14 Practice paper 1

Time allowed: 1 hour 30 minutes

- Answer all the questions
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

Practice papers on CD: *IB examination papers include spaces for you to write your answers. There is a version of this practice paper with space for you to write your answers on the CD. You can also find an additional set of papers for further practice.*

Worked solutions on CD: *Detailed worked solutions for this practice paper are given as a PowerPoint presentation on the CD.*

1 a Find the exact value of $\frac{\sqrt{b^2 - ac}}{3125}$, given that $a = 6.4$, $b = 7$ and $c = -5$. *[2 marks]*

b Write your answer to **a**

i correct to 3 decimal places;

ii correct to 2 significant figures;

iii in the form $a \times 10^k$, where $1 \leq a < 10$, $k \in \mathbb{Z}$. *[4 marks]*

2 The table below shows the number of children in the families of a class in a school.

Number of children	1	2	3	4	5
Frequency	3	8	7	4	2

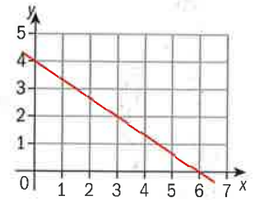
a Write down the number of families in the class. *[1 mark]*

b Calculate the mean number of children per family. *[2 marks]*

c Calculate the standard deviation of the number of children per family. *[1 mark]*

d Find the median number of children per family. *[2 marks]*

3 The diagram shows the straight line L_1.



a Calculate the gradient of L_1. *[2 marks]*

b Write down the equation of L_1. *[1 mark]*

A second line L_2 is perpendicular to L_1 and passes through the point (3, 2). The equation of L_2 is $y = mx + c$.

c Find the value of m and of c. *[3 marks]*

4 a Complete the next two columns of the truth table. *[2 marks]*

p	q	$\neg p$	$\neg p \Rightarrow q$	Inverse
T	T			
T	F			
F	T			
F	F			

b Write down the inverse of the statement $\neg p \Rightarrow q$ *[2 marks]*

c Complete the final column of the truth table with the truth values for **b**. *[1 mark]*

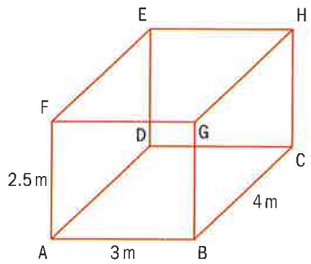
d The statement $\neg p \Rightarrow q$ and its inverse are **not** equivalent. State the reason why not. *[1 mark]*

5 The second term, u_2, of a geometric sequence is 162. The fifth term, u_5, of the same sequence is –6.

a Find the common ratio of the sequence *[4 marks]*

b Find u_1, the first term in the sequence. *[2 marks]*

6 A room is in the shape of a cuboid. Its floor measures 3 m by 4 m and its height is 2.5 m.

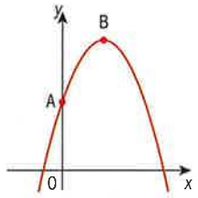


a Calculate the length of BD, the diagonal of the floor of the room. *[2 marks]*

b Calculate the length of BE, the diagonal of the room. *[2 marks]*

c Calculate the angle of depression of B from E. *[2 marks]*

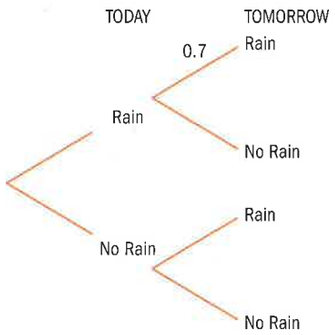
7 The quadratic function $f(x) = 5 + 6x - 2x^2$ intersects the y-axis at point A and has its vertex at point B.



a Write down the coordinates of A. *[1 mark]*
b Find the coordinates of B. *[2 marks]*

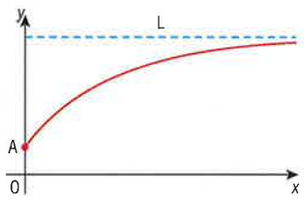
Point C has the same y coordinate as A.
c Label point C on the diagram. *[1 mark]*
d Write down the coordinates of C. *[2 marks]*

8 The probability that it rains today is 0.8. If it rains today, the probability that it will rain tomorrow is 0.7. If it does not rain today, the probability that it will rain tomorrow is 0.9.
a Complete the tree diagram below. *[3 marks]*



b Calculate the probability that it does not rain tomorrow. *[3 marks]*

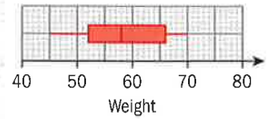
9 The graph shows the function $f(x) = 10 - (8)a^{-x}$. It intersects the y-axis at point A and has the line L as a horizontal asymptote.



a Find the y coordinate of A. *[2 marks]*
b Write down the equation of L. *[2 marks]*

$f(x)$ passes through the point (1, 8).
c Calculate the value of a. *[2 marks]*

10 The weights in kg of 40 adult females were collected and summarized in the box and whisker plot shown below.



a Write down the median weight of the females. *[1 mark]*
b Calculate the interquartile range. *[2 marks]*

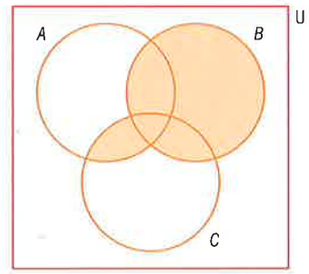
Two females are chosen at random.
c Find the probability that both females chosen weighed more than 66 kg. *[3 marks]*

11 Jing Yi invests 4000 euros in an account which pays a nominal annual interest rate of 3%, **compounded monthly**.
Give all answers correct to two decimal places.
Find:
a the value of the investment after 5 years *[3 marks]*
b the difference in the final value of the investment if the interest was compounded quarterly at the same nominal rate. *[3 marks]*

12 Given the sequence: 437, 422, 407, 392, ...
a Write down the common difference of the sequence. *[1 mark]*
b Calculate the sum of the first 50 terms of the sequence. *[2 marks]*

u_k is the first term in the sequence which is negative.
c Find the value of k. *[3 marks]*

13 a Express in set notation the shaded region on the Venn diagram below. *[2 marks]*



b Given that $x \in (A \cap B' \cap C')$, place x in its correct position on the Venn diagram. *[2 marks]*
c Shade carefully on the above Venn diagram the region which represents $(A \cup B)' \cap C$. *[2 marks]*

14 Consider $f(x) = x^2 - kx$.
a Find $f'(x)$. *[2 marks]*

The graph of $y = f(x)$ has a minimum point with coordinates $(3, p)$.
b Find the value of k. *[2 marks]*
c Find the value of p. *[2 marks]*

15 Consider the statement p:
"If a quadrilateral is a rhombus then the four sides of the quadrilateral are equal".

a Write down the contrapositive of statement p in words. *[2 marks]*
b Write down the converse of statement p in words. *[2 marks]*
c Determine whether the converse of statement p is always true. Give an example to justify your answer. *[2 marks]*

Use the mark scheme in the Answer section at the back of this book to mark your answers to this practice paper.

Practice paper 2

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1 The 350 students of an international school play three sports: hockey (H), football (F) and tennis (T).

150 play hockey
220 play football
35 play tennis
80 play hockey and football but not tennis
10 play football and tennis but not hockey
8 play tennis and hockey but not football
5 play all three sports.

a Draw a Venn diagram that illustrates the above information. *[4 marks]*
b Find the number of students that play tennis only. *[1 mark]*
c Find the number of students that do not play any sport. *[2 marks]*

One student is chosen at random. Calculate the probability that this student
d plays hockey or football but not both, *[2 marks]*
e plays hockey given that the student plays tennis. *[2 marks]*

Two students are chosen at random.
f Calculate the probability that these two students play both football and tennis. *[3 marks]*

2 University students were given a number of additional Physics lessons before they took the Physics exam. The following table shows the results (y) in this exam of 10 of these students with the number of additional lessons they took (x).

Number of additional lessons (x)	2	3	4	5	7	8	9	10	12	14
Result (y)	70	72	75	76	79	80	79	82	87	91

a i Use your graphic display calculator to find r, the correlation coefficient between x and y.

ii Use your answer for r to describe the correlation between x and y. *[4 marks]*

b Write down the equation of the regression line y on x. *[2 marks]*

c Use your equation in **b** to estimate the score for a student who took 6 additional lessons. *[2 marks]*

Peter believes that the time when the students took the additional lessons (morning or afternoon) influenced their result in the Physics exam. He records the number of students attending these lessons in the table below and performs a chi-squared test at the 5% significance level to determine whether he is correct.

	Students' result on the Physics exam (y)		
	$y \leq 40$	$40 < y < 60$	$60 \leq y \leq 100$
Morning	35	22	14
Afternoon	48	18	9

d Write down the null hypothesis, H_0. *[1 mark]*

e Write down the number of degrees of freedom. *[1 mark]*

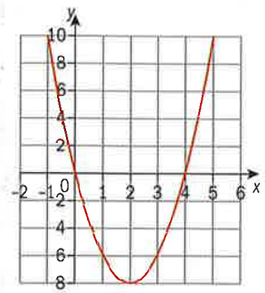
f Show that the expected number of students that took the additional lessons during the **morning** and had a result in the Physics exam **between 40 and 60** is 19 correct to the nearest integer. *[2 marks]*

g Use your graphic display calculator to find the chi-test statistic. *[2 marks]*

The χ^2 value at the 5% is 5.991.

h Peter accepts H_0. Give a reason for his decision. *[1 mark]*

3 The following is the graph of the function $f(x) = 2x^2 - tx$ where t is a constant.



a i Factorize the expression $f(x) = 2x^2 - tx$.

ii Using the graph of $f(x)$ write down the solutions to the equation $f(x) = 0$.

iii Hence or otherwise find the value of t. *[6 marks]*

The function $f(x)$ is increasing for $x > a$.

b Write down the value of a. *[1 mark]*

The graph of the function $g(x) = mx + c$ intersects the graph of $f(x)$ at the points A and B, where $x = 1$ and $x = 5$ respectively.

c Write down the y-coordinate of

i A

ii B. *[2 marks]*

d Hence write down two equations in m and c. *[2 marks]*

e Find the value of m and of c. *[2 marks]*

f Find the x coordinate of the point at which $g(x)$ intersects the x-axis. *[2 marks]*

g Write down the interval of values of x for which $g(x) > f(x)$. *[2 marks]*

4 Three cities, Pemberley (P), Vimy (V) and Ridge (R) are the vertices of a triangle; the distance between Pemberley and Vimy is 45 km, the distance between Vimy and Ridge is 60 km. The angle PVR is 75°. This information is given in the diagram.

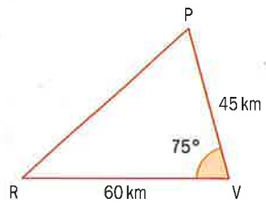


DIAGRAM NOT TO SCALE

a Calculate the area of triangle PVR. Give your answer correct to the nearest km^2. *[4 marks]*

b Find the length of PR. *[3 marks]*

c Find the angle RPV. *[3 marks]*

A road is constructed from R and meets PV at T such that RT is perpendicular to PV. A company wants to build a water reservoir for the three cities at M, the midpoint of RT.

d Show that the distance MR is 29 km correct to the nearest km. *[4 marks]*

This water reservoir will be in the shape of a square of side 150 m and have a depth of 2.85 m

e Calculate the volume of the reservoir. *[2 marks]*

To construct the reservoir, the company will pay a fee of 1.25 Swiss Francs (CHF) per m^3 of its volume.

f Calculate this fee in CHF. Give your answer correct to **two decimal places**. *[2 marks]*

One third of the capacity of the reservoir will be used by Pemberley. 1 m^3 is equal to 1000 litres.

g Calculate the number of litres of water that Pemberley will use. *[2 marks]*

h Give your answer to **g** in the form $a \times 10^k$, where $1 \leq a < 10$, and $k \in \mathbb{Z}$. *[2 marks]*

5 Consider the function $f(x) = x^2 + \frac{2}{x}$, $x \neq 0$

a Sketch the graph of $f(x)$ for $-3 \leq x \leq 3$ and $-10 \leq y \leq 10$. Indicate clearly any asymptotes to the graph. *[4 marks]*

b Write down the x-intercept of the graph of $f(x)$. *[1 mark]*

c Find $f'(x)$. *[3 marks]*

The graph of $f(x)$ has a local minimum at point P.

d Use your answer to **c** to show that the x coordinate of P is 1. *[3 marks]*

e Write down the y coordinate of P. *[1 mark]*

f Describe the behavior of the graph of $f(x)$ in the interval $x > 1$. *[2 marks]*

Let T be the tangent to the graph of $f(x)$ at $x = -2$.

g i Find the gradient of the graph of $f(x)$ at $x = -2$.

ii Write down the equation of T. Give your answer in the form $ax + by + d = 0$. *[5 marks]*

h Find the distance between P and the point of intersection of T with the y-axis. *[3 marks]*

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Answers

Chapter 1

Skills check

1 a −0.033 b −12.1
c 0.88

2 a $x = 7$ b $x = 8$
c $x = 1$ d $x = 4, x = -4$

3 a 96 b 0.234

4 a $x \geq 9$ 

b $x > 6$ 

c $x \leq 0$ 

5 a 5 b $\frac{1}{2}$
c 2 d 50

Exercise 1A

a i 8 ii 12
iii −12 iv 4

b i Natural ii Natural
iii Not natural iv Natural

Investigation - natural numbers

a T b T
c F eg. 3 − 8 = −5. Negative numbers are not natural.

Exercise 1B

1 a $x = -\frac{1}{2}$ b Not an integer

2 a $x = 2; x = -2$
b Both are integers

3 a i −3 ii 9.75
b i Integer ii Not an integer

Investigation - integers

a T b T
c F eg. $\frac{1}{2} = 0.5$ d T

Exercise 1C

1 a $\frac{2}{3} = 0.6666...$, $-\frac{5}{4} = -1.25$, $\frac{2}{9} = 0.2222...$, $\frac{4}{7} = 0.57114285...$, $\frac{-11}{5} = -2.2$

b i $-\frac{5}{4}, \frac{-11}{5}$ ii $\frac{2}{3}, \frac{2}{9}, \frac{4}{7}$

2 a $\frac{5}{9}$ b $\frac{17}{9}$ c $\frac{22}{9}$

3 a For example 0.8
b For example 0.12
c For example 3.4578

Exercise 1D

1 For example 2.1, 2.2, 2.23

2 a 2.5 b It is rational

3 a For example 1.81; 1.82; 1.83
b i For example −2.14; −2.12; −2.1
ii infinite

Investigation - rational numbers

a T b T
c T b F eg. $\sqrt{2}$

Exercise 1E

1 a 2.5 cm b rational

2 a (25π) cm² b irrational

Exercise 1F

1 a i $1 < x \leq 3$ ii $x \leq 2$
b i 

ii 

c i Both are solutions
ii q is solution and t isn't.

2 a i $x > -1$ ii $3 \leq x \leq 7$
iii $x < 3$
b i 

ii 

iii 

p \ Inequality	$2x+1>-1$	$4 \leq x+1 \leq 8$	$2-x>-1$
$-\frac{2}{3}$	✓		✓
$\sqrt{10}$	✓	✓	
2π	✓	✓	

Exercise 1G

1 a 358 b 25
c 109 d 10 016

2 a 250 b 110
c 1020 d 270

3 a 100 b 200
c 1200 d 3100

4 a 106 000 b 2000
c 10 000 d 1000

5 Any x where $150 \leq x < 250$

6 Any x where $2500 \leq x < 3500$

7 Any x where $5.5 \leq x < 6.5$

Exercise 1H

1 a 45.7 b 301.1
c 2.4 d 0.1

2 a 0.00 b 201.31
c 9.62 d 28.08

3 a 10.049 b 3.900
c 201.781 d 0.008

4 a 3025.0 b 3024.98
c 3024.984 d 3000
e 3000

5 a 15.60 b 15.603
c 16 d 20

6 Any x where $2.365 \leq x < 2.375$

7 Any x where $4.05 \leq x < 4.15$

Exercise 1I

1 a 3 b 1, 2 or 3
c 1 d 3 or 4
e 4

2 a 300 b 0.07
c 400 d 0.001

3 a 360 b 0.080
c 1.1 d 1600

4 a 2970 b 0.326
c 10 400 d 0.501

5 a 400 b 426
c 425.9 d 425.88

6 a 3 b 3.14
c 3.1 d 3.142

7 a 200 b 4610
c 2.70

8 a 0.3703703704
b i 0.37
ii 0.370
iii 0.3704

Exercise 1J

1 a 1.828 cm b 11 cm

2 a 2.288 b 20.9
c 4.5 cm²

Exercise 1K

1 a 3000 b 16
c 15 d 10

2 4000 pipes

3 300 people per km²

4 20 reams

5 15 km h⁻¹

6 20 000 000 visitors per year

7 Peter is not correct.
An estimate of the area is 10 000 m².