| 3.95 |   |

7. Here is a table with the number of doctor's appointments each student in a school has.

appointments	students	Calculate: (give your answers with 3sf)
1	2	
2	15	a) The overall number of students [1]
3	48	
4	133	b) The average number of appointments students have [2]
5	93	
6	41	c) The standard deviation in the number of appointments [2]
7	16	
8	6	d) The variance in the number of appointments [2]
9	2	
10	1	e) The mode in the number of appointments [1]

8. 4. Here are the total distance (in thousands of Km) different cars of a given model are driven before they are thrown for scrap.

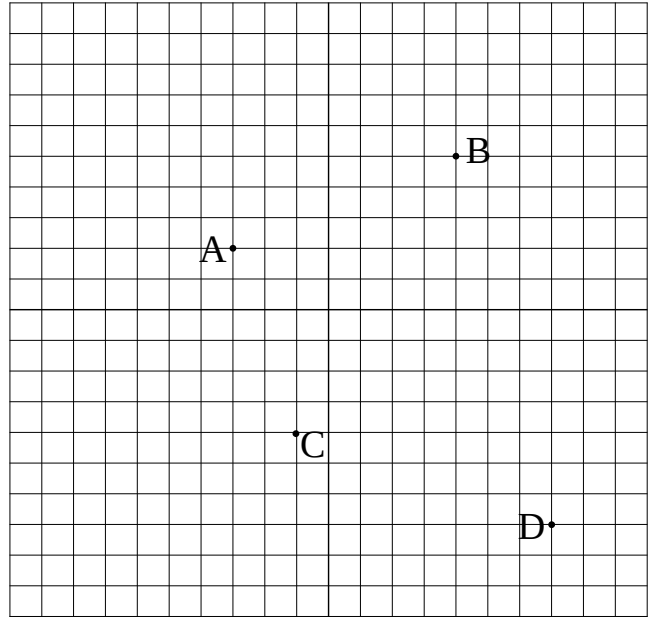
230	246	204	217	271	a) Group them by total distance (make at least 5 groups) [3]
206	232	248	284	251	
231	242	276	290	213	
261	254	229	243	258	
242	228	218	263	221	
256	237	248	241	263	
234	271	289	253	257	
268	212	244	246	235	
288	220	290	208	227	
230	222	237	221	239	

b) Draw a histogram [2]



9. This the location of four hospitals in a city.

- a) Draw the Voronoi diagram (calculate and show the equations of two of them and sketch the rest). [6]
- b) Explain the meaning of the area around each hospital. [3]
- c) Discuss whether a Voronoi diagram is the best way to solve the issue of hospital proximity. [3]



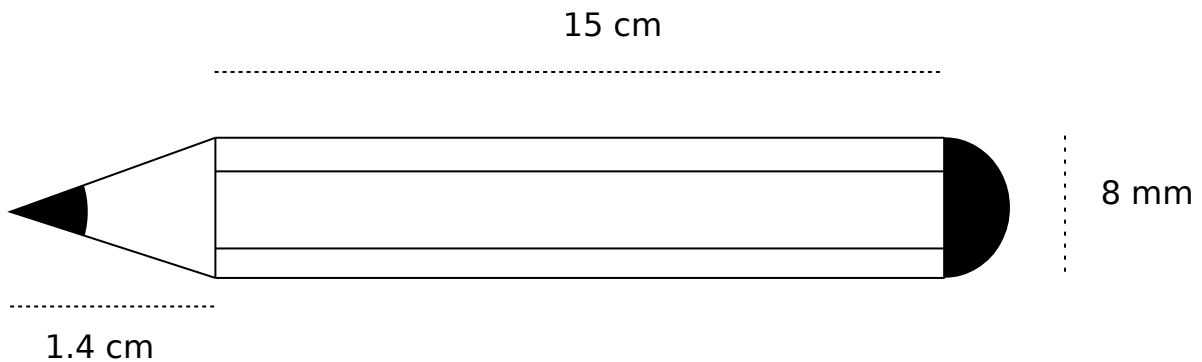
10. A lime (that we can consider spherical) is cut into four equal pieces. If the diameter of the lime was 5.5 cm.

- a) Calculate the volume of the lime.[2]
- b) Calculate the surface of each piece of lime [4]

Limes are often cut in order to give flavour to a drink. This is because the outer skin of the lime doesn't release any flavour.

- c) Calculate how much more flavour it will release if cut into 8 equal pieces compared to what it releases cut in 4 equal pieces (give your answer in relative terms, as in "three times as much" or "30% more") [6]

11. Calculate the volume of the pencil in the picture assuming that the shaft is a hexagonal prism.



[12]