13 Prior learning

CHAPTER OBJECTIVES:

This chapter contains short explanations, examples and practice exercises on topics that you should know before starting the course. You do not need to work through the whole of this chapter in one go. You can dip into it for help when you need it.

For example, before you start Chapter 2 Descriptive Statistics, work through Section 4 Statistics in this chapter.

Chapter contents

1 Number

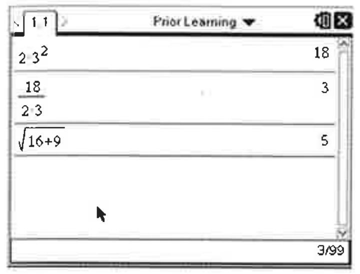
1.1 Calculation

There are several versions of the rules for the order of operations. They all amount to the same thing:

- Brackets or parentheses are calculated first.
- Next come exponents, indices or orders.
- Then multiplication and division, in order from left to right.
- Additions and subtractions, in order from left to right.

A fraction line or the line above a square root counts as a bracket too.

Your GDC follows the rules, so if you enter a calculation correctly you should get the correct answers.



GDC help on CD: *Alternative demonstrations for the TI-84 Plus and Casio FX-9860GII GDCs are on the CD.*

The GDC shows divisions as fractions, which makes the order of operations clearer.

BEDMAS: Brackets, exponents, division, multiplication, addition, subtraction.
BIDMAS: Brackets, indices, division, multiplication, addition, subtraction.
BEMDAS: Brackets, exponents, multiplication, division, addition, subtraction.
BODMAS: Brackets, orders, division, multiplication, addition, subtraction.
BOMDAS: Brackets, orders, multiplication, division, addition, subtraction.
PEMDAS: Parentheses, exponents, multiplication, division, addition, subtraction.

Simple calculators, like the ones on phones, do not always follow the calculation rules.

Example 1

a Evaluate $\frac{11+(-1)^2}{4-(3-5)}$

$= \frac{11+1}{4-(-2)}$ *Brackets first*

$= \frac{12}{6}$ *Simplify numerator and denominator.*

$= 2$

b Evaluate $\frac{-3+\sqrt{9-8}}{4}$

$= \frac{-3+\sqrt{1}}{4}$ *Simplify the terms inside the square root.*

$= \frac{-3+1}{4}$ *Evaluate the root.*

$= \frac{-2}{4}$ *Simplify the numerator and denominator.*

$= -\frac{1}{2}$

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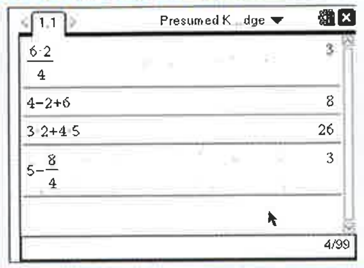
On your GDC you can either use templates for the fractions and roots or you can use brackets.



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Exercise 1A

Do the questions by hand first, then check your answers with your GDC.

1 Calculate

a $12 - 5 + 4$ **b** $6 \div 3 \times 5$ **c** $4 + 2 \times 3 - 2$

d $8 - 6 \div 3 \times 2$ **e** $4 + (3 - 2)$ **f** $(7 + 2) \div 3$

g $(1 + 4) \times (8 - 4)$ **h** $1 - 3 + 5 \times (2 - 1)$

2 Find

a $\frac{6+9}{4-1}$ **b** $\frac{2 \times 9}{3 \times 4}$ **c** $\frac{2-(3+4)}{4 \times (2-3)}$ **d** $\frac{6 \times 5 \times 4}{3 \times 2 - 1}$

3 Determine

a $3 \times (-2)^2$ **b** $2^2 \times 3^3 \times 5$ **c** $4 \times (5-3)^2$ **d** $(-3)^2 - 2^2$

4 Calculate

a $\sqrt{3^2 + 4^2}$ **b** $\left(\sqrt{4}\right)^3$ **c** $\sqrt{4^3}$ **d** $\sqrt{2 + \sqrt{2 + 2}}$

5 Find

a $\sqrt{\frac{13^2 - (3^2 + 4^2)}{2 \times 18}}$ **b** $2\sqrt{\frac{3 + 5^2}{7}}$ **c** $2(3^2 - 4(-2)) - (2 - \sqrt{7 - 3})$

1.2 Primes, factors and multiples

A **prime** number is an integer, greater than 1, that is not a multiple of any other number apart from 1 and itself.

In 2009, the largest known prime was a 12 978 189-digit number. Prime numbers have become big business because they are used in cryptography.

Example 2

List all the factors of 42.

Answer	
$42 = 1 \times 42$, $42 = 2 \times 21$, $42 = 3 \times 14$, $42 = 6 \times 7$ The factors of 42 are 1, 2, 3, 6, 7, 14, 21 and 42.	*Write 42 as a product of two numbers every way you can.*

Example 3

Write the number 24 as a product of prime factors.

Answer	
2)24 $24 = 2 \times 2 \times 2 \times 3$ 2)12 $= 2^3 \times 3$ 2)6 3)3 1	*Begin dividing by the smallest prime number. Repeat until you reach an answer of 1.*

Example 4

Find the **lowest common multiple** (LCM) of 12 and 15.

Answer	
The multiples of 12 are 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144, ... The multiples of 15 are 15, 30, 45, 60, 75, 90, 105, 120, 135, ... The common multiples are 60, 120, ... The LCM is 60.	*List all the multiples until you find some in both lists. The LCM is the smallest number in each of the lists.*

Example 5

Find the **highest common factor** (HCF) of 36 and 54.

Answer

2)36 $36 = 2 \times 2 \times 3 \times 3$ 2)54 $54 = 2 \times 3 \times 3 \times 3$

2)18   3)27

3)9   3)9

3)3   3)3

1   1

The HCF of 36 and 54 is $2 \times 3 \times 3 = 18$.

Write each number as a product of prime factors. Find the product of all the factors that are common to both numbers.

Exercise 1B

1 List all the factors of
  a 18 b 27 c 30 d 28 e 78

2 Write as products of prime factors.
  a 36 b 60 c 54 d 32 e 112

3 Find the LCM of
  a 8 and 20 b 6, 10 and 16

4 Find the HCF of
  a 56 and 48 b 36, 54 and 90

1.3 Fractions and decimals

There are two types of fraction:

- **common** fractions (often just called 'fractions') like $\frac{4}{5}$ $\frac{\text{numerator}}{\text{denominator}}$
- **decimal** fractions (often just called 'decimals') like 0.125.

Fractions can be:

- **proper** like $\frac{2}{3}$ where the numerator is less than the denominator
- **improper** like $\frac{4}{3}$ where the numerator is greater than the denominator
- **mixed numbers** like $6\frac{7}{8}$.

Fractions where the numerator and denominator have no common factor are in their **lowest terms**.

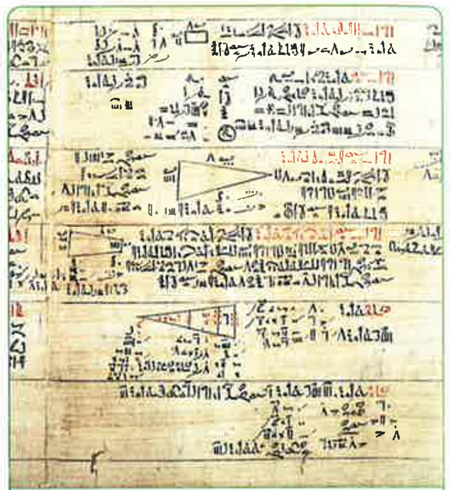
$\frac{1}{3}$ and $\frac{4}{12}$ are **equivalent** fractions.

0.675 is a **terminating** decimal.

0.32... or $0.\overline{32}$ or $0.\dot{3}\dot{2}$ are different ways of writing the **recurring** decimal 0.323 232 323 2...

Non-terminating, non-recurring decimals are **irrational** numbers, like π or $\sqrt{2}$.

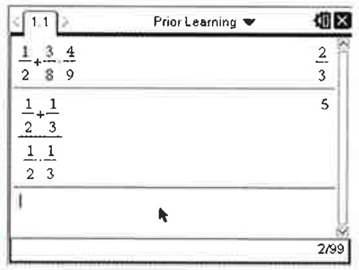
Using a GDC, you can either enter a fraction using the fraction template $\frac{\square}{\square}$ or by using the divide key ÷. Take care – you will sometimes need to use brackets.



The Rhind Papyrus from ancient Egypt in around 1600 BCE shows calculations using fractions. Egyptians used **unit** fractions, so for $\frac{4}{5}$ they would write $\frac{1}{2}+\frac{1}{4}+\frac{1}{20}$. This is not generally regarded as a very helpful way of writing fractions.

$\pi \approx$ 3.14159265358979323846264338327950288419716939937 5...
$\sqrt{2} \approx$ 1.41421356237309504880168872420969807856967187537...
They do not terminate and there are no repeating patterns in the digits.

Example 6

a Evaluate $\frac{1}{2}+\frac{3}{8}\times\frac{4}{9}$	× *before* +.
$=\frac{1}{2}+\frac{1}{6}$	
$=\frac{4}{6}$	*Simplify.*
$=\frac{2}{3}$	
b Evaluate $\dfrac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{2}\times\frac{1}{3}}$	*Evaluate the numerator and denominator first.*
$=\dfrac{\frac{5}{6}}{\frac{1}{6}}$	
$=5$	

GDC help on CD: *Alternative demonstrations for the TI-84 Plus and Casio FX-9860GII GDCs are on the CD.*

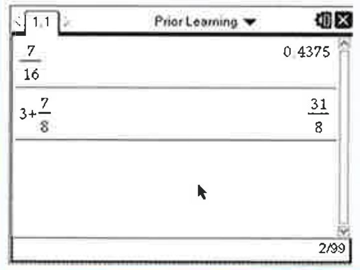
Example 7

a Convert $\frac{7}{16}$ to a decimal. **b** Write $3\frac{7}{8}$ as an improper fraction.

Answers

a $\frac{7}{16}=0.4375$

b $3\frac{7}{8}=\frac{24}{8}+\frac{7}{8}$
$=\frac{31}{8}$



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Exercise 1C

1 Calculate

  a $\frac{1}{2}+\frac{3}{4}\times\frac{5}{9}$ b $\frac{2}{3}\div\frac{5}{6}\times 1\frac{1}{3}$

  c $\sqrt{\left(\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2}$ d $\dfrac{1-\left(\frac{2}{3}\right)^5}{1-\frac{2}{3}}$

2 Write the following fractions in their lowest terms.

a $\frac{16}{36}$ **b** $\frac{35}{100}$ **c** $\frac{34}{51}$ **d** $\frac{125}{200}$

3 Write these mixed numbers as improper fractions.

a $3\frac{3}{5}$ **b** $3\frac{1}{7}$ **c** $23\frac{1}{4}$ **d** $2\frac{23}{72}$

4 Write these improper fractions as mixed numbers.

a $\frac{32}{7}$ **b** $\frac{100}{3}$ **c** $\frac{17}{4}$ **d** $\frac{162}{11}$

5 Convert to decimals.

a $\frac{8}{25}$ **b** $\frac{5}{7}$ **c** $3\frac{4}{5}$ **d** $\frac{45}{17}$

There are some useful tools for working with fractions. Look in menu 2:Number.

To convert a fraction to a decimal, divide the numerator by the denominator. Pressing ctrl ≈ will give the result as a decimal instead of a fraction.

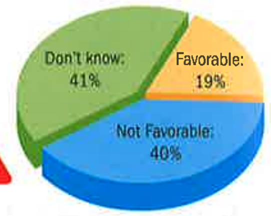
1.4 Percentages

A percentage is a way of expressing a fraction or a ratio as part of a hundred.

For example 25% means 25 parts out of 100.

As a fraction, $25\% = \frac{25}{100} = \frac{1}{4}$.

As a decimal, 25% = 0.25.



Example 8

Lara's mark in her Mathematics test was 25 out of 40. What was her mark as a percentage?

Answer	
$\frac{25}{40} \times 100 = 62.5\%$	*Write the mark as a fraction. Multiply by 100. Use your GDC.*

Example 9

There are 80 students taking the IB in a school. 15% take Mathematical Studies. How many students is this?

Answer	
Method 1	
$\frac{15}{100} \times 80 = 12$	*Write the percentage as a fraction out of a hundred and then multiply by 80.*
Method 2	
15% = 0.15 0.15 × 80 = 12	*Write the percentage as a decimal. Multiply by 80.*

International currencies

Questions in the Mathematical Studies examination may use international currencies. For example: Swiss franc (CHF); US dollar (USD); British pound sterling (GBP); euro (EUR); Japanese yen (JPY) and Australian dollar (AUD).

Exercise 1D

1 Write as percentages

a 13 students from a class of 25

b 14 marks out of 20

2 Find the value of

a 7% of 32 CHF

b $4\frac{1}{2}$% of 12.00 GBP

c 25% of 750.28 EUR

d 130% of 8000 JPY

7% = 0.07

Percentage increase and decrease

Consider an increase of 35%.

The new value after the increase will be 135% of the original value.

So, to increase an amount by 35%, find 135% of the amount.

Multiply by $\frac{135}{100}$ or 1.35.

Now consider a decrease of 15%.

After a 15% decrease, the new value will be 85% of the original. So to decrease an amount by 15% find 85%. Multiply by $\frac{85}{100}$ or by 0.85.

Example 10

a The manager of a shop increases the prices of CDs by 12%. A CD originally cost 11.60 CHF. What will it cost after the increase?

b The cost of a plane ticket is decreased by 8%. The original price was 880 GBP. What is the new price?

c The rent for an apartment has increased from 2700 EUR to 3645 EUR per month. What is the percentage increase?

Answer	
a 11.60 × 1.12 = 12.99 CHF (to the nearest 0.01 CHF)	
b 880 × 0.92 = 809.60 GBP	
c **Method 1**	
The increase is 3645 − 2700 = 945 EUR	*Find the increase.*
The percentage increase is $\frac{945}{2700} \times 100 = 35\%$	*Work out the increase as a percentage of the original amount.*
Method 2	
$\frac{3645}{2700} = 1.35 = 135\%$ Percentage increase is 35%.	*Calculate the new price as a percentage of the old price.*

After a 12% increase, the amount will be 112% of its original value.

After an 8% decrease, the amount will be 92% of its original value.

$$\text{Percentage increase} = \frac{\text{actual increase}}{\text{original amount}} \times 100\%$$

Example 11

In a shop, an item's price is given as 44 AUD, **including** tax. The tax rate is 10%. What was the price without the tax?	
Answer Call the original price x. After tax has been added, the price will be $1.10x$. Hence $1.10x = 44$ $x = 44 \div 1.10$ $= 40$ The price without tax is 40 AUD.	*110% = 1.10* *Solve for x.* *Divide both sides by 1.10.*

Exercise 1E

1 In the UK, prices of some goods include a government tax called VAT, which is at 20%.
A TV costs 480 GBP before VAT. How much will it cost including VAT?

2 In a sale in a shop in Tokyo, a dress that was priced at 17 000 JPY is reduced by 12.5%. What is the sale price?

3 The cost of a weekly train ticket goes up from 120 GBP to 128.40 GBP. What is the percentage increase?

4 Between 2004 and 2005, oil production in Australia fell from 731 000 to 537 500 barrels per day. What was the percentage decrease in the production?

5 Between 2005 and 2009 the population of Venezuela increased by 7%. The population was 28 400 000 in 2009. What was it in 2005 (to the nearest 100 000)?

6 An item appears in a sale marked as 15% off with a price tag of 27.20 USD. What was the original price before discount?

7 The rate of GST (goods and service tax) that is charged on items sold in shops was increased from 17% to 20%. What would the price increase be on an item that costs 20 GBP before tax?

8 A waiter mistakenly adds a 10% service tax onto the cost of a meal which was 50.00 AUD. He then reduces the price by 10%. Is the price now the same as originally? If not, what is the percentage change from the original price?

1.5 Ratio and proportion

The **ratio** of two numbers r and s is $r : s$. It is equivalent to the fraction $\frac{r}{s}$. Like the fraction, it can be written in its lowest terms.

For example, 6 : 12 is equivalent to 1 : 2 (dividing both numbers in the ratio by 6).

When you write a ratio in its lowest terms, both numbers in the ratio should be positive whole numbers.

In a **unitary ratio**, one of the terms is 1.

For example 1 : 4.5 or 25 : 1.

If two quantities a and b are in **proportion**, then the ratio $a : b$ is fixed.

When you write a unitary ratio, you can use decimals.

We also write $a \propto b$ (a is proportional to b).

Example 12

200 tickets were sold for a school dance. 75 were bought by boys and the rest by girls. Write down the ratio of boys to girls at the dance, in its lowest terms.
Answer The number of girls is $200 - 75 = 125$ The ratio of boys to girls is $75 : 125 = 3 : 5$

Always give the ratio in its lowest terms.

Map scales are often written as a ratio. A scale of 1 : 50 000 means that 1 cm on the map represents 50 000 cm = 0.5 km on the earth.

Example 13

An old English map was made to the scale of 1 inch to a mile. Write this scale as a ratio.	
Answer 1 mile = $1760 \times 3 \times 12$ = 63 360 inches The ratio of the map is 1 : 63 360.	*Always make sure that the units in ratios match each other.*

12 inches = 1 foot
3 feet = 1 yard
1760 yards = 1 mile

Example 14

Three children, aged 8, 12 and 15, win a prize of 140 USD. They decide to share the prize money in the ratio of their ages. How much does each receive?	
Answer 140 USD is divided in the ratio 8 : 12 : 15. This is a total of 8 + 12 + 15 = 35 parts. $140 \div 35 = 4$ USD $8 \times 4 = 32$, $12 \times 4 = 48$ and $15 \times 4 = 60$ The children receive 32 USD, 48 USD and 60 USD.	*Divide the money into 35 parts.* *One part is 4 USD.*

Exercise 1F

1 Aspect ratio is the ratio of an image's width to its height. A photograph is 17.5 cm wide by 14 cm high. What is its aspect ratio, in its lowest terms?

2 Gender ratio is expressed as the ratio of men to women in the form n : 100. Based on the figures for 2008, the gender ratio of the world was 102 : 100. In Japan, there were 62 million men and 65.2 million women in 2008. What was the gender ratio in Japan?

3 Ryoka was absent for a total of 21 days during a school year of 32 weeks. What is the ratio of the number of days that she was absent to the number of possible days she could have spent at the school during the year, in its simplest terms? (A school week is 5 days.)

4 A model airplane has a wingspan of 15.6 cm. The model is built to a scale of 1 : 72. What is the wingspan of a full-sized airplane (in metres)?

5 On a map, a road measures 1.5 cm. The actual road is 3 km long. What is the scale of the map? How long would a footpath that is 800 m long be on the map?

6 A joint collection is made for two charities and it is agreed that the proceeds should be split in the ratio 5 : 3 between an animal charity and one for sick children. 72 USD is collected. How much is donated to the two charities?

7 For a bake sale, a group of students decide to make brownies, chocolate chip cookies and flapjacks in the ratio 5 : 3 : 2. They plan to make 150 items all together. How many of each will they need to make?

Leonardo da Vinci drew this famous drawing of Vitruvian Man around 1487. The drawing is based on ideal human proportions described by the ancient Roman architect Vitruvius.

1.6 The unitary method

In the unitary method, you begin by finding the value of **one** part or item.

Example 15

A wheelbarrow full of concrete is made by mixing together 6 spades of gravel, 4 spades of sand, 2 spades of cement and water as required. When there are only 3 spades of sand left, what quantities of the other ingredients will be required to make concrete?

▶ Continued on next page

Answer

The ratio is gravel : sand : cement

6 : 4 : 2

or $\frac{6}{4}:\frac{4}{4}:\frac{2}{4}$

$=\frac{3}{2}:1:\frac{1}{2}=\frac{9}{2}:3:\frac{3}{2}$

Hence the mixture requires $4\frac{1}{2}$ spades of gravel to 3 spades of sand to $1\frac{1}{2}$ spades of cement.

Since the value you want to change is the sand, make sand equal to 1 by dividing through by 4. Then multiply through by 3 to make the quantity of sand equal to 3.

Exercise 1G

1 Josh, Jarrod and Se Jung invested 5000 USD, 7000 USD and 4000 USD to start up a company. In the first year, they make a profit of 24 000 USD which they share in the ratio of the money they invested. How much do they each receive?

2 Amy is taking a Mathematics test. She notices that there are three questions worth 12, 18 and 20 marks. The test lasts one hour and fifteen minutes. She decides to allocate the time she spends on each question in the ratio of the marks. How long does she spend on each question?

2 Algebra

The word **algebra** comes from the title of a book *Hisab al-jabr w'al-muqabala* written by Abu Ja'far Muhammad ibn Musa Al-Khwarizmi in Baghdad around 800 CE. It is regarded as the first book to be written about algebra.

2.1 Expanding brackets and factorization

The **distributive law** is used to expand brackets and factorize expressions.

$a(b + c) = ab + ac$

Two other laws used in algebra are the **commutative law** $ab = ba$ and the **associative law** $(ab)c = a(bc)$.

Example 16

Expand $2y(3x + 5y - z)$

Answer

$2y(3x + 5y - z) = 2y(3x) + 2y(5y) + 2y(-z)$

$= 6xy + 10y^2 - 2yz$

Example 17

Factorize $6x^2y - 9xy + 12xz^2$

Answer

$6x^2y - 9xy + 12xz^2 = 3x(2xy - 3y + 4z^2)$

Look for a common factor. Write this outside the bracket. Find the terms inside the bracket by dividing each term by the common factor.

Exercise 2A

1 Expand

a $3x(x-2)$ **b** $\frac{x}{y}(x^2y - y^2 + x)$ **c** $a(b-2c) + b(2a+b)$

2 Factorize

a $3pq - 6p^2q^3r$ **b** $12ac^2 + 15bc - 3c^2$ **c** $2a^2bc + 3ab^2c - 5abc^2$

2.2 Formulae

Rearranging formulae

Example 18

The formula for the area of a circle is $A = \pi r^2$, where A is the area and r is the radius.
The subject of the formula is A.
Rearrange the formula to make r the subject.

Answer

$A = \pi r^2$

$r^2 = \frac{A}{\pi}$

$r = \sqrt{\frac{A}{\pi}}$

Use the same techniques as for solving equations. Whatever you do to one side of the formula, you must do to the other.
Divide both sides by π.
Take the square root of both sides.

The subject of a formula is the letter on its own on one side of the = sign.

You can use this formula to work out the radius of a circle when you know its area.

Exercise 2B

Rearrange these formulae to make the quantity shown in brackets the subject.

1 $v = u - gt$ (t) **2** $a = \sqrt{b^2 + c^2}$ (c) **3** $c = 2\pi r$ (r)

4 $\frac{\sin A}{a} = \frac{\sin B}{b}$ (b) **5** $a^2 = b^2 + c^2 - 2bc\cos A$ $(\cos A)$

Substituting into formulae

You can always use your GDC in Mathematical Studies.

When using formulae, let the calculator do the calculation for you. You should still show your working.

1 Find the formula you are going to use (from the Formula booklet, from the question or from memory) and write it down.
2 Identify the values that you are going to substitute into the formula.
3 Write out the formula with the values substituted for the letters.
4 Enter the formula into your calculator. Use templates to make the formula look the same on your GDC as it is on paper.
5 If you think it is necessary, use brackets. It is better to have too many brackets than too few!
6 Write down, with units if necessary, the result from your calculator (to the required accuracy).

Example 19

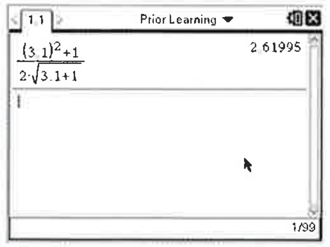
x and y are related by the formula $y = \frac{x^2+1}{2\sqrt{x+1}}$.
Find y when x is 3.1.

Answer

$y = \frac{3.1^2+1}{2\sqrt{3.1+1}}$

$= 2.62$ (to 3 sf)

Write the formula with 3.1 instead of x.



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Exercise 2C

1 If $a = 2.3$, $b = 4.1$ and $c = 1.7$, find d where

$d = \frac{3a^2 + 2\sqrt{b}}{ac + b}$

2 If $b = 8.2$, $c = 7.5$ and $A = 27°$, find a where

$a = \sqrt{b^2 + c^2 - 2bc\cos A}$

3 If $u_1 = 10.2$, $r = 0.75$ and $n = 14$, find the value of S, where

$S = u_1\frac{1-r^n}{1-r}$

2.3 Solving linear equations

'Solve an equation' means 'find the value of the unknown variable' (the letter).

Rearrange the equation so that the unknown variable x becomes the subject of the equation. To keep the equation 'balanced' always do the same to both sides.

Example 20

Solve the equation $3x + 5 = 17$

Answer

$3x + 5 = 17$

$3x + 5 - 5 = 17 - 5$ — *Subtract 5.*

$3x = 12$

$\frac{3x}{3} = \frac{12}{3}$ — *Divide by 3.*

$x = 4$

Add, subtract, multiply or divide on both sides of the equation until the x is by itself on one side. (This can be either the left or the right-hand side.)

Example 21

Solve the equation $4(x - 5) = 8$

Answer

$4(x - 5) = 8$

$\frac{4(x-5)}{4} = \frac{8}{4}$ — *Divide by 4.*

$x - 5 = 2$

$x - 5 + 5 = 2 + 5$ — *Add 5.*

$x = 7$

Always take care with – signs.

Example 22

Solve the equation $7 - 3x = 1$

Answer

$7 - 3x = 1$

$7 - 3x - 7 = 1 - 7$ — *Subtract 7.*

$-3x = -6$

$\frac{-3x}{-3} = \frac{-6}{-3}$ — *Divide by –3.*

$x = 2$

An alternative method for this equation would be to start by **adding** 3x. Then the x would be positive, but on the right-hand side.

Example 23

Solve the equation $3(2 + 3x) = 5(4 - x)$

Answer

$3(2 + 3x) = 5(4 - x)$

$6 + 9x = 20 - 5x$

$6 + 9x + 5x = 20 - 5x + 5x$ — *Add 5x.*

$6 + 14x = 20$

$6 + 14x - 6 = 20 - 6$ — *Subtract 6.*

$14x = 14$

$\frac{14x}{14} = \frac{14}{14}$ — *Divide by 14.*

$x = 1$

Compare this method with the one in Example 21. Sometimes it can be quicker to **divide** first rather than expanding the brackets.

Exercise 2D

Solve these equations.

1 $3x - 10 = 2$

2 $\frac{x}{2} + 5 = 7$

3 $5x + 4 = -11$

4 $3(x + 3) = 18$

5 $4(2x - 5) = 20$

6 $\frac{2}{5}(3x - 7) = 8$

7 $21 - 6x = 9$

8 $12 = 2 - 5x$

9 $2(11 - 3x) = 4$

10 $4(3 + x) = 3(9 - 2x)$

11 $2(10 - 2x) = 4(3x + 1)$

12 $\frac{5x+2}{3} = \frac{3x+10}{4}$

2.4 Simultaneous linear equations

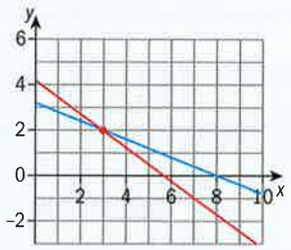
Simultaneous equations involve two variables.
There are two methods which you can use to solve them, called substitution and elimination. You can also sometimes solve them graphically.

Example 24

Solve the equations $3x + 4y = 17$ and $2x + 5y = 16$.

Answer

Graphical method



Geometrically you could consider these two linear equations as the equations of two straight lines. Finding the solution to the equation is equivalent to finding the point of intersection of the lines. The coordinates of the point will give you the values for x and y.

The solution is $x = 3$, $y = 2$.

Substitution method

$3x + 4y = 17$

$2x + 5y = 16$

$5y = 16 - 2x$ — *Rearrange one of the equations to make y the subject.*

$y = \frac{16}{5} - \frac{2}{5}x$

$3x + 4\left(\frac{16}{5} - \frac{2}{5}x\right) = 17$ — *Substitute for y in the other equation.*

$3x + \frac{64}{5} - \frac{8}{5}x = 17$

$15x + 64 - 8x = 85$ — *Solve the equation for x.*

$15x - 8x = 85 - 64$

$7x = 21$

$x = 3$

▶ Continued on next page

$3(3) + 4y = 17$ $9 + 4y = 17$ $4y = 8$ $y = 2$ The solution is $x = 3$, $y = 2$.	*Substitute for x in one of the original equations and solve for y.*
Elimination method $3x + 4y = 17$ (1) $2x + 5y = 16$ (2) Multiply equation (1) by 2 and equation (2) by 3. $6x + 8y = 34$ (3) $6x + 15y = 48$ (4)	*This is to make the coefficients of x equal.*
Subtract the equations. [(4) − (3)] $7y = 14$ $y = 2$	*Subtracting now eliminates x from the equations.*
$3x + 4(2) = 17$ $3x + 8 = 17$ $3x = 17 - 8$ $3x = 9$ $x = 3$ The solution is $x = 3$, $y = 2$.	*Substitute for y in one of the original equations and solve for x.*

Exercise 2E

1 Solve these simultaneous equations using substitution.
a $y = 3x - 2$ and $2x + 3y = 5$ **b** $4x - 3y = 10$ and $2y + 5 = x$
c $2x + 5y = 14$ and $3x + 4y = 7$

2 Solve these simultaneous equations using elimination.
a $2x - 3y = 15$ and $2x + 5y = 7$ **b** $3x + y = 5$ and $4x - y = 9$
c $x + 4y = 6$ and $3x + 2y = -2$ **d** $3x + 2y = 8$ and $2x + 3y = 7$
e $4x - 5y = 17$ and $3x + 2y = 7$

2.5 Exponential expressions

Repeated multiplication can be written as an **exponential** expression. For example, squaring a number:

$3 \times 3 = 3^2$ or $5.42 \times 5.42 = 5.42^2$

If we multiply a number by itself three times then the exponential expression is a cube. For example

$4.6 \times 4.6 \times 4.6 = 4.6^3$

You can also use exponential expressions for larger integer values. So, for example,

$3^7 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

Index and **power** are other names for **exponent**.

You use squares in Pythagoras' theorem $c^2 = a^2 + b^2$ or in the formula for the area of a circle $A = \pi r^2$.
You use a cube in the formula for the volume of a sphere $V = \frac{4}{3}\pi r^3$.

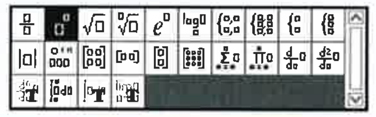
Where the exponent is not a positive integer, these rules apply:

$a^0 = 1, a \neq 0$

$a^{-n} = \frac{1}{a^n}$

Example 25

Write down the values of 10^2, 10^3, 10^1, 10^0, 10^{-2}, 10^{-3}.

Answer $10^2 = 10 \times 10 = 100$ $10^3 = 10 \times 10 \times 10 = 1000$ $10^1 = 10$ $10^0 = 1$ $10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$ $10^{-3} = \frac{1}{10^3} = \frac{1}{1000} = 0.001$	*To evaluate an exponential function with the GDC use either the ^ key or the template key and the exponent template.* 

GDC help on CD: Alternative demonstrations for the TI-84 Plus and Casio FX-9860GII GDCs are on the CD.

Exercise 2F

Evaluate these expressions.

1 **a** $2^3 + 3^2$ **b** $4^2 \times 3^2$ **c** 2^6
2 **a** 5^0 **b** 3^{-2} **c** 2^{-4}
3 **a** 3.5^5 **b** 0.495^{-2} **c** $2\frac{(1-0.02)^{10}}{1-0.02}$

2.6 Solving inequalities

Inequalities can be solved in a similar way to equations.

Example 26

Solve the inequalities **a** $2x + 5 < 7$ **b** $3(x - 2) \geq 4$

Answers **a** $2x + 5 < 7$ $2x < 2$ $x < 1$	**b** $3(x - 2) \geq 4$ $x - 2 \geq 1\frac{1}{3}$ $x \geq 3\frac{1}{3}$	*Add, subtract, multiply or divide on both sides until the x is by itself on one side.*

Multiplying or dividing by a negative number reverses the inequality.

If you multiply or divide an inequality by a negative value, change the signs on both sides of the inequality and reverse the inequality sign. For example, $4 > 2$, but $-4 < -2$.

Example 27

Solve the inequality $7 - 2x \le 5$

Answer

$7 - 2x \le 5$	*Subtract 7.*
$-2x \le -2$	*Divide by −2.*
$x \ge 1$	*Change ≤ to ≥.*

Example 28

Solve the inequality $19 - 2x > 3 + 6x$

Answer

$19 - 2x > 3 + 6x$	
$19 > 3 + 8x$	
$16 > 8x$	
$2 > x$	
$x < 2$	*Reverse the inequality.*

Sometimes the x ends up on the right-hand side of the inequality. In this case reverse the inequality as in this example.

Exercise 2G

1 Solve the inequality for x and represent it on the number line.

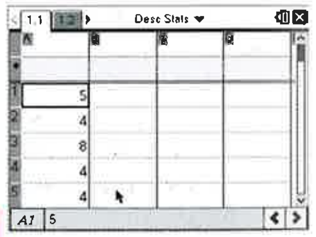
a $3x + 4 \le 13$ **b** $5(x - 5) > 15$ **c** $2x + 3 < x + 5$

2 Solve for x.

a $2(x - 2) \ge 3(x - 3)$ **b** $4 < 2x + 7$ **c** $7 - 4x \le 11$

Properties of inequalities

→ When you add or subtract a real number from both sides of an inequality the direction of the inequality is unchanged.



For example:

- $4 > 6 \Rightarrow 4 + 2 > 6 + 2$
- $15 \le 20 \Rightarrow 15 - 6 \le 20 - 6$
- $x - 7 \ge 8 \Rightarrow x - 7 + 7 \ge 8 + 7$
- $x + 5 < 12 \Rightarrow x + 5 - 5 < 12 - 5$

→ When you multiply or divide both sides of an inequality by a positive real number the direction of the inequality is unchanged. When you multiply or divide both sides of an inequality by a negative real number the direction of the inequality is reversed.

For example:

- $4 > 5 \Rightarrow 2(4) > 2(5)$
- $6 \le 10 \Rightarrow -2(6) \ge -2(10)$
- $10 \le 30 \Rightarrow \frac{10}{5} \le \frac{30}{5}$
- $18 > 24 \Rightarrow \frac{18}{-3} < \frac{24}{-3}$
- $-12 > -20 \Rightarrow \frac{-12}{4} > \frac{-20}{4}$

2.7 Absolute value

The absolute value (or modulus) of a number $|x|$ is the numerical part of the number without its sign. It can be written as $|x| = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \ge 0 \end{cases}$

Example 29

Write down $|a|$ where $a = -4.5$ and $a = 2.6$

Answer

If $a = -4.5$ then $|a| = 4.5$

If $a = 2.6$ then $|a| = 2.6$

Example 30

Write the value of $|p - q|$ where $p = 3$ and $q = 6$.

Answer

$|p - q| = |3 - 6| = |-3| = 3$

Exercise 2H

1 Write the value of $|a|$ when a is

a 3.25 **b** −6.18 **c** 0

2 Write the value of $|5 - x|$ when $x = 3$ and when $x = 8$.

3 If $x = 6$ and $y = 4$, write the values of

a $|x - y|$ **b** $|x - 2y|$ **c** $|y - x|$

3 Geometry

Euclid's *Elements*, written around 300 BCE, was one of the first mathematics textbooks and remained a required text until the 20th century. Euclid began his first book with postulates: self-evident truths.

A point is that which has no part.

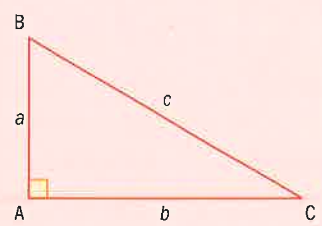
A line is breadthless length.

A plane is a surface which lies evenly with the straight lines on itself. An angle is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line.

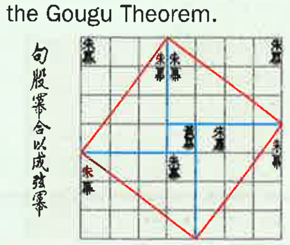
3.1 Pythagoras' theorem

→ In a right-angled triangle ABC with sides a, b and c, where a is the *hypotenuse*:

$c^2 = a^2 + b^2$

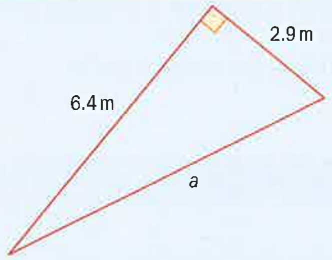


Although the theorem is named after the Greek mathematician Pythagoras, it was known several hundred years earlier to the Indians in their Sulba Sutras and thousands of years before to the Chinese as the Gougu Theorem.



Example 31

Find the length labeled a.



Answer

$a^2 = 6.4^2 + 2.9^2$

$a = \sqrt{6.4^2 + 2.9^2}$

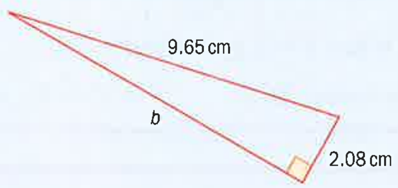
$a = 7.03$ cm (to 3 sf)

> You can use Pythagoras' theorem to calculate the length of one side of a right-angled triangle when you know the other two.

Sometimes you have to find a shorter side.

Example 32

Find the length labeled b.



Answer

$9.65^2 = b^2 + 2.08^2$

$b^2 = 9.65^2 - 2.08^2$

$b = \sqrt{9.65^2 - 2.08^2}$

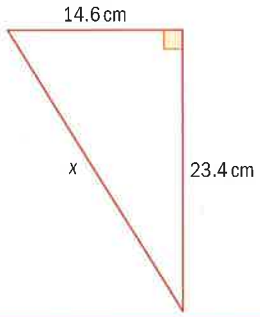
$b = 9.42$ cm (to 3 sf)

> Check your answer by making sure that the hypotenuse is the longest side of the triangle.

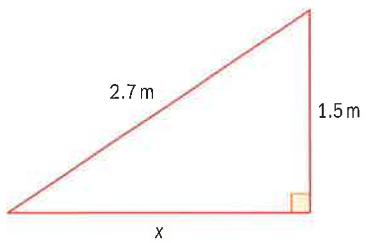
Exercise 3A

In each diagram, find the length of the side marked x.
Give your answer to 3 signifigant figures.

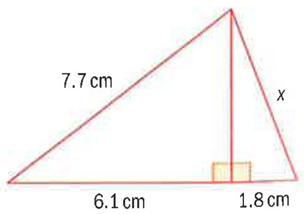
1



2

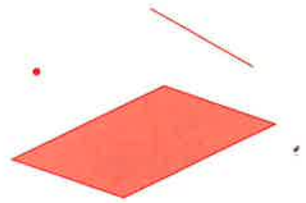


3

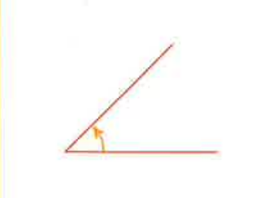
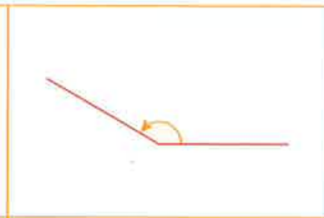
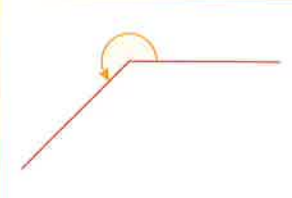


3.2 Points, lines, planes and angles

The most basic ideas of geometry are points, lines and planes. A **straight line** is the shortest distance between two points. Planes can be **finite**, like the surface of a desk or a wall, or can be **infinite**, continuing in every direction.



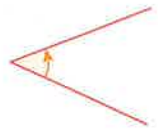
We say that a point has zero dimensions, a line has one dimension and a plane has two dimensions.
Angles are measured in degrees.

			
Acute angle between 0° and 90°	Right angle 90°	Obtuse angle between 90° and 180°	Reflex angle between 180° and 360°

Exercise 3B

1 Draw a sketch of:
- **a** a reflex angle **b** an acute angle
- **c** a right angle **d** an obtuse angle.

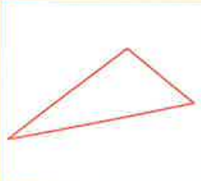
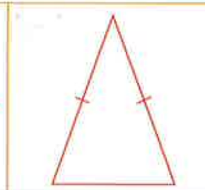
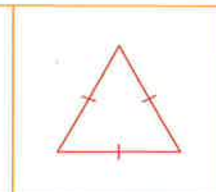
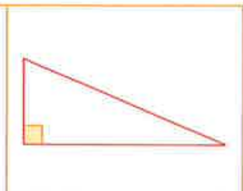
2 State whether the following angles are acute, obtuse or reflex.

a  **b**  **c** 

3 State whether the following angles are acute, obtuse or reflex.
- **a** 173° **b** 44° **c** 272°
- **d** 82° **e** 308° **f** 196°

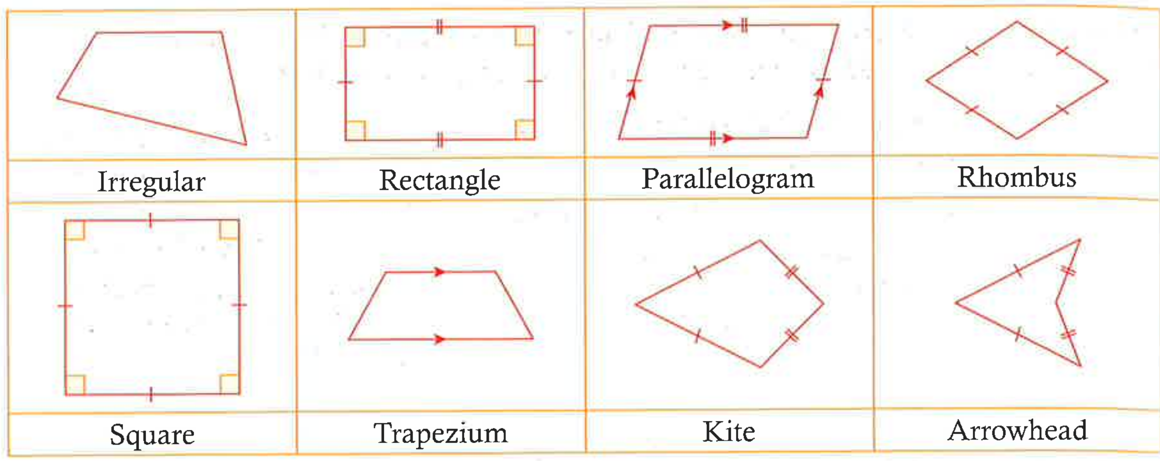
3.3 Two-dimensional shapes

Triangles

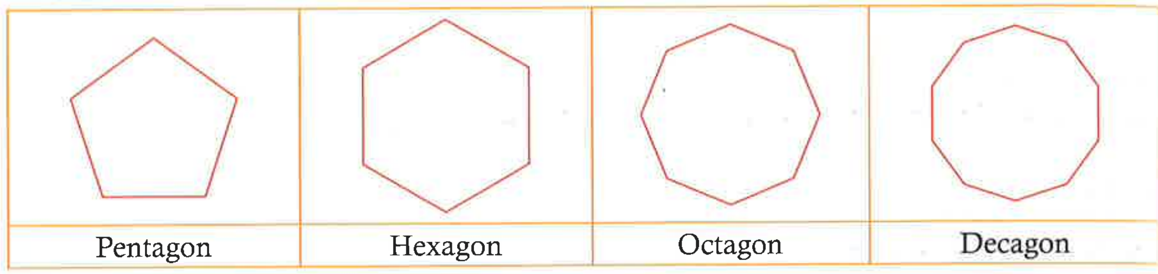
			
Scalene triangle	Isosceles triangle	Equilateral triangle	Right-angled triangle

> The small lines on these diagrams show equal lines and the arrows show parallel lines.

Quadrilaterals



Polygons



Exercise 3C

1 Sketch the quadrilaterals from the table above, with their diagonals. Copy and complete the following table.

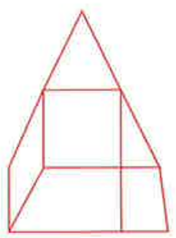
Diagonals	Irregular	Rectangle	Parallelogram	Rhombus	Square	Trapezium	Kite
Perpendicular					▯		
Equal					▯		
Bisect					▯		
Bisect angles					▯		

For example, the diagonals of a square are perpendicular to each other, equal in length, bisect each other and bisect the angles of the square.

2 List the names of all the shapes that are contained in each of these figures.

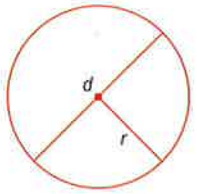
a

b



3.4 Perimeter

The **perimeter** of a figure is defined as the length of its boundary. The perimeter of a polygon is found by adding together the sum of the lengths of its sides.



The perimeter of a circle is called its **circumference**.

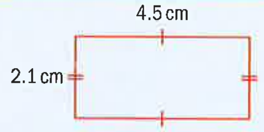
In the circle on the left, r is the radius and d is the diameter. If C is the circumference.

$C = 2\pi r$ or $C = \pi d$

> π = 3.14159265358979323846 2...
> Many mathematical enthusiasts around the world celebrate Pi day on March 14 (3/14). The use of the symbol π was popularized by the Swiss mathematician Leonhard Euler (1707–1783).

Example 33

Find the perimeter of this shape.

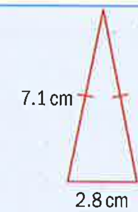


Answer

Perimeter = 4.5 cm + 2.1 cm + 4.5 cm + 2.1 cm = 13.2 cm

Example 34

Find the perimeter of this shape.



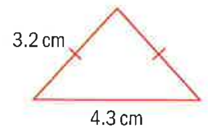
Answer

Perimeter = 2 × 7.1 cm + 2.8 cm = 17.0 cm

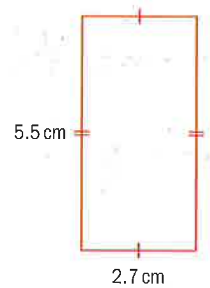
Exercise 3D

Find the perimeters of these shapes.

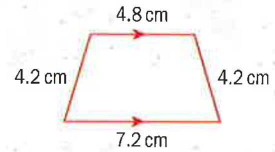
a



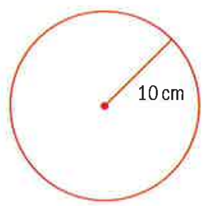
b



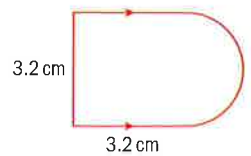
c



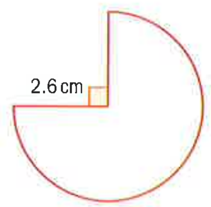
d



e

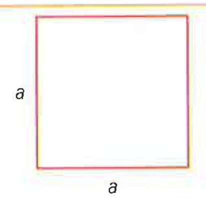
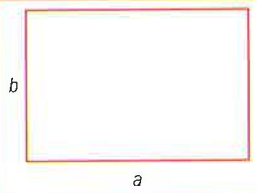
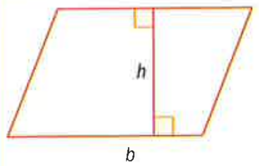
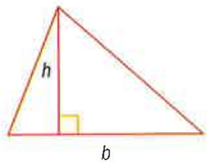
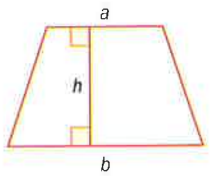
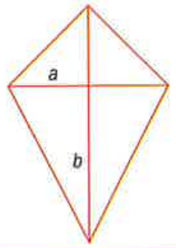
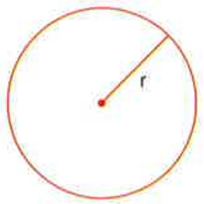


f



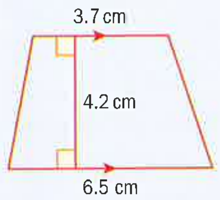
3.5 Area

These are the formulae for the areas of a number of plane shapes.

			
$A = a^2$	$A = ab$	$A = bh$	$A = \frac{1}{2}bh$
			
$A = \frac{1}{2}(a + b)h$	$A = \frac{1}{2}ab$	$A = \pi r^2$	

Example 35

Find the area of this shape.

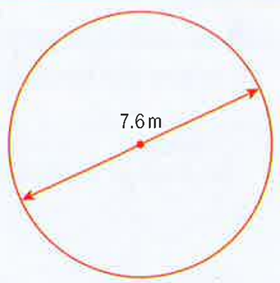


Answer

Area $= \frac{1}{2}(3.7 + 6.5)(4.2) = 21.42\ \text{cm}^2$

Example 36

Find the area of this shape, giving your answer to 3 significant figures.



Answer

Area $= \pi(3.8)^2 = 45.4\ \text{cm}^2$ (to 3 sf)

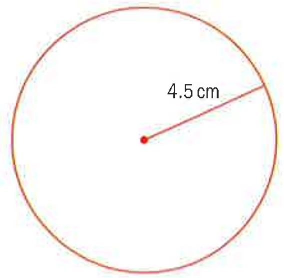
Diameter = 7.6 m, so radius = 7.6 ÷ 2 = 3.8 m

Use the π button on your calculator to enter π.

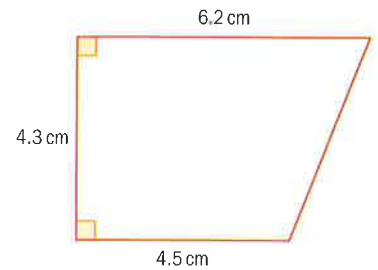
Exercise 3E

Find the areas of these shapes. Give your answer to 3 signifigant figures.

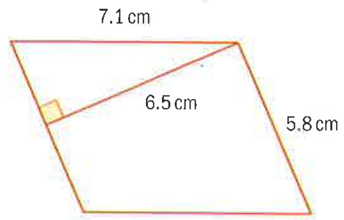
1



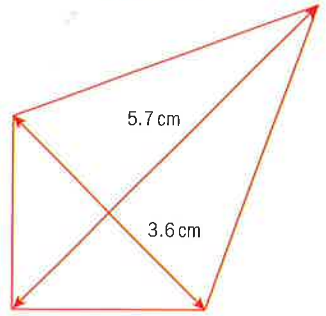
2



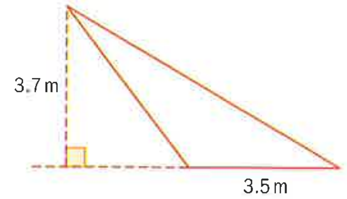
3



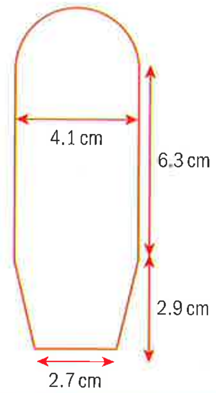
4



5



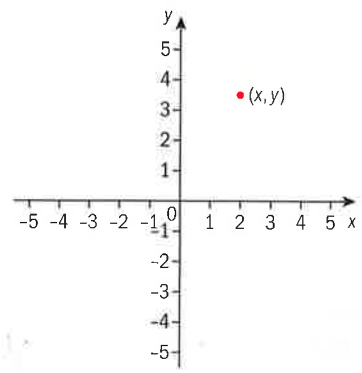
6



3.6 Coordinate geometry

Coordinates

Coordinates describe the position of points in the plane. Horizontal positions are shown on the x-axis and vertical positions on the y-axis.

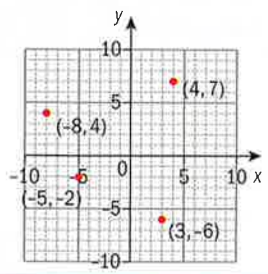


René Descartes introduced the use of coordinates in a treatise in 1637. You may see axes and coordinates described as Cartesian axes and Cartesian coordinates.

Example 37

Draw axes for and $-10 \le x \le 10$ and $-10 \le y \le 10$.
Plot the points with coordinates: (4, 7), (3, −6), (−5, −2) and (−8, 4).

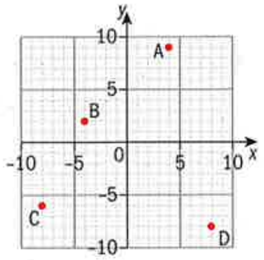
Answer



Exercise 3F

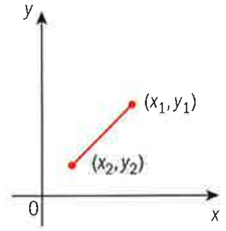
1 Draw axes for $-8 \le x \le 8$ and $-5 \le y \le 10$.
Plot the points with coordinates:
$(5, 0)$, $(2, -2)$, $(-7, -4)$ and $(-1, 9)$.

2 Write down the coordinates of the points shown in this diagram.



Midpoints

The midpoint of the line joining the points with coordinates (x_1, y_1) and (x_2, y_2) is given by $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.



Example 38

Find the midpoint of the line joining the points with coordinates $(1, 7)$ and $(-3, 3)$.

Answer

The midpoint is $= \left(\frac{1+(-3)}{2}, \frac{7+3}{2}\right) = (-1, 5)$

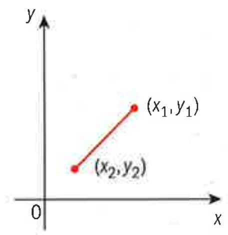
Exercise 3G

Calculate the midpoints of the lines joining these pairs of points.

1 $(2, 7)$ and $(8, 3)$ **2** $(-6, 5)$ and $(4, -7)$ **3** $(-2, -1)$ and $(5, 6)$.

Distance between two points

The distance between points with coordinates (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.



Example 39

Find the distance between the points with coordinates $(2, -3)$ and $(-5, 4)$.

Answer

Distance $= \sqrt{(-5-2)^2+(4-(-3))^2} = \sqrt{(-7)^2+7^2} = 9.90$ (to 3 sf)

Exercise 3H

Calculate the distance between the following pairs of points.
Give your answer to 3 signifigant figures where appropriate.

1 $(1, 2)$ and $(4, 6)$

2 $(-2, 5)$ and $(3, -3)$

3 $(-6, -6)$ and $(1, 7)$

4 Statistics

4.1 Statistical graphs

In a statistical investigation we collect information, known as **data**. To represent the data in a clear way we can use graphs. Three types of statistical graph are bar charts, pie charts and pictograms.

Bar charts

A **bar chart** is a graph made from rectangles, or bars, of equal width whose length is proportional to the quantity they represent, or frequency. Sometimes we leave a small gap between the bars.

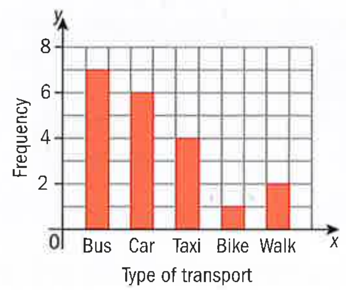
Example 40

Juliene collected some data about the ways in which her class travel to school.

Type of transport	Bus	Car	Taxi	Bike	Walk
Frequency	7	6	4	1	2

Represent this information in a bar chart.

Answer



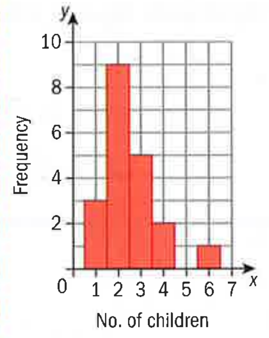
Example 41

Lakshmi collected data from the same class about the number of children in each of their families.

No. of children	1	2	3	4	6
Frequency	3	9	5	2	1

Represent this information in a bar chart.

Answer



Pie charts

A **pie chart** is a circle divided into sectors, like slices from a pie.
The sector angles are proportional to the quantities they represent.

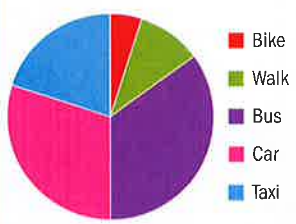
Example 42

Use Juliene's data from Example 40 to construct a pie chart.

Answer

Type of transport	Frequency		Sector angle
Bus	7	$\frac{7}{20} \times 360°$	126°
Car	6	$\frac{6}{20} \times 360°$	108°
Taxi	4	$\frac{4}{20} \times 360°$	72°
Bike	1	$\frac{1}{20} \times 360°$	18°
Walk	2	$\frac{2}{20} \times 360°$	36°

The total of the frequencies is 20. The total angle for the whole circle is 360°.



Start by drawing a radius and then measure, with your protractor, each angle in turn. The total of the sector angles should be 360°.

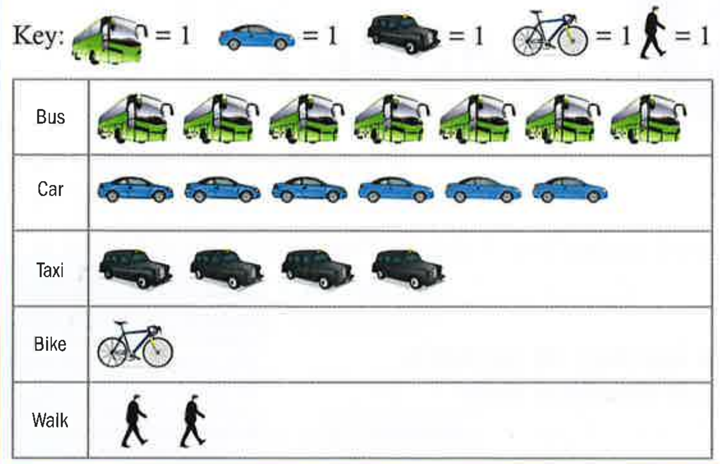
Pictograms

Pictograms are similar to bar charts, except that pictures are used. The number of pictures is proportional to the quantity they represent. The pictures can be relevant to the items they show or just a simple character such as an asterisk.

Example 43

Use Juliene's data from Example 40 to construct a pictogram.

Answer



In this pictogram, different symbols are used for each category but the symbols describe the category as well.

Example 44

Use these data on the number of children in a sample of families to construct a pictogram.

Number of children	1	2	3	4	6
Frequency	4	9	6	2	1

Answer

No. of children	
1	△△△△
2	△△△△△△△△△
3	△△△△△△
4	△△
6	△

Key: △ = 1

Exercise 4A

1 Adam carried out a survey of the cars passing by his window on the road outside. He noted the colors of the cars that passed by for 10 minutes and collected the following data.

Color	Black	Red	Blue	Green	Silver	White
Frequency	12	6	10	7	14	11

Draw a bar chart, a pie chart and a pictogram to represent the data.

2 Ida asked the members of her class how many times they had visited the cinema in the past month. She collected the following data.

Number of times visited	1	2	3	4	8	12
Number of students	4	7	4	3	1	1

Draw a bar chart, a pie chart and a pictogram to represent the data.