

d Show that the distance MR is 29 km correct to the nearest km.

This water reservoir will be in the shape of a square of side 150 m and have a depth of 2.85 m

e Calculate the volume of the reservoir.

To construct the reservoir, the company will pay a fee of 1.25 Swiss Francs (CHF) per m^3 of its volume.

f Calculate this fee in CHF. Give your answer correct to **two decimal places**.

One third of the capacity of the reservoir will be used by Pemberley. 1 m^3 is equal to 1000 litres.

g Calculate the number of litres of water that Pemberley will use.

h Give your answer to g in the form $a \times 10^k$, where $1 \leq a < 10$, and $k \in \mathbb{Z}$.

5 Consider the function $f(x) = x^2 + \frac{2}{x}$, $x \neq 0$

a Sketch the graph of $f(x)$ for $-3 \leq x \leq 3$ and $-10 \leq y \leq 10$.

Indicate clearly any asymptotes to the graph.

b Write down the x -intercept of the graph of $f(x)$.

c Find $f'(x)$.

The graph of $f(x)$ has a local minimum at point P.

d Use your answer to c to show that the x coordinate of P is 1.

e Write down the y coordinate of P.

f Describe the behavior of the graph of $f(x)$ in the interval $x > 1$.

Let T be the tangent to the graph of $f(x)$ at $x = -2$.

g i Find the gradient of the graph of $f(x)$ at $x = -2$.

ii Write down the equation of T . Give your answer in the form $ax + by + d = 0$.

h Find the distance between P and the point of intersection of T with the y -axis.

[4 marks]

[2 marks]

[2 marks]

[2 marks]

[2 marks]

[4 marks]

[1 mark]

[3 marks]

[3 marks]

[1 mark]

[2 marks]

[5 marks]




[3 marks]

Use the mark scheme in the Answer section at the back of this book to mark your answers to this practice paper.

Answers

Chapter 1

Skills check

- 1 a -0.033 b -12.1
c 0.88
- 2 a $x = 7$ b $x = 8$
c $x = 1$ d $x = 4, x = -4$
- 3 a 96 b 0.234
- 4 a $x \geq 9$ 
b $x > 6$ 
c $x \leq 0$ 
- 5 a 5 b $\frac{1}{2}$
c 2 d 50

Exercise 1A

- a i 8 ii 12
iii -12 iv 4
- b i Natural ii Natural
iii Not natural iv Natural

Investigation - natural numbers

- a T b T
c F eg. $3 - 8 = -5$. Negative numbers are not natural.

Exercise 1B

- 1 a $x = -\frac{1}{2}$ b Not an integer
2 a $x = 2; x = -2$
b Both are integers
3 a i -3 ii 9.75
b i Integer ii Not an integer

Investigation - integers

- a T b T
c F eg. $\frac{1}{2} = 0.5$ d T

Exercise 1C

- 1 a $\frac{2}{3} = 0.6666\dots$, $-\frac{5}{4} = -1.25$,
 $\frac{2}{9} = 0.2222\dots$,
 $\frac{4}{7} = 0.5714285\dots$,
 $-\frac{11}{5} = -2.2$
b i $-\frac{5}{4}, -\frac{11}{5}$ ii $\frac{2}{3}, \frac{2}{9}, \frac{4}{7}$
- 2 a $\frac{5}{9}$ b $\frac{17}{9}$ c $\frac{22}{9}$

- 3 a For example 0.8
b For example 0.12
c For example 3.4578

Exercise 1D

- 1 For example $2.1, 2.2, 2.23$
- 2 a 2.5 b It is rational
- 3 a For example $1.81; 1.82; 1.83$
b i For example $-2.14; -2.12; -2.1$
ii infinite






Investigation - rational numbers

- a T b T
c T b F eg. $\sqrt{2}$

Exercise 1E

- 1 a 2.5 cm b rational
2 a $(25\pi) \text{ cm}^2$ b irrational

Exercise 1F

- 1 a i $1 < x \leq 3$ ii $x \leq 2$
b i 
ii 
c i Both are solutions
ii q is solution and t isn't.
- 2 a i $x > -1$ ii $3 \leq x \leq 7$
iii $x < 3$
b i 
ii 
iii 

Inequality	$2x+1 > -1$	$4 \leq x+1 \leq 8$	$2-x > -1$
$\frac{2}{3}$	✓		✓
$\sqrt{10}$	✓	✓	
2π	✓	✓	

Exercise 1G

- 1 a 358 b 25
c 109 d $10\,016$
- 2 a 250 b 110
c 1020 d 270
- 3 a 100 b 200
c 1200 d 3100
- 4 a $106\,000$ b 2000
c $10\,000$ d 1000
- 5 Any x where $150 \leq x < 250$
- 6 Any x where $2500 \leq x < 3500$
- 7 Any x where $5.5 \leq x < 6.5$

Exercise 1H

- 1 a 45.7 b 301.1
c 2.4 d 0.1
- 2 a 0.00 b 201.31
c 9.62 d 28.08
- 3 a 10.049 b 3.900
c 201.781 d 0.008
- 4 a 3025.0 b 3024.98
c 3024.984 d 3000
e 3000
- 5 a 15.60 b 15.603
c 16 d 20
- 6 Any x where $2.365 \leq x < 2.375$
- 7 Any x where $4.05 \leq x < 4.15$

Exercise 1I

- 1 a 3 b $1, 2$ or 3
c 1 d 3 or 4
e 4

- 2 a 300 b 0.07
c 400 d 0.001
- 3 a 360 b 0.080
c 1.1 d 1600
- 4 a 2970 b 0.326
c $10\,400$ d 0.501
- 5 a 400 b 426
c 425.9 d 425.88
- 6 a 3 b 3.14
c 3.1 d 3.142
- 7 a 200 b 4610
c 2.70
- 8 a 0.3703703704
b i 0.37
ii 0.370
iii 0.3704

Exercise 1J

- 1 a 1.828 cm b 11 cm
2 a 2.288 b 20.9
c 4.5 cm^2

Exercise 1K

- 1 a 3000 b 16
c 15 d 10
- 2 4000 pipes
- 3 300 people per km^2
- 4 20 reams
- 5 15 km h^{-1}
- 6 $20\,000\,000$ visitors per year
- 7 Peter is not correct.
An estimate of the area is $10\,000 \text{ m}^2$.

Exercise 1L

- 1 a 119.423 b 17.2% (3 sf)
 2 a 8.17 (3 sf) b 8
 c 2.04% (3 sf)
 3 a 18.5832 m²
 b 5.3 m and 3.5 m
 c 0.179% (3 sf)
 4 a 5.323 m b 33.4 m (3 sf)
 c 10% (2 sf)

Exercise 1M

- 1 2.5×10^{-3} , 10^{10}
 2 a 1.356×10^5 b 2.45×10^{-3}
 c 1.6×10^{10} d 1.08×10^{-4}
 e 2.3×10^2
 3 3.4×10^9 , 0.21×10^7 ,
 215×10^4 , 2.3×10^6
 4 3.621×10^4 , 0.3621×10^4 ,
 3.261×10^3 , 31.62×10^2

Exercise 1N

- 1 a 1.764×10^{17} b 2.25×10^{-4}
 c 1.5×10^{-2}
 2 a 2.99×10^6
 b 3 000 000 or 3×10^6
 3 a 2.205×10^9 b 700
 c 7×10^2
 4 a 2.25×10^{10}
 b True because
 $x^2 = 5.06 \times 10^{20} > 10^{20}$
 c i 150 000 ii 1.5×10^5

Investigation - SI units

- a Many different names
 eg. mm, cm, km
 b $10^6 = M$ (mega), $10^{-6} = \mu$ (micro)
 c length eg. Millimetres or
 centimetres weight eg. Kilograms

Exercise 1O

- 1 a km h^{-2} b kg m^{-3}
 c ms^{-1}
 2 a decagram
 b centisecond
 c millimetre
 d decimetre
 3 a 32 000 m b 0.087 dam
 c 1.28 m
 4 a 0.5 kg b 35 700 dag
 c 1.080 hg
 5 a 80 ms b 120 das
 c 800 ds
 6 a 68 kg b 36 km
 c 6.54×10^2 mg

Exercise 1P

- 1 a $23\,600 \text{ cm}^2$
 b $0.000\,15 \text{ dam}^2$
 c 54 cm^2
 d $60\,000 \text{ mm}^2$
 e 80 hm^2
 f 0.035 km^2
 2 a $5 \times 10^6 \text{ cm}^3$
 b 10^2 m^3
 c $3.5 \times 10^9 \text{ dm}^3$
 d $2.55 \times 10^{11} \text{ mm}^3$
 e $1.2 \times 10^1 \text{ dam}^3$
 f $7.802 \times 10^2 \text{ dam}^3$
 3 a 169 cm^2 b 0.0169 m^2
 4 a $0.614\,125 \text{ m}^3$
 or 0.614 m^3 (3 sf)
 b $614\,125 \text{ cm}^3$ or
 $614\,000 \text{ cm}^3$ (3 sf)
 5 7560 cm^2 , 0.8 m^2 , 82 dm^2 ,
 $8\,000\,000 \text{ mm}^2$, 0.081 dam^2
 6 1200 dm^3 , 0.01 dam^3 , $10\,900\,000 \text{ cm}^3$,
 $11\,020\,000\,000 \text{ mm}^3$, 11.2 m^3

Exercise 1Q

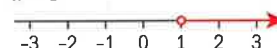
- 1 a 94 980 s b 95 000 s
 2 a 173 100 s
 b $1.731 \times 10^5 \text{ s}$ or $1.73 \times 10^5 \text{ s}$
 3 a 5000 ml
 b 0.000 005 6 hl
 c 4 500 000 cl
 4 a $5 \times 10^5 \text{ cm}^3$
 b $1.458 \times 10^1 \text{ dm}^3$
 c $8 \times 10^5 \text{ cm}^3$
 5 a 13 l b 4 hl
 c 81 cl
 6 a 75 min b 4500 s
 7 a 3.375 m^3 b 3375 dm^3
 c No, only 3375 l can be poured.
 8 a 0.176 l b 8 tea cups
 9 a 8.625 h b 696.5 km h^{-1}
 c 10:08 p.m. (nearest minute,
 Buenos Aires time)

Exercise 1R

- 1 a 6.9°C b 26.7°C
 2 a 70°F b 36°F
 3 a 16.85°C b 62.33°F
 4 a $t_K = t_C + 273.15$
 b $t_C = \frac{5}{9}(t_F - 32)$

Review exercise

Paper 1 style questions

- 1
- | | | | | | |
|---|---|-----------------|----|---------------|-----|
| | 5 | $\frac{\pi}{2}$ | -3 | $\frac{5}{4}$ | 2.3 |
| N | ✓ | | | | |
| Z | ✓ | | ✓ | | |
| Q | ✓ | | ✓ | ✓ | ✓ |
| R | ✓ | ✓ | ✓ | ✓ | ✓ |
- 2 a $\sqrt{2}$ b 1.4142
 c $0.001\,39 \times 10^2$, 14.1×10^{-1} , $\sqrt{2}$,
 1414×10^{-2} , 1.4×10^2
 3 a $2.69 \times 10^3 \text{ kg}$
 b i 2700 kg ii 0.372% (3 sf)
 4 a $300\,000\,000 \text{ ms}^{-1}$
 b 300 000 km
 c $1.08 \times 10^9 \text{ km h}^{-1}$
 5 a 0.58 kg b 0.6 kg
 c 33.3%
 6 a 1.56 dm^3 b 1.17 l
 c i 21 ii 0.43 l
 7 a 31.25 b 31.3
 c 3.13×10^1
 8 a $A = x^2$
 b i 1600 m ii 6400 m
 9 a 80.33°F b 311 K
 10 a $x > 1$
 b 
 c $\sqrt{3}$; 2.06; $\frac{101}{100}$
 11 a $62\,370 \text{ mm}^2$
 b $0.062\,370 \text{ m}^2$
 c 4.68 g d 2.34 kg

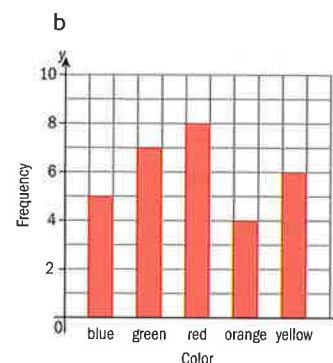
Paper 2 style questions

- 1 a 7.52 km b $\$2463.85$
 c 1.06% d 3.15 km^2
 2 a 2857 m b 4 laps
 c 0.150 h d 44.842 minutes
 e 1.88%
 3 a 8.18 cm^3 b 73.63 cm^3
 c $\frac{15}{2.5} = 6$ d 24.5 cm^3
 e $24\,500 \text{ mm}^3$ f $2.45 \times 10^4 \text{ mm}^3$
 or
 $2.45 \times 10^1 \text{ cm}^3$

Chapter 2

Skill check

- 1 (Example)
 Which age group do you belong to?
 ‡ under 16 † over 16
 Are you
 ‡ male? † female?
 2 a blue * * * * *
 green * * * * *
 red * * * * *
 orange * * * * *
 yellow * * * * *
 Key: * = 1 sweet



- 3 Axes drawn to scale of 1 cm to 2 units
 on the x-axis, 1 cm to 10 units on the
 y-axis

Investigation - population distribution

Tower Hamlets has a greater proportion
 of younger people compared to the UK
 population, whereas Christchurch has
 a greater proportion of older people
 compared to the UK population.

Tower Hamlets is in the city of London
 where there are many jobs and schools
 and therefore younger people are more
 likely to live there.

Exercise 2A

- 1 a Discrete b Continuous
 c Discrete d Discrete

- e Continuous f Discrete
 g Continuous h Continuous
 i Continuous j Discrete
 k Continuous l Discrete
 2 a Biased b Random
 c Biased d Random
 e Biased

- 6 $m = 6$, $n = 3$

Exercise 2C

- 1 Answers will depend on width of
 class intervals chosen. Example:

a

Number	Frequency
$0 \leq x < 5$	1
$5 \leq x < 10$	7
$10 \leq x < 15$	3
$15 \leq x < 20$	4
$20 \leq x < 25$	6
$25 \leq x < 30$	1
$30 \leq x < 35$	5
$35 \leq x < 40$	0
$40 \leq x < 45$	2
$45 \leq x < 50$	1

b

Number	Frequency
$10 \leq x < 20$	7
$20 \leq x < 30$	5
$30 \leq x < 40$	7
$40 \leq x < 50$	5
$50 \leq x < 60$	7
$60 \leq x < 70$	5
$70 \leq x < 80$	5
$80 \leq x < 90$	2
$90 \leq x < 100$	2

c

Number	Frequency
$1 \leq x < 3$	3
$3 \leq x < 5$	7
$5 \leq x < 7$	4
$7 \leq x < 9$	3
$9 \leq x < 11$	6
$11 \leq x < 13$	3
$13 \leq x < 15$	4
$15 \leq x < 17$	3
$17 \leq x < 19$	1
$19 \leq x < 21$	1

Exercise 2D

1 a

Class	Lower boundary	Upper boundary
9-12	8.5	12.5
13-16	12.5	16.5
17-20	16.5	20.5
21-24	20.5	24.5

b

Time (t seconds)	Lower boundary	Upper boundary
$2.0 \leq t < 2.2$	2.0	2.2
$2.2 \leq t < 2.4$	2.2	2.4
$2.4 \leq t < 2.6$	2.4	2.6

Exercise 2B

1

Number of goals	Frequency
0	4
1	7
2	7
3	4
4	1
5	2

2

Number of heads	Frequency
0	1
1	1
2	4
3	4
4	3
5	7
6	9
7	4
8	5
9	2
10	4
11	3
12	3

3

Age	Frequency
9	4
10	9
11	8
12	7
13	4
14	1
15	4
16	3

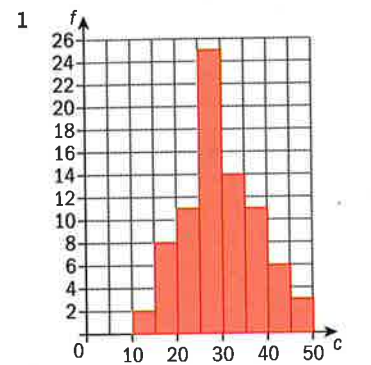
4

Number of crisps	Frequency
88	3
89	6
90	16
91	3
92	2

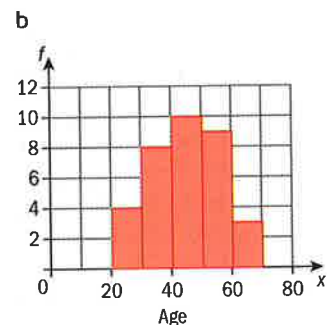
5

Number	Frequency
1	7
2	9
3	11
4	6
5	7
6	10

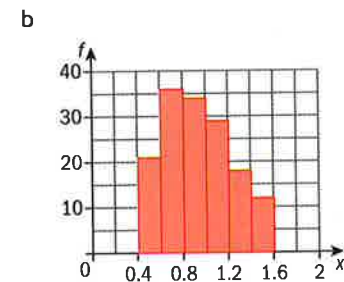
Exercise 2E



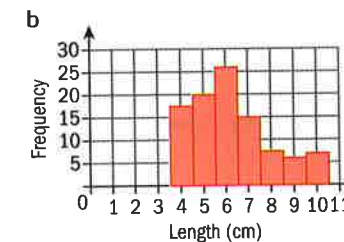
- 2 a Lower boundaries are 20, 30, 40, 50, 60
Upper boundaries are 30, 40, 50, 60, 70



- 3 a Lower boundary of the third class is 0.8 and the upper boundary is 1.0

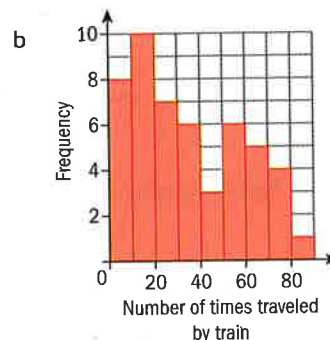


- 4 a Lower boundaries are 3.5, 4.5, 5.5, 6.5, 7.5, 8.5, 9.5
Upper boundaries are 4.5, 5.5, 6.5, 7.5, 8.5, 9.5, 10.5



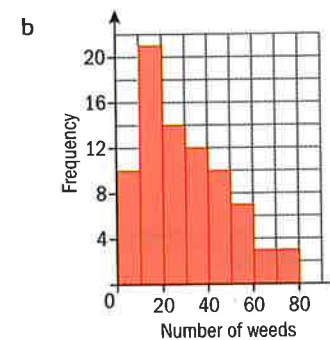
5 a

Number	Frequency
$0 \leq x < 10$	8
$10 \leq x < 20$	10
$20 \leq x < 30$	7
$30 \leq x < 40$	6
$40 \leq x < 50$	3
$50 \leq x < 60$	6
$60 \leq x < 70$	5
$70 \leq x < 80$	4
$80 \leq x < 90$	1

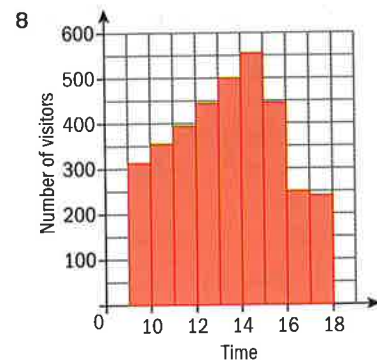
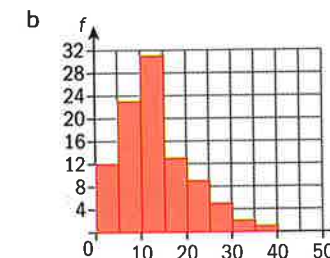


6 a

Number of weeds	frequency
$0 \leq x < 10$	10
$10 \leq x < 20$	21
$20 \leq x < 30$	14
$30 \leq x < 40$	12
$40 \leq x < 50$	10
$50 \leq x < 60$	7
$60 \leq x < 70$	3
$70 \leq x < 80$	3



- 7 a The lower boundary of the fourth group is 15.5 and the upper boundary is 20.5



Exercise 2F

- 1 a mode = 1, median = 7, mean = 5.57 (3 sf)
b mode = 5, median = 5, mean = 5.92 (3 sf)
- 2 a 1.71, b 1.52 c 31.3
d 54.9 e 58.0
- 3 a median = 24.45 kg
b mean = 25.0 kg
- 4 $s = 5, t = 11$
- 5 a mean = 65 b 77
- 6 a 75 b 86

Exercise 2G

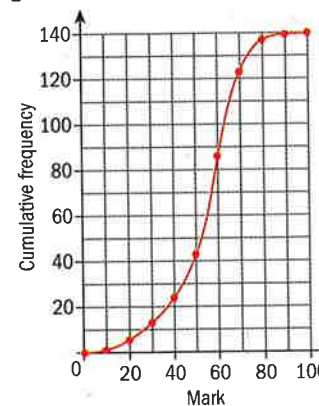
- 1 a 4 b 4 c 3.31 (3 sf)
- 2 a 25 b 2 c 2.2
- 3 a $n = 7$ b 3.7 c 4
- 4 a 4.53 (3 sf) b 55% c 4

Exercise 2H

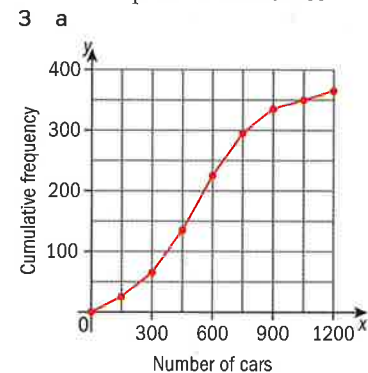
- 1 a modal class is 24–26, $24 \leq t < 26$
b mean = 25.88 min
- 2 a $70 \leq s < 80$
b mean = 88.3 km h^{-1} (3 sf)
- 3 a modal class is 40–50
b mean = 51.8

Exercise 2I

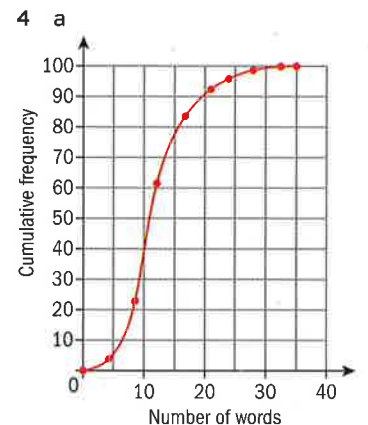
- 1 a 50 b $a = 8, b = 14, c = 38$
- 2 a $s = 13, t = 122$



- c i Median mark is about 60
ii Lower quartile approximately 46
iii 60% of 140 = 84 therefore pass mark is about 60



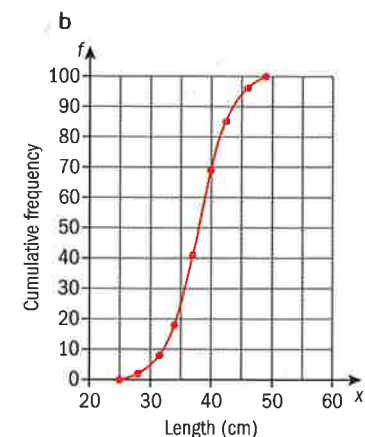
- b Median is approximately 525 and interquartile range $690 - 370 = 320$
- c More than 800 cars on approximately $365 - 310 = 55$ days so, $\frac{55}{365} \times 100 = 15.1\%$ of the days.



- b Lower quartile is approximately 8.5, upper quartile is approximately 14.5, median is approximately 11.5

5 a

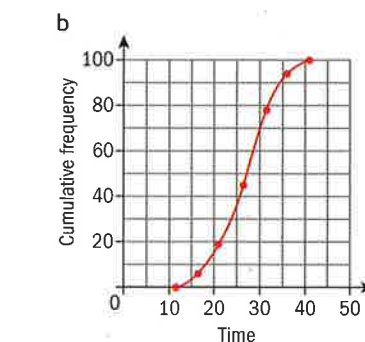
Length (x cm)	Cumulative frequency
≤ 28	3
≤ 31	7
≤ 34	18
≤ 37	41
≤ 40	69
≤ 43	84
≤ 46	96
≤ 49	100



- c i 38 cm
ii $41 - 35 = 6$ cm

6 a

Time (t min)	Cumulative frequency
< 15.5	6
< 20.5	19
< 25.5	46
< 30.5	77
< 35.5	92
< 40.5	100



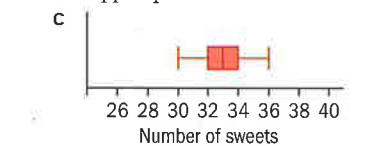
- c i 26 min ii $30 - 22 = 8$ min
iii 30 min

Exercise 2J

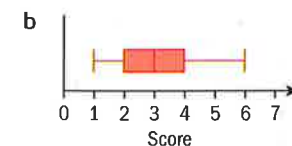
1 a

Sweets	Frequency
30	1
31	3
32	10
33	16
34	9
35	4
36	2

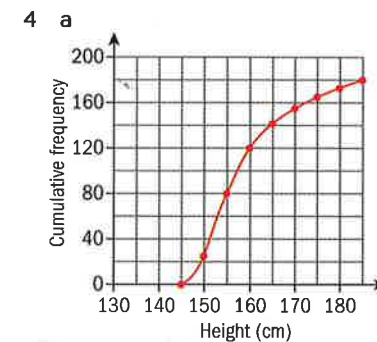
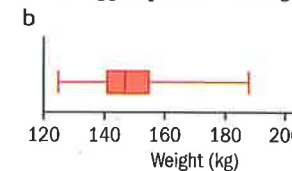
- b Median = 33, lower quartile = 32, upper quartile = 34



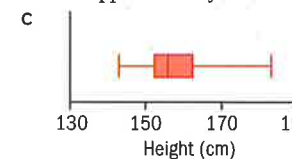
- 2 a Median = 3, lower quartile = 2, upper quartile = 4



- 3 a i median = 147 kg
ii lower quartile = 141 kg
iii upper quartile = 155 kg

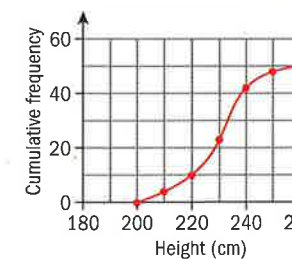


- b i median is approximately 156 cm
ii Upper quartile approximately 163 cm, lower quartile approximately 152 cm

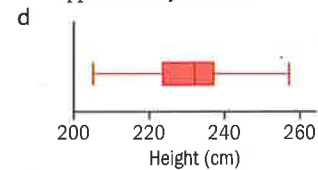


5 a

Height (x cm)	Cumulative frequency
< 210	4
< 220	10
< 230	21
< 240	43
< 250	48
< 260	50



- b Median is about 232 cm
 c Lower quartile approximately 223 cm, upper quartile approximately 237 cm



Exercise 2K

- 1 a median for boys = 55
 median for girls = 55
 b IQR boys = $64 - 40 = 24$
 IQR girls = $68 - 45 = 23$
 c 50% d 25%
- 2 a 0 b 12
 c 14 d 28 e 25%
- 3 a 22 b 44
 c 53 d 22

Exercise 2L

- 1 a i 19 ii 6
 b i 13 ii 4
 c i 7 ii 4.5

Exercise 2M

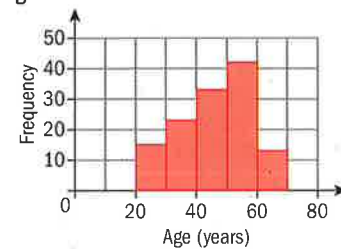
- 1 a standard deviation = 3.17
 b standard deviation = 2.29
- 2 mean = 8.8
 standard deviation = 5.44
- 3 a range = 5 b IQR = 2
 c mean = 3.26
 standard deviation = 1.28
- 4 a range = 6 b IQR = 2
 c mean = 7.32
 standard deviation = 1.41
- 5 a mean = 67.2
 b standard deviation = 4.94
 c range = 18 d IQR = 6
- 6 a $x = 45$
 b standard deviation = 15.6
 c range = 46
 d IQR = 27
- 7 a $m = 9$ b mean = 12.7
 c standard deviation = 1.49
 d IQR = 2
- 8 a range = 7, IQR = 3
 b mean = 7.92
 standard deviation = 1.87
- 9 mean = 32 min
 standard deviation = 7.57 min
- 10 a girls' mean = 55.4 and
 standard deviation = 11.5
 boys' mean = 51.8 and
 standard deviation = 23.1

- b There is a big difference in the standard deviation implying that the boys' marks are much more widespread than the girls' marks.

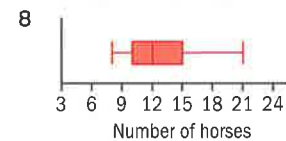
Review exercise

Paper 1 style questions

- 1 a 9 b 5.5
 2 a 1 b 5.5
 3 a 6.62 b 6 c 6
 4 a i 6.54 m ii 3.08 m
 b 6.1 m
 5 a discrete b 1.93
 c 1.25
 6 a 46.2
 b



- 7 a 41 b 31
 c 49 d 18



Paper 2 style questions

- 1 a i mean = 98
 ii mode = 96

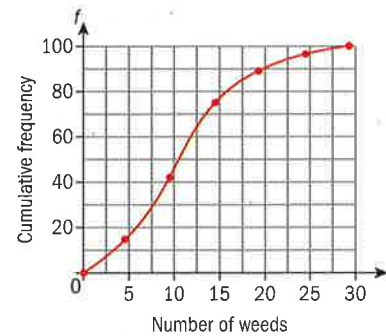
b i

Number	Frequency
94	1
96	4
97	3
98	3
99	3
100	3
101	2

- ii median = 98, IQR = 4

2 a i

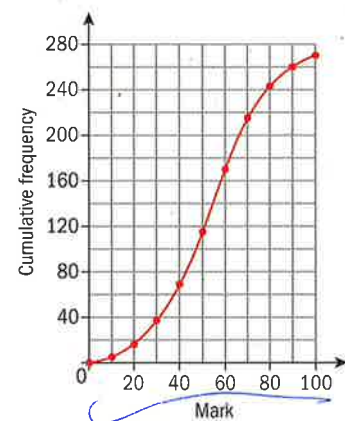
Number	Cumulative frequency
< 4.5	18
< 9.5	43
< 14.5	75
< 19.5	89
< 24.5	96
< 29.5	100



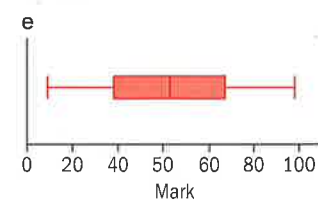
- ii Median is approximately 10.6
 iii 11%
- b i Mean is approximately 10.95
 ii Standard deviation is approximately 6.53
 iii Total number of weeds is approximately 8760000

3 a

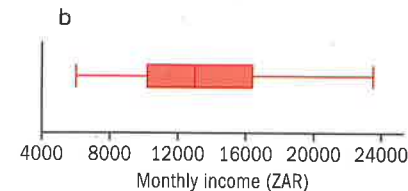
Mark	Cumulative frequency
< 10	3
< 20	17
< 30	38
< 40	73
< 50	115
< 60	170
< 70	213
< 80	245
< 90	260
< 100	270



- b 53 c $67 - 38 = 29$
 d 48



- 4 a Median is approximately 13 000, IQR is approximately 6200



c

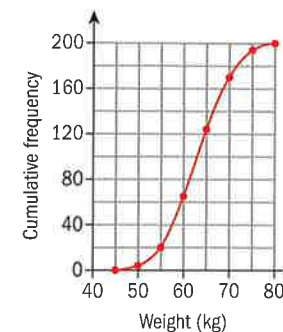
Monthly Income (ZAR)	Frequency
$6000 \leq x < 8000$	10
$8000 \leq x < 10000$	19
$10000 \leq x < 12000$	30
$12000 \leq x < 14000$	29
$14000 \leq x < 16000$	20
$16000 \leq x < 18000$	15
$18000 \leq x < 20000$	11
$20000 \leq x < 22000$	9
$22000 \leq x < 24000$	7

- d Mean is approximately 13 747 and standard deviation 4237

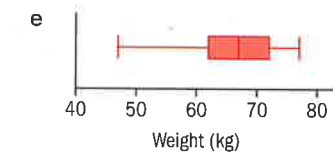
- 5 a Modal group is 60–65, $60 \leq w < 65$
 b Mean is approximately 63.2 and standard deviation 6.62

c

Weight (kg)	Cumulative frequency
< 50	4
< 55	20
< 60	65
< 65	123
< 70	166
< 75	194
< 80	200



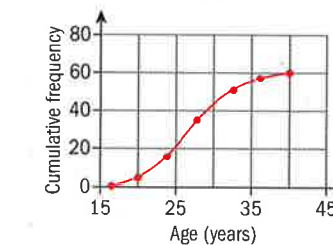
- d median = 63, lower quartile = 59, upper quartile = 68



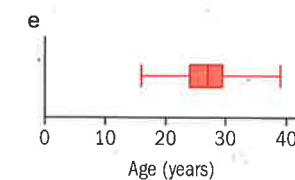
- 6 a Mean is approximately 26.9 and standard deviation 4.40
 b Modal class is 24–28

c

Age (years)	Cumulative frequency
≤ 20	3
≤ 24	15
≤ 28	37
≤ 32	52
≤ 36	59
≤ 40	60



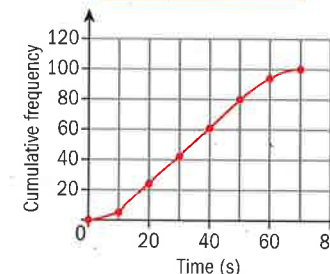
- d median = 27, IQR = 5.5



- 7 a Modal class is 30–40
 b Estimate of mean is 34.3 and standard deviation is 16.6

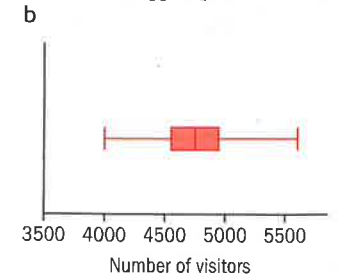
c

Time (s)	Cumulative frequency
< 10	5
< 20	24
< 30	42
< 40	64
< 50	80
< 60	92
< 70	100



- d median = 33, IQR = 25.5

- 8 a median = 4750, lower quartile = 4570, upper quartile = 5000



c

Visitors	Frequency
$4000 \leq x < 4200$	1
$4200 \leq x < 4400$	3
$4400 \leq x < 4600$	5
$4600 \leq x < 4800$	9
$4800 \leq x < 5000$	6
$5000 \leq x < 5200$	3
$5200 \leq x < 5400$	2
$5400 \leq x < 5600$	1
$5600 \leq x < 5800$	1

- d Modal class is 4600–4800
 e Estimate of mean is 4784 and standard deviation is 355

9 a

Weight (x kg)	Frequency
$120 \leq x < 130$	10
$130 \leq x < 140$	35
$140 \leq x < 150$	75
$150 \leq x < 160$	50
$160 \leq x < 170$	15
$170 \leq x < 180$	10
$180 \leq x < 190$	5

- b Modal class is $140 \leq x < 150$
 c Estimate of mean is 149

Chapter 3

Skills check

- 1 a $h = 20$ cm
 b $\sqrt{50}$ cm = 7.07 (3 sf)
- 2 a i (0, 6) ii $\sqrt{40} = 6.32$ (3 sf)
 b $q = 3, p = 6$

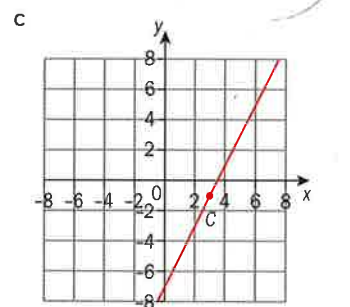
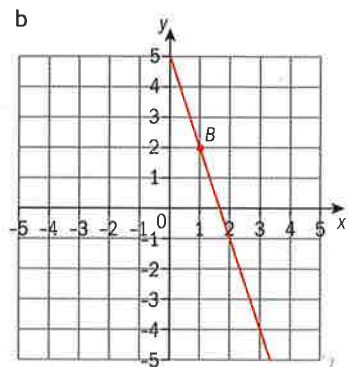
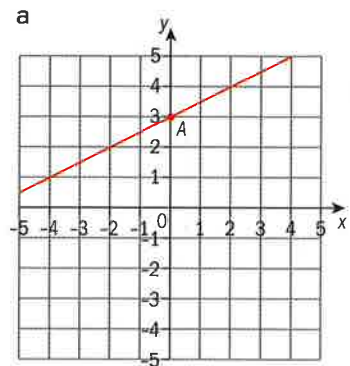
Exercise 3A

- 1 a -1 b 8 c -8 d 1
 2 a i A(1, 5), B(0, 1) ii 4
 b i A(-1, 5), B(0, 1), ii -4
 c i A(0, 3), B(3, 2), ii $-\frac{1}{3}$
 d i A(0, -1), B(1, 0), ii 1

- e i $A(-1, -2), B(2, 0)$ ii $\frac{2}{3}$
 f i $A(2, 4), B(4, 1)$ ii $-\frac{3}{2}$

Exercise 3B

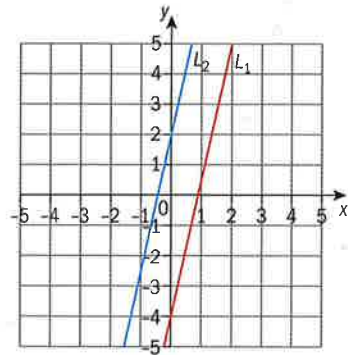
1



- 2 a i 2 ii $p = 9$
 b i 4 ii $t = 10$
 c i -5 ii $q = -10$
 d i 1 ii $s = 3$
 e i -3 ii $r = -2$
 3 a $\frac{5}{a+1}$ b $a = \frac{1}{4}$
 4 a 0.5 b $\frac{t-6}{-5}$ c $t = 3.5$

Exercise 3C

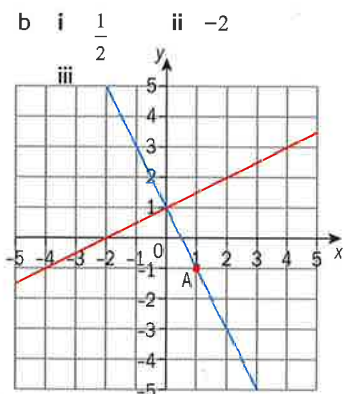
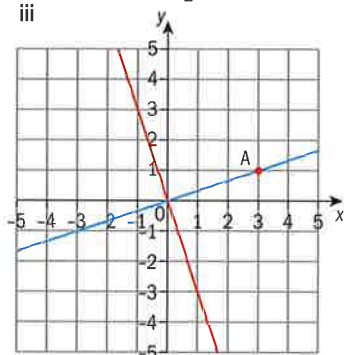
- 1 a 4.5
 b & c



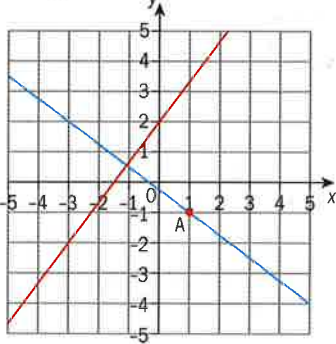
- 2 a Parallel to x -axis
 b Parallel to y -axis c Neither
 3 a x b y c zero
 4 a = 3 b $m = -5$

Exercise 3D

- 1 a, b, d
 2 b, d
 3 a $\frac{1}{3}$ b -1.5
 c 4 d -1 e 1
 4 a $\frac{3}{7}$ b $-\frac{5}{12}$
 5 a i -3 ii $\frac{1}{2}$
 iii



- c i $\frac{4}{3}$ ii $-\frac{3}{4}$
 iii



- 6 a $\frac{a-3}{-2}$ b $-\frac{1}{2}$ c $a = 4$
 7 a $-\frac{13}{2}$ b $\frac{2}{13}$ c $t = 18$

Exercise 3E

- 1 a $y = 3x + 1$ b $y = \frac{5}{3}x + \frac{4}{3}$
 c $y = -2x - 6$
 2 a i 2 ii (0, 1)
 iii $(-\frac{1}{2}, 0)$
 b i -3 ii (0, 2)
 iii $(\frac{2}{3}, 0)$
 c i -1 ii (0, 3)
 iii (3, 0)
 d i $-\frac{2}{5}$ ii (0, -1)
 iii $(-2.5, 0)$
 3 a $y = 1.5x - 9$ b 1.5
 c -9 d (6, 0)
 4 a -5 b $y = -5x + 6$
 5 a 2 b $y = 2x + 1$
 c -0.5 d $y = -0.5x + 2$
 6 a $-\frac{1}{3}$ b $y = -\frac{1}{3}x + \frac{8}{3}$
 c 8
 7 a $y = x + 1$ b $y = -x + 1$
 c $y = -0.5x - 1$
 d $y = 2x - 2$
 e $y = \frac{1}{3}x - \frac{1}{3}$
 f $y = \frac{4}{5}x - \frac{3}{5}$

Exercise 3F

- 1 a $4x + y - 20 = 0$
 b $x - 2y + 4 = 0$
 c $-5x - 4y + 7 = 0$
 d $x - y + 5 = 0$

- 2 a $y = -3x$ b $y = -x - 1$
 c $y = -2x + 1$ d $y = 0.5x$
 e $y = -2x + 3$
 3 a $y = 0.5x + 1$
 b $x = -2$ c $y = 1$
 4 a A, C, D, F
 b $a = 6.5$ c $t = 8$
 5 a A, B, E
 b $a = \frac{2}{3}$ c $t = 31$
 6

Line	Conditions
A	H
B	G
C	F
D	E

- 7 a 2 b 6 c $c = -2.25$
 d $t = 16$ e 2
 f $y = 2x + 4$ ($2x - y + 4 = 0$)
 8 a $y = -2x + 4$
 ($2x + y - 4 = 0$)
 b Yes, A, B and C are collinear.
 The coordinates of A, B and C
 all satisfy the equation of L_1 .

Investigation-vertical and horizontal lines

- 1 a $(-3, -1), (-3, 0), (-3, 1), (-3, 2)$
 and $(-3, 3)$
 b All the coordinates have the x -coordinate as -3.
 c To lie on L_1 , the x -coordinate must be -3. ie. $x = -3$
 2 a $(2, -1), (2, 0), (2, 1), (2, 2)$
 and $(2, 3)$
 b All the coordinates have the x -coordinate as 2.
 c To lie on L_3 , the x -coordinate must be 2. ie. $x = 2$
 3 $x = 1$
 4 a $(-1, 1), (0, 1), (1, 1), (2, 1)$
 and $(3, 1)$
 b All the coordinates have the y -coordinate as 1.
 c To lie on L_3 , the y -coordinate must be 1. ie. $y = 1$
 5 a $(-1, -2), (0, -2), (1, -2), (2, -2)$
 and $(3, -2)$
 b All the coordinates have the y -coordinate as -2.
 c To lie on L_4 , the y -coordinate must be -2. ie. $y = -2$
 6 $y = -3$

Exercise 3G

- 1 a $x = 3$ b $y = 1$
 2 a $(-2, 0)$ b (5, 1) c $(-7, 3)$
 d $(-2, 1)$ e (4, -1) f (0, 4)
 3 $L_1: y = 5x - 1$
 $L_2: y = 5x + 2$
 L_1 and L_2 have same gradient but different y -intercepts.
 4 a no point
 b an infinite number of points
 c only one point
 d infinite number of points
 5 a $y = 5x - 5$ ($5x - y - 5 = 0$)
 b $y = -\frac{1}{5}x + \frac{1}{5}$ ($x + 5y - 1 = 0$)

Investigation - right-angled triangles

- 1 Angles are identical
 2 $\frac{AE}{AC} = 1.5$
 3 $\frac{AD}{AB} = 1.5$
 4 $\frac{DE}{BC} = 1.5$

All the ratios are identical.

Exercise 3H

1

H	Opp	Adj
XY	YZ	XZ
CB	AB	AC
RQ	PQ	PR

- 2 a $\cos \delta = \frac{AC}{AB}$
 $\sin \delta = \frac{BC}{AB}$
 $\tan \delta = \frac{BC}{AC}$
 b $\cos \delta = \frac{QR}{PQ}$
 $\sin \delta = \frac{PR}{PQ}$
 $\tan \delta = \frac{PR}{QR}$
 c $\cos \delta = \frac{EF}{DF}$
 $\sin \delta = \frac{ED}{DF}$
 $\tan \delta = \frac{ED}{EF}$

- 3 a i $\sin \alpha = \frac{4}{\sqrt{41}}$
 ii $\cos \alpha = \frac{5}{\sqrt{41}}$
 iii $\tan \alpha = \frac{4}{5}$

- b i $\sin \alpha = \frac{\sqrt{28}}{8}$
 ii $\cos \alpha = \frac{6}{8}$
 iii $\tan \alpha = \frac{\sqrt{28}}{6}$
 c i $\sin \alpha = \frac{10}{14}$
 ii $\cos \alpha = \frac{\sqrt{96}}{14}$
 iii $\tan \alpha = \frac{10}{\sqrt{96}}$

- 4 a $\sin \beta = \frac{x}{10}$
 b $\cos \beta = \frac{x}{5}$
 c $\tan \beta = \frac{x}{12}$
 d $\tan \beta = \frac{7}{x}$
 e $\sin \beta = \frac{14}{x}$
 f $\cos \beta = \frac{3}{x}$

Exercise 3I

- 1 $h = 3.11$ cm 2 $x = 6.41$ cm
 3 $m = 4.88$ cm 4 $y = 13.94$ cm
 5 $t = 386.37$ m 6 $s = 86.60$ m

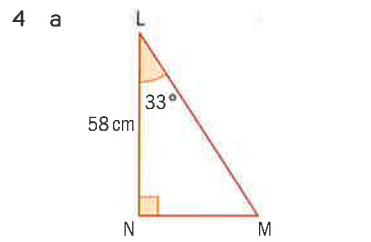
Exercise 3J

- 1 a P

 b $\hat{Q} = 69^\circ$ c $QR = 5.76$ cm
 2 a U

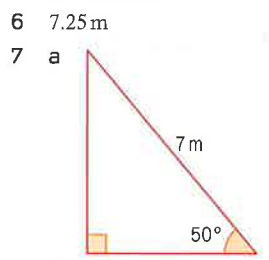
 b $\hat{S} = 35^\circ$ c $TU = 20.1$ cm
 3 a Z

 b $\hat{Z} = 75^\circ$ c $VZ = 8.04$ cm



b $\hat{M} = 57^\circ$ c $LM = 69.2$ cm

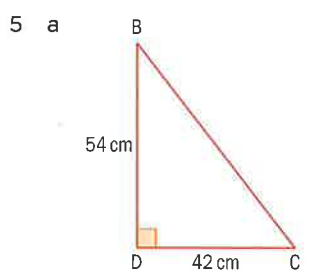
5 a 6.93 cm
b 37.9 cm
c 83.1 cm²



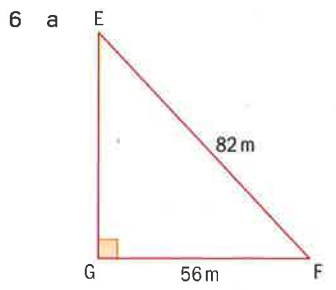
b 5.36 m c 4.50 m

Exercise 3K

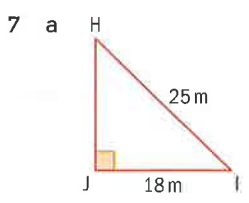
- a The angle with a sine of 0.6.
b The angle with a tangent of $\frac{1}{2}$.
c The angle with a cosine of $\frac{2}{3}$.
- a 36.9° b 26.6° c 48.2°
- a 11.5° b 48.2° c 45°
- a $\hat{A} = 53.6^\circ, \hat{C} = 36.4^\circ$
b $\hat{R} = 41.4^\circ, \hat{Q} = 48.6^\circ$
c $\hat{M} = 36.9^\circ, \hat{Q} = 53.1^\circ$
d $\hat{Y} = 41.4^\circ, \hat{Z} = 48.6^\circ$
- e $\hat{J} = 70.1^\circ, \hat{I} = 19.9^\circ$
f $\hat{D} = 25.9^\circ, \hat{F} = 64.1^\circ$



b $\hat{C} = 52.1^\circ$



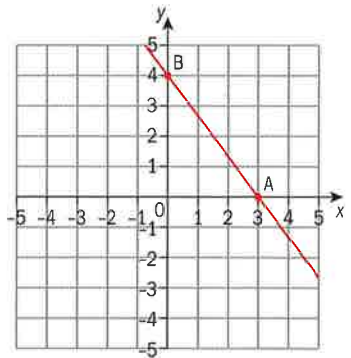
b $\hat{F} = 46.9^\circ$



b $\hat{H} = 46.1^\circ$

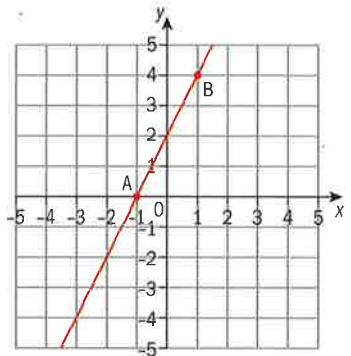
8 26.6°
9 57.0°
10 22.0°

11 a & b



c 53.1°

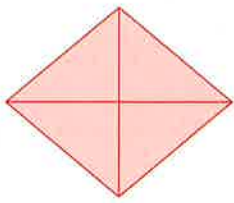
12 a & b



c 63.4°

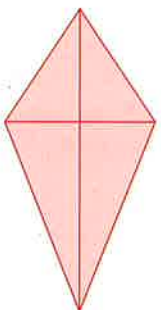
Investigation - 2-D shapes

1 The diagonals of a rhombus are perpendicular to each other.

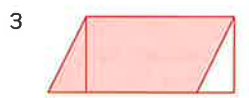


There are 4 right-angled triangles in the rhombus. They are congruent because all the sides are the same length.

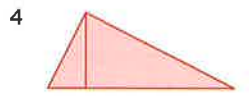
2 The diagonals of a kite are perpendicular to each other.



There are 4 right-angled triangles in the kite. They are not congruent because the diagonals are not of equal length.



Cutting the parallelogram as shown gives 3 shapes with 2 right-angled triangles. This explains why the area of a parallelogram is the same as the area of a rectangle, length x width.



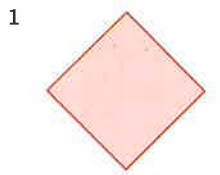
The height of the triangle is perpendicular to the base. The two right-angled triangles formed would be congruent if the triangle was equilateral or isosceles.

5 Triangles are congruent if $AD = BC$ on the trapezium.

6 ABO is an isosceles triangle since the two radii are the same length.

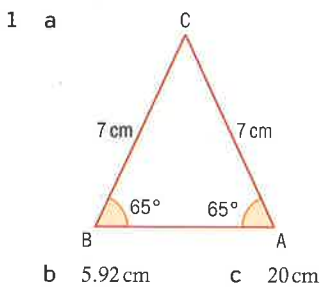
The two triangles formed are congruent because all the sides are of equal length.

Investigation - rhombus

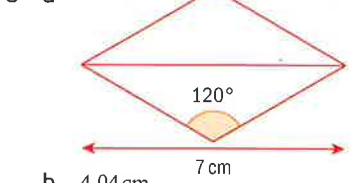


3 There are infinitely many different rhombuses with a side length of 6 cm. They differ in that their diagonals are not of equal length.

Exercise 3L

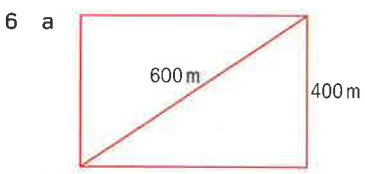


b 5.92 cm c 20 cm

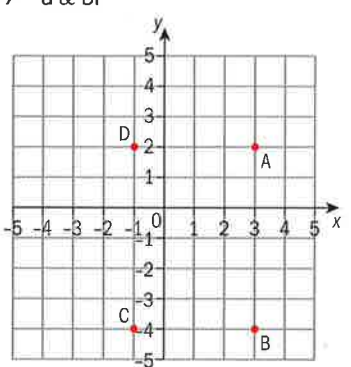


b 4.04 cm

4 b 70.5°
5 a 4 cm b 34 cm²
c 53.1°



b 41.8°



b ii $B(3, -4)$
c i 6 ii 4 d 56.3°

Exercise 3M

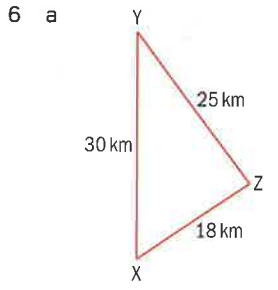
1 27.5° 2 52.1°
3 125 m 4 6.89°
5 21.5 m 6 32.3 m
7 a 12° b 425 m

Exercise 3N

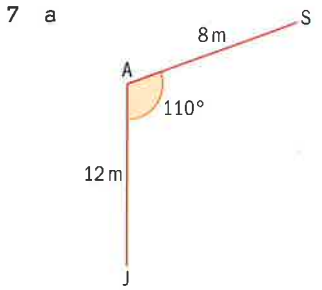
1 a $y = 13.7$ km
b $r = 3.47$ cm
c $c = 11.0$ km
2 8.34 cm 3 2.65 cm
4 7.32 km
5 a $\hat{C} = 37.9^\circ$ b $\hat{R} = 58.6^\circ$
c $\hat{Y} = 27.6^\circ$
6 $\hat{C} = 42.9^\circ$ 7 $\hat{R} = 46.3^\circ$
8 a 150° b 5.08 m
c 2.54 m

Exercise 3O

1 a $y = 13.5$ km
b $p = 9.74$ cm
c $c = 6.84$ m
2 a $x = 38.0^\circ$ b $y = 59.4^\circ$
c $a = 50.1^\circ$
3 193 m 4 7.14 cm 5 55.8°



b $\hat{Z} = 86.9^\circ$



b $JS = 16.5$ m c 2.74 m
8 113°
9 a $PR = 15.9$ m
b $\hat{P}RQ = 30.5^\circ$ c 2.54 m

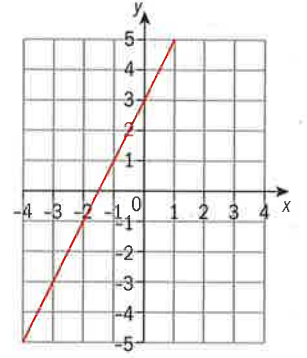
Exercise 3P

1 a 41.6 km² b 1890 m²
2 a 100° b 49.2 cm²
3 a 80° b 4.43 m²
4 60.8 km²
5 a 50.5° b 1930 m²
6 a $A = 0.25x^2$ or equivalent
b $x = 4$
7 a $\sqrt{61}$ cm or 7.81 cm (3 sf)
b 15.4 cm c 56.5 cm²
d 71.5 cm²

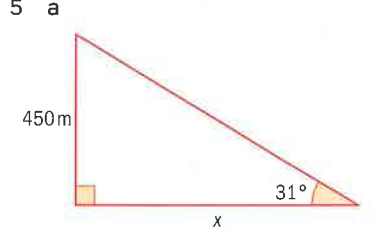
Review exercise

Paper 1 style questions

- a $-\frac{1}{2}$
b $y = -\frac{1}{2}x + 4$ or $x + 2y - 8 = 0$
- a -1 b 1
c $y = x$ or $y - x = 0$
- a i $(-1.5, 0)$ ii $(0, 3)$
b



c 63.4°
4 a $a = 1$
b $b = -19$
c $(1, 4)$



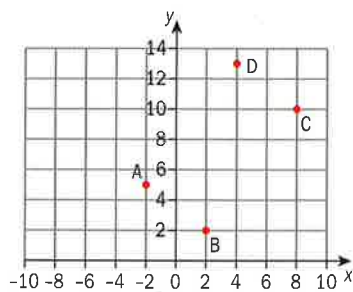
b $x = 749$ m

6 a 116°
b 11.8 cm
c 62.5 cm²
7 a 9 m
b 38.9°
c 14.1 m²

- 8 a 97.2°
 b 12.4 cm^2
 c 66.1 cm^2

Paper 2 style questions

- 1 a & b i



- b ii (4, 13) c $\frac{4}{3}$
 d $\frac{3}{4}$
 e $3x + 4y - 64 = 0$
 f i 5 ii 10
 g 26.6°
 2 a 4m
 b 3.46m
 c 1.96m
 d 3.48m
 e 44.8°
 3 a 500m b 36.87°
 c 71.1° d 1140 m
 e i 3040 m
 ii 13 minutes
 f 0.344 km^2

Chapter 4

Skills check

- 1 a 18.5 b 2 c $\frac{31}{4}$
 2 a 1.30, -2.30 b -0.781, 1.28
 c $x = -19, y = -11$
 3 a $-\frac{3}{4}$ b $\frac{5}{2}$

Exercise 4A

- 1 a function since each student is in only one mathematics class
 b not a function since each teacher teaches two of the students

- 2 a function since each element of A is related to one and only one element of B
 b not a function since one element of B (16) is not related to any element of A
 c function since each element of C is related to one and only one element of A

- 3 a i not a function since one element of A (4) is not related to any element of B
 ii not a function since one element of A (4) is not related to any element of C
 iii not a function since one element of C (1) is not related to any element of A
 iv function since each element of B is related to one and only one element of C
 v not a function since one element of C (6) is not related to any element of A

	B	C
	2	1
	4	2
	6	4

- 4 a $y = 2x$ b $y = \frac{x}{2}$
 c $y = \sqrt[3]{x}$ d $y = \frac{x^3}{2}$
 5 a function b function
 c not a function since negative elements in the first set are not related to any element in the second set
 d function

Exercise 4B

x	$-\frac{1}{2}$	0	1	3.5	6
$y = 2x$	-1	0	2	7	12

- ii the set of all real numbers
 iii yes, $y = 0$ is the image of $x = 0$

x	-3	0	2	$\frac{1}{4}$	-2	x
$y = x^2 + 1$	10	1	5	$\frac{17}{16}$	5	5

- ii the set of all real numbers

- iii no, there is no solution to the equation $0 = x^2 + 1$

x	-2	-1	0	$\frac{1}{2}$	3	5
$y = \frac{1}{x+1}$	-1	x	1	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{1}{6}$

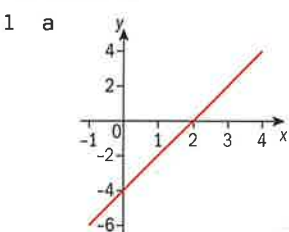
- ii the set of all real numbers except $x = -1$
 iii no, there is no solution to the equation $0 = \frac{1}{x+1}$

x	-3	0	$\frac{1}{4}$	1	9	100
$y = \sqrt{x}$	x	0	$\frac{1}{2}$	1	3	10

- ii the set of all non-negative real numbers
 iii yes, $y = 0$ is the image of $x = 0$

- 2 a false, there is no solution to the equation $0 = \frac{2}{x}$
 b true, $y = x^2 \geq 0$ for all values of x
 c true, $y = x^2 = 3 \geq 3$ for all values of x
 d true, $y = 3$ when $x = \pm 2$
 e true, $\frac{-3}{3} - 1 = -2$
 f false, the image of $x = -1$ is $y = 4$

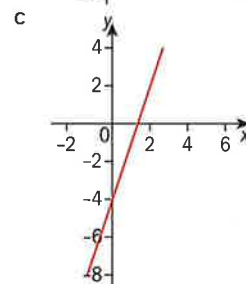
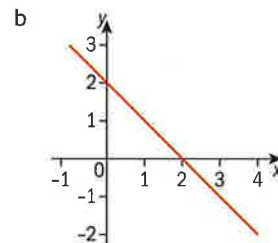
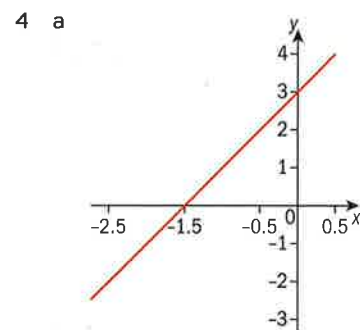
Exercise 4C



- 1 a i (2, 0) ii (0, -4)
 c no, $490 \neq 2 \times 250 - 4$
 d -10
 2 a i $\{x \mid -4 \leq x \leq 6\}$
 ii $\{y \mid -4 \leq y \leq 1\}$
 iii (4, 0) iv (0, -2)
 b i $\{x \mid x \in \mathbb{R}\}$
 ii $\{y \mid y \leq 8\}$
 iii (-4, 0), (0, 0)
 iv (0, 0)

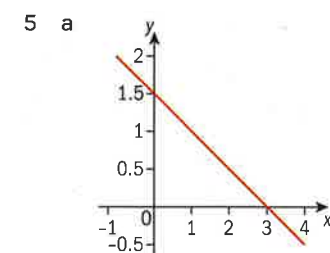
- c i $\{x \mid -1 \leq x \leq 1\}$
 ii $\{y \mid 0 \leq y \leq 1\}$
 iii (-1, 0), (1, 0)
 iv (0, 1)
 d i $\{x \mid x \geq -1\}$
 ii $\{y \mid y \geq 4\}$
 iii no points
 iv (0, 8)

- 3 a i false ii false iii true
 b i false ii true iii false
 c i false ii true iii false
 d i false ii false iii true



Exercise 4D

- 1 a 10 b $-\frac{7}{8}$
 d $f(-1) = -1(-2)(2) = 4$, so (-1, -4) lies on the graph of f
 2 a t b 6.25 c 4
 d $d(1) = d(4) = 4$
 3 a 80 b 70 c 10
 4 a i 3 ii -3
 b -1 c 2 d $t > 2$



- b (3, 0) c (0, 1.5)
 d $x = -1$
 6 a i 3 ii 6
 b $x = -3$

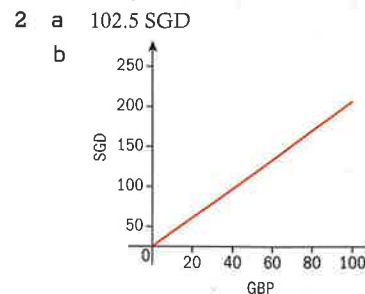
Exercise 4E

- 1 a i $l = 30 - 2x$
 ii $w = 15 - 2x$
 b $V = (30 - 2x)(15 - 2x)x$
 i $V(3)$ is the volume of the box when the squares cut from each corner have side length 3 cm
 ii 648 cm^3
 iii 646.816 cm^3
 iv no, $x < 7.5$ since the width of the card is only 15 cm
 2 a $12 - x$
 b $A = x(12 - x)$
 c i $A(2)$ is the area of the rectangle when the length is 2 cm
 ii 20 cm^2
 d no, if $x = 12$ the width of the rectangle would be 0
 3 a $C = 300 + 150n$
 b 4800 USD
 c i $300 + 150n \leq 2300$
 ii no
 iii 13 days
 4 b a loss of 576 AUD
 c i 3300 AUD
 ii 136 AUD
 d 10 or 150

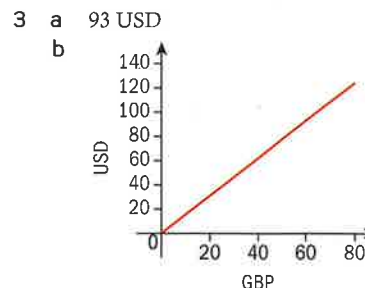
Exercise 4F

- 1 a 110 pounds
 b

- c gradient = 2.2 $p(x) = 2.2x$
 d $p(75) = 165$ $p(125) = 275$
 e $k(x) = \frac{x}{2.2}$
 f $k(75) = 34.1$ $k(100) = 45.5$

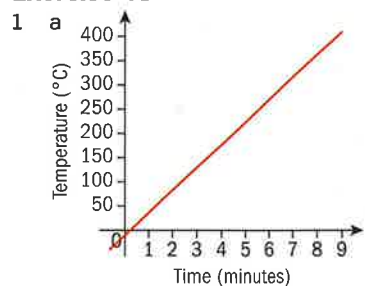


- c gradient = 2.05 $s(x) = 2.05x$
 d $s(80) = 164$ $s(140) = 287$
 e $p(x) = \frac{x}{2.05}$
 f $p(180) = 87.8$



- 3 a 93 USD
 b

Exercise 4G






- 1 a 10°C c $T(x) = 40x + 10$
 2 a

- b 18 mm
 c 20 mm
 d 0.5 mm
 e $L(x) = 0.5x + 18$
- 3 b $T(x) = \frac{2}{3}x + 10$
 c 66.7°C
- 4 b 20 cm
 c 20 cm
 d 350 g
 e $L(x) = 0.08x + 20$

Exercise 4H

- 1 a flour = $80s + 60f$
 b fat = $50s + 90f$
 c 5 sponge cakes, 7 fruit cakes
- 2 3 tables, 9 chairs
- 3 7 vans, 8 cars
- 4 4 passenger planes, 6 transport planes
- 5 16 volume 1, 8 volume 2

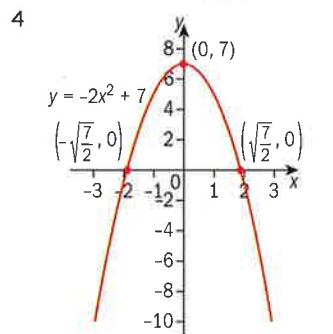
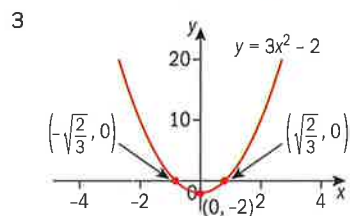
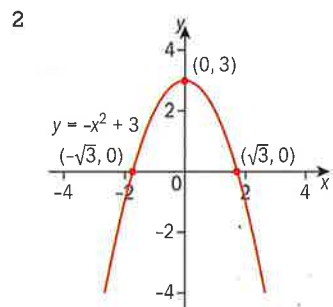
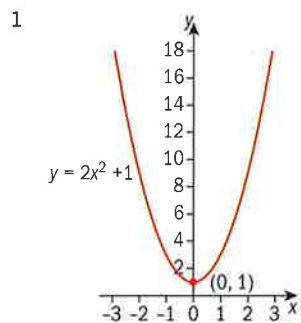
Investigation - the curve $y = ax^2$

- 1 The curves are related by being a reflection of each other in the x -axis.
- 2 a The curve is still a parabola. Positive numbers give a U-shaped parabola. Negative numbers give a -shaped graph.
- b There is a vertical line of symmetry in each graph at $x = 0$.
- c The vertex is $(0,0)$. It is a minimum point for the U-shaped graphs and a maximum point for the -shaped graph.
- 4 'a' represents the steepness of the graph. Higher values of a give a steeper curve. Smaller values for a give a shallower graph. Negative values for a give a -shaped graph.

Investigation - the curve $x^2 + c$

Changing the value of c translates the graph of $y = x^2$ vertically up and down. The value of c is the y -intercept of the graph.

Exercise 4I



Investigation - the curves $y = (x + p)^2$ and $y = (x + p)^2 + q$

- 1 Changing the value of p shifts the graph of $y = x^2$ to the left if p is positive and to the right if p is negative.
- 2 The axis of symmetry is $x = -p$. The coordinates of the vertex is $(-p, q)$.

Exercise 4J

- 1 $(-3, -2)$ $x = -3$
 2 $(-5, 4)$ $x = -5$
 3 $(4, -1)$ $x = 4$
 4 $(5, 7)$ $x = 5$

- 5 $(-3, 4)$ $x = -3$

Investigation - the curves $y = kx - x^2$ and $y = x^2 - kx$

Part A:

- 1 The axis of symmetry is $x = 2$. The coordinates of the vertex is $(2, 4)$. X -axis intercepts are $(0, 0)$ and $(4, 0)$.
- 2 Graphs of required curves.
- 3 Varying the value of k changes where the graph intersects the x -axis.

The axis of symmetry is $x = \frac{1}{2}k$

X -axis intercepts are $(0, 0)$ and $(k, 0)$

Part B:

Graphs of required curves.

- 4 Varying the value of k changes where the graph intersects the x -axis. This time the shape of the graph is U-shaped. The axis of symmetry is $x = \frac{1}{2}k$. X -axis intercepts are $(0, 0)$ and $(k, 0)$.

Investigation - curves of the form $y = (x - k)(x - l)$

- 1 The curve intersects the x -axis at $(1, 0)$ and $(3, 0)$. The axis of symmetry is at $x = 2$. The co-ordinate of the vertex is $(2, -1)$.
- 2 The curve intersects the x -axis at $(k, 0)$ and $(l, 0)$. The axis of symmetry is at $x = \frac{(k+l)}{2}$.

Exercise 4K

- 1 a $x = 2$ b $(0, 0), (4, 0)$
 c $(2, -4)$
- 2 a $x = -3$ b $(0, 0), (-6, 0)$
 c $(-3, -9)$
- 3 a $x = 4$ b $(0, 0), (8, 0)$
 c $(4, 16)$
- 4 a $x = \frac{3}{2}$ b $(0, 0), (3, 0)$
 c $(\frac{3}{2}, \frac{9}{4})$
- 5 a $x = 1$ b $(0, 0), (2, 0)$
 c $(1, -1)$
- 6 a $x = \frac{1}{2}$ b $(0, 0), (1, 0)$
 c $(\frac{1}{2}, -\frac{1}{4})$

- 7 a $x = -2$ b $(0, 0), (-4, 0)$
 c $(-2, -4)$
- 8 a $x = -\frac{1}{2}$ b $(0, 0), (-1, 0)$
 c $(-\frac{1}{2}, -\frac{1}{4})$
- 9 a $x = 1$ b $(-1, 0), (3, 0)$
 c $(1, -4)$
- 10 a $x = 1$ b $(5, 0), (-3, 0)$
 c $(1, -16)$
- 11 a $x = 4$ b $(2, 0), (6, 0)$
 c $(4, -4)$
- 12 a $x = 1$ b $(4, 0), (-2, 0)$
 c $(1, -9)$

Investigation - the general quadratic curve $y = ax^2 + bx + c$

Part A

- 1 The curve intersects the x -axis at $(1, 0)$ and $(3, 0)$.

The axis of symmetry is at $x = 2$. The co-ordinate of the vertex is $(2, -1)$.

- 2 In the case $a = 1$:

The curve intersects the x -axis at

$$\left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}, 0 \right) \text{ and } \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}, 0 \right)$$

The axis of symmetry is at $x = \frac{-b}{2}$

The co-ordinate of the vertex is

$$\left(\frac{-b}{2}, c - \frac{b^2}{4} \right)$$

Part B

- 1 The curve does not intercept the x -axis. The axis of symmetry is at $x = 1$. The co-ordinate of the vertex is $(1, 1)$.

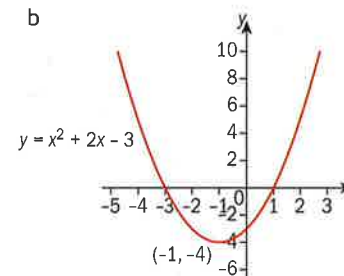
Exercise 4L

- 1 a $x = 1$ b no points
 c $(1, 2)$
- 2 a $x = -2$ b $(1, 0), (-5, 0)$
 c $(-2, -9)$
- 3 a $x = -3$ b $(-0.764, 0), (-5.24, 0)$
 c $(-3, -5)$
- 4 a $x = 1$ b $(0.423, 0), (1.58, 0)$
 c $(1, -1)$

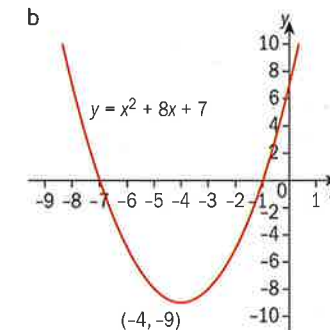
- 5 a $x = 2$
 b $(-0.121, 0), (4.12, 0)$
 c $(2, -9)$
- 6 a $x = -\frac{3}{2}$
 b $(0.898, 0), (-3.90, 0)$
 c $(-\frac{3}{2}, -\frac{23}{2})$
- 7 a $x = 1$
 b no points
 c $(1, \frac{3}{2})$
- 8 a $x = -3$
 b $(1.12, 0), (-7.12, 0)$
 c $(-3, -\frac{17}{2})$

Exercise 4M

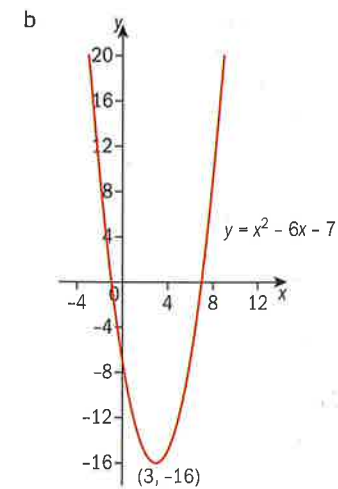
- 1 a i $(0, -3)$ ii $x = -1$
 iii $(-1, -4)$ iv $(-3, 0), (1, 0)$
 v $y \geq -4$



- 2 a i $(0, 7)$ ii $x = -4$
 iii $(-4, -9)$ iv $(-7, 0), (-1, 0)$
 v $y \geq -9$



- 3 a i $(0, -7)$ ii $x = 3$
 iii $(3, -16)$
 iv $(7, 0), (-1, 0)$
 v $y \geq -16$

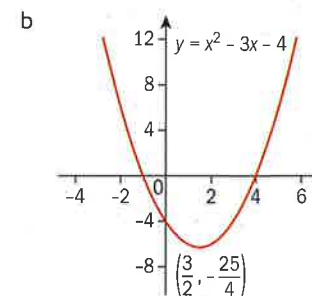


- 4 a i $(0, -4)$ ii $x = \frac{3}{2}$

iii $(\frac{3}{2}, -\frac{25}{4})$

iv $(4, 0), (-1, 0)$

v $y \geq -\frac{25}{4}$

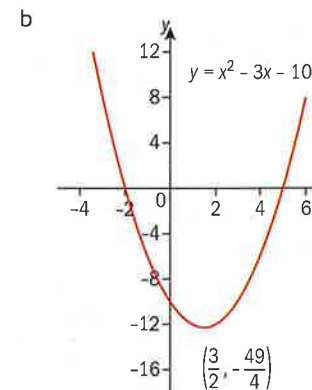


- 5 a i $(0, -10)$ ii $x = \frac{3}{2}$

iii $(\frac{3}{2}, -\frac{49}{4})$

iv $(5, 0), (-2, 0)$

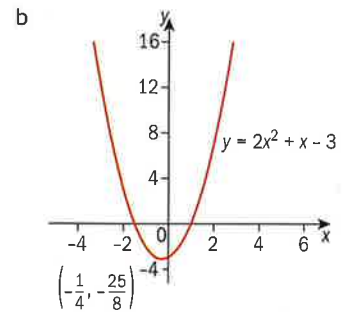
v $y \geq -\frac{49}{4}$



- 6 a i (0, -3)
 ii $x = -\frac{1}{4}$ iii $(-\frac{1}{4}, -\frac{25}{8})$

iv (1, 0), $(-\frac{3}{2}, 0)$

v $y \geq -\frac{25}{8}$

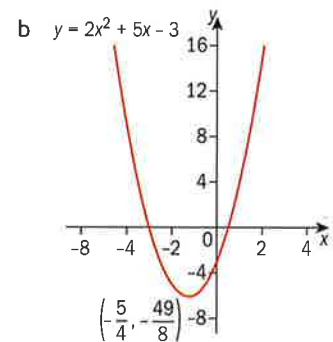


- 7 a i (0, -3) ii $x = -\frac{5}{4}$

iii $(-\frac{5}{4}, -\frac{49}{8})$

iv (-3, 0), $(\frac{1}{2}, 0)$

v $y \geq -\frac{49}{8}$

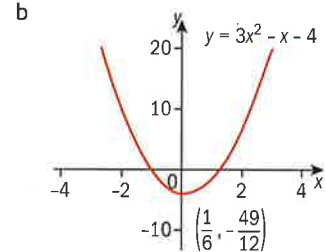


- 8 a i (0, -4) ii $x = \frac{1}{6}$

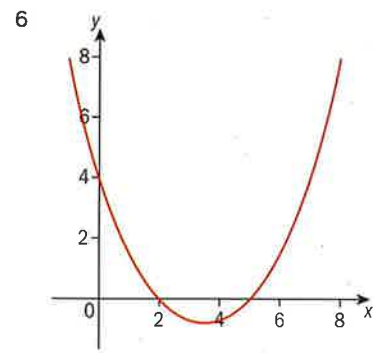
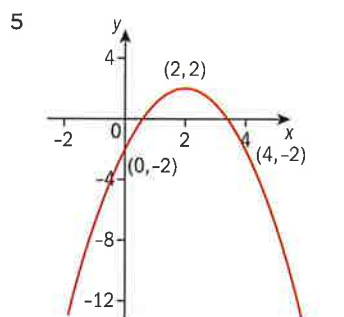
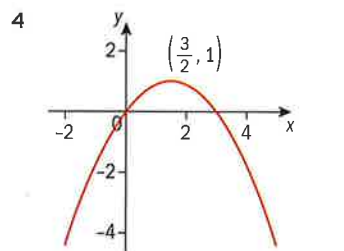
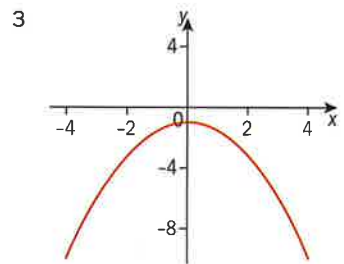
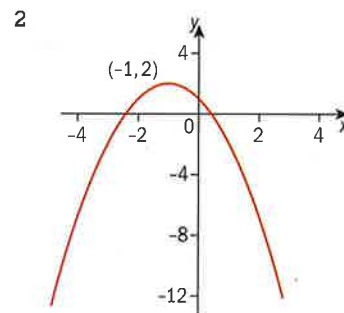
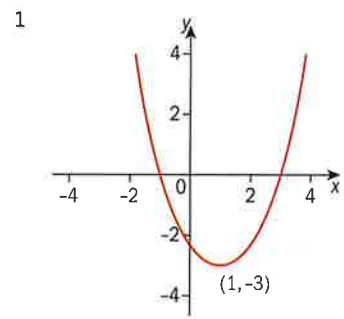
iii $(\frac{1}{6}, -\frac{49}{12})$

iv (-1, 0), $(\frac{4}{3}, 0)$

v $y \geq -\frac{49}{12}$



Exercise 4N



Exercise 4O

- 1 a (-3, -5), (1, -1)
 b yes: $x = -3$ or $x = 1$
 c (-2, -7), (1, -1)
 2 (0, -5), (-4, -1)
 3 a (4, 1), (-1, 1)
 b (2, 7), (-1, 1)
 4 b $f: \{y \mid -3.125 \leq y \leq 18\}$
 $g: \{y \mid -2 \leq y \leq 4\}$
 c $x = -1$ or $x = 2$
 e $x = -1$ or $x = \frac{5}{2}$
 f (-2, 7), (2, 3)
 5 a (2.12, 1.5), (-2.12, 1.5)
 b $-2.12 < x < 2.12$

Exercise 4P

- 1 $f(x) = x^2 + 4x - 1$
 $g(x) = x^2 + 2x - 2$
 2 $f(x) = x^2 - 4x + 5$
 $g(x) = x^2 - 2x + 3$
 3 $f(x) = -x^2 + 4x + 5$
 $g(x) = -x^2 + 2x + 3$
 4 $f(x) = -3x^2 - 6x + 2$
 $g(x) = -2x^2 - 8x - 3$
 5 $f(x) = 2x^2 + 2x$
 $g(x) = -x^2 + 3$

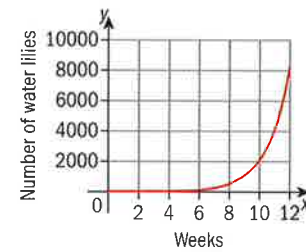
Exercise 4Q

- 1 a length = 42.5 m, width = 42.5 m
 b length = 31.25 m, width = 31.25 m
 2 a 13531.25 riyals
 b 3000 riyals
 c 69 or 1369 units
 3 a 270 m
 b 342.25 m
 c 37 s

Investigation - exponential graphs

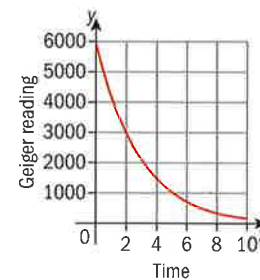
1

Weeks	Number of water lilies
1	4
2	8
3	16
4	32
5	64
6	128
7	256
8	512
9	1024
10	2048
11	4096
12	8192



2

Time	Geiger reading
0	6000
2	3000
4	1500
6	750
8	375
10	187.5



Exercise 4R

For all questions: y-intercept is (0, 1), horizontal asymptote is $y = 0$

Investigation - graphs of $f(x) = ka^x$

- 1 Graph of $y = 2(3)^x$
 a $k = 2$ b (0, 2)
 c $y = 0$

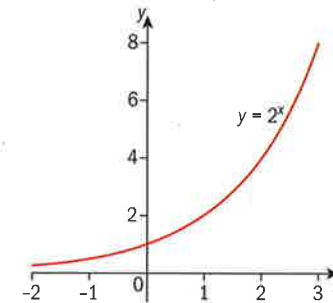
- 2 Graph of $y = 3(\frac{1}{2})^x$
 a $k = 3$ b (0, 3) c $y = 0$
 3 Graph of $y = -3(2)^x$
 a $k = -3$ b (0, -3) c $y = 0$
 The y-intercept is given by (0, k) and all graphs have a horizontal asymptote of $y = 0$.

Investigation - graphs of $f(x) = ka^x + c$

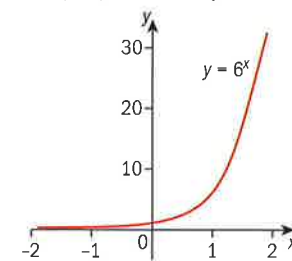
- 1 Graph of $y = 2^x + 3$
 a $k = 1, c = 3$
 b (0, 4) c $y = 3$
 2 Graph of $y = 3(\frac{1}{2})^x - 4$
 a $k = 3, c = -4$
 b (0, -1) c $y = -4$
 3 Graph of $y = -2(3)^x + 5$
 a $k = -2, c = 5$
 b (0, 3) c $y = 5$
 The y-intercept is given by (0, k + c) and the horizontal asymptote is $y = c$.

Exercise 4S

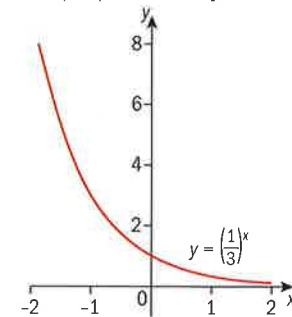
- 1 a (0, 1) b $y = 0$



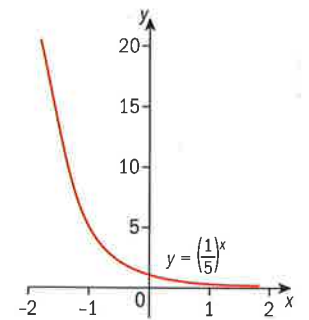
- 2 a (0, 1) b $y = 0$



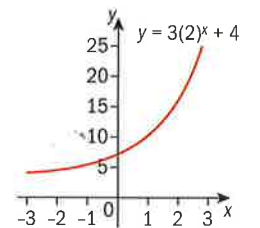
- 3 a (0, 1) b $y = 0$



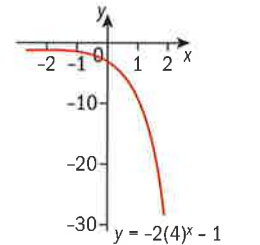
- 4 a (0, 1) b $y = 0$



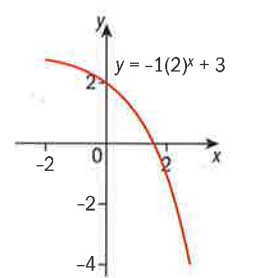
- 5 a (0, 7) b $y = 4$



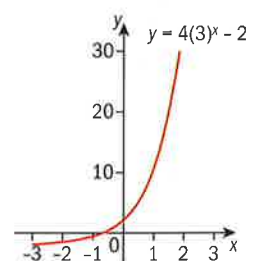
- 6 a (0, -3) b $y = -1$



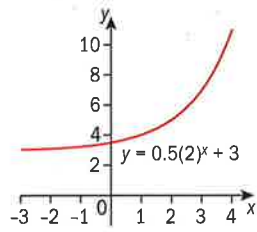
- 7 a (0, 2) b $y = 3$



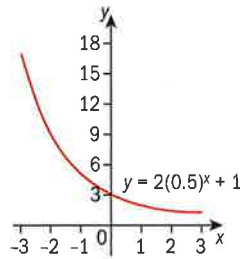
- 8 a (0, 2) b $y = -2$



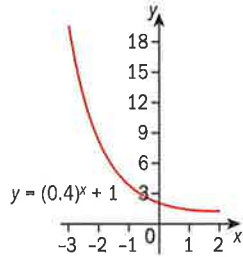
- 9 a (0, 3.5) b $y = 3$



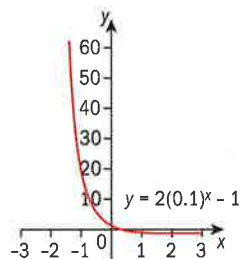
- 10 a (0, 3) b $y = 1$



- 11 a (0, 2) b $y = 1$

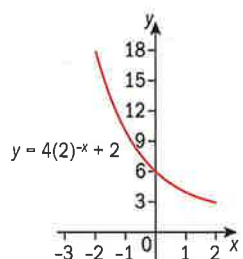


- 12 a (0, 1) b $y = -1$

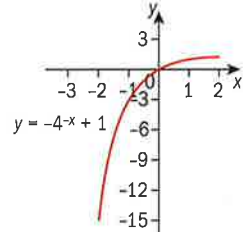


Exercise 4T

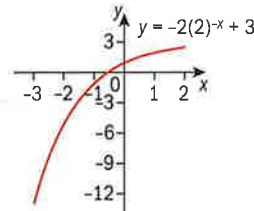
- 1 a (0, 6) b $y = 2$



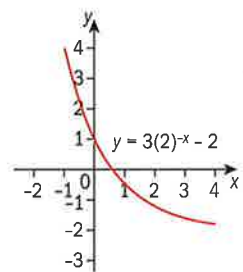
- 2 a (0, 0) b $y = 1$



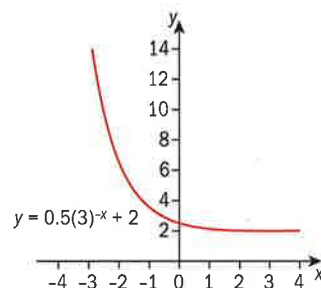
- 3 a (0, 1) b $y = 3$



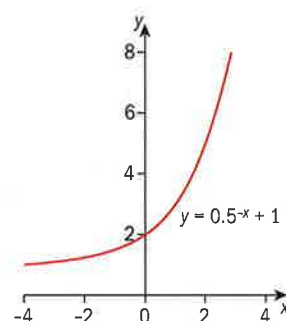
- 4 a (0, 1) b $y = -2$



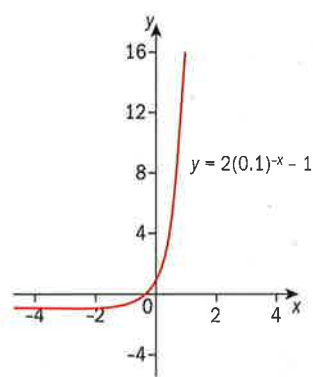
- 5 a (0, 2.5) b $y = 2$



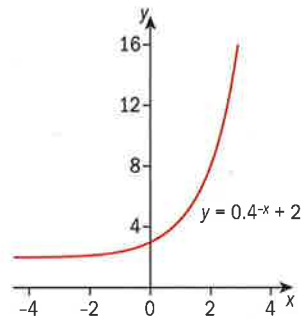
- 6 a (0, 2) b $y = 1$



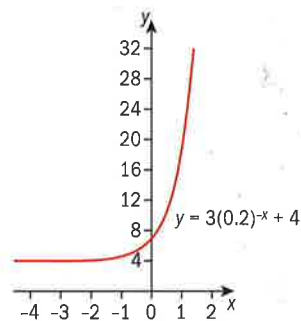
- 7 a (0, 1) b $y = -1$



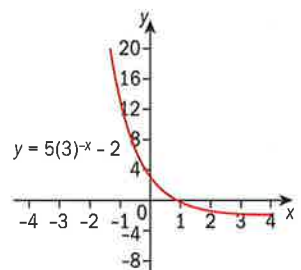
- 8 a (0, 3) b $y = 2$



- 9 a (0, 7) b $y = 4$

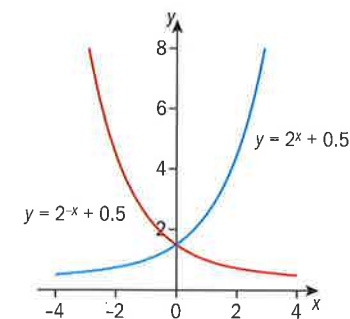


- 10 a (0, 3) b $y = -2$



Exercise 4U

1

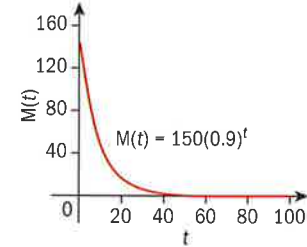


- a (0, 1.5) b $y = 0.5$

- 2 a 26 000 euros b 0.85

- c 10 years

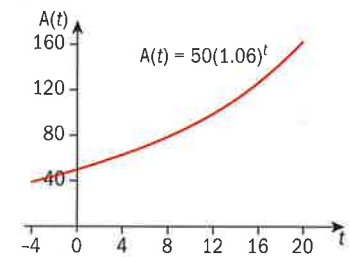
3 a



- b $M(t) = 0$ c 18.2 g

- d 7 years

4 a

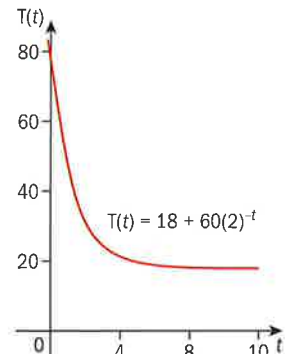


- b days before 1st June

- c 113 m² d $t = 8$

- 5 $c = -10, k = 5$

6 a



- b 78°C

- c 19.875°C

- d 1.45 minutes

- e 18°C, $T = 18$ is an asymptote, as t increases T gets closer to 18°C

- 7 a 18 000 USD

- b 10 628.82 USD

- c 7 years

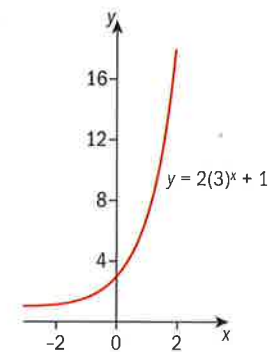
- 8 $a = 5, b = 0.2$

- 9 a $a = 4, b = 5$

- b $y = 3$

- 10 a $a = 1.667, b = 19$

b



- c $f(x) > 1$

Investigation - quartic functions

In general, a quartic graph has 3 turning points and intercepts the x -axis a total of 4 times. It intercepts the y -axis just once.

Exercise 4V

- 1 b 8.77 hours

- c 1.80 hours, 17.4 hours

- 2 a 6

- b 2

- c $f(x) \geq 6$

Investigation - graphs of $y = ax^{-n}$

- 1 The graphs of $y = x^{-3}$ and $y = x^{-1}$ (the odd powers) have very similar shapes to each other. The graphs of $y = x^{-4}$ and $y = x^{-2}$ (even powers) also have very similar shapes to each other.

- 2 The graphs with odd powers all have similar shapes to each other but the graph of $y = 2x^{-3}$ is further out than the graph of $y = x^{-3}$, which is further out than the graph of $y = x^{-1}$.

The graphs with even powers all have similar shapes to each other but the graph of $y = 3x^{-4}$ is further out than the graph of $y = x^{-4}$, which is further out than the graph of $y = x^{-2}$.

Exercise 4W

- 1 b 28.9°C

- c 2.72 minutes

- d $x = 0$

- e $y = 21$

- f 21°C

- 2 b 90°C

- c 1.43 minutes

- d 100°C

- 3 b ± 0.791

- c $x = 0, y = 0$

- d $f(x) > 0$

- 4 b 3.75

- c 3

- d $x = 0, y = 3$

- e $\{y \mid y \in \mathbb{R}, y \neq 3\}$

Exercise 4X

- 1 b minimum value = 17.5 (when $x = 1.71$)

- c 75.3 ms⁻¹

- d 0.403 s, 4.79 s

- 2 a $V = 2x^2y$

- b $A = 2x^2 + \frac{900}{x}$

- d length = 6.08, breadth = 12.2 cm, height = 4.05 cm

- 3 a $V = \frac{1}{3}x^2h$

- c $A = x^2 + 2x\sqrt{h^2 + \left(\frac{x}{2}\right)^2}$

- d $A = x^2 + 2x\sqrt{\frac{4500^2}{x^4} + \frac{x^2}{4}}$

- f side length = 14.7 m, height = 20.8 m

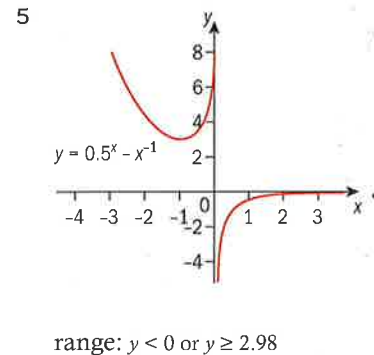
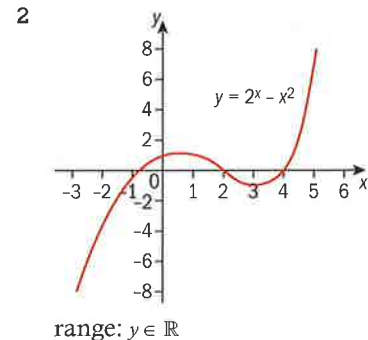
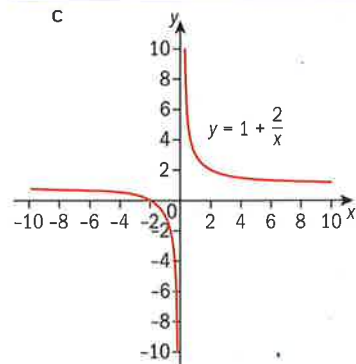
- 4 2670 cm²

Exercise 4Y

1 a $\{x \mid x \in \mathbb{R}, x \neq 0\}$

b

x	-10	-5	-4	-2	-1	-0.5	-0.2	0	0.2	0.5	1	2	4	5	10
$f(x)$	0.8	0.6	0.5	0	-1	-3	-9	x	11	5	3	2	1.5	1.4	1.2

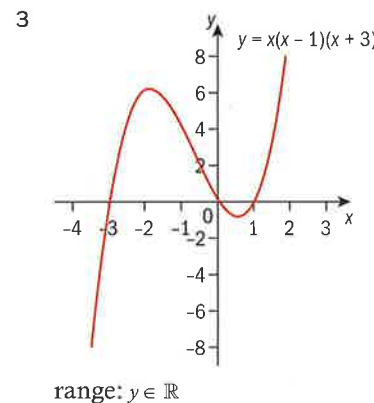
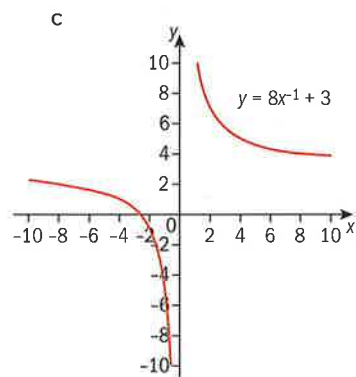


d ii $x=0$ e ii $y=3$

2 a $\{x \mid x \in \mathbb{R}, x \neq 0\}$

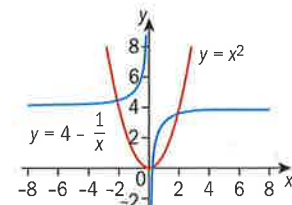
b

x	-10	-8	-5	-4	-2	-1	0	1	2	4	5	8	10
$f(x)$	2.2	2	1.4	1	-1	-5	x	11	7	5	4.6	4	3.8



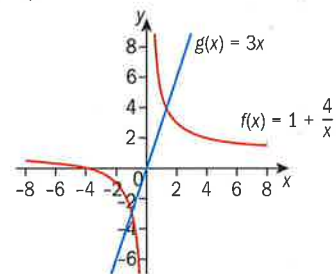
Exercise 4AA

1 a



b (0.254, 0.0646), (1.86, 3.46), (-2.11, 4.47)

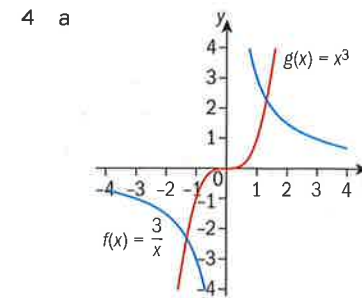
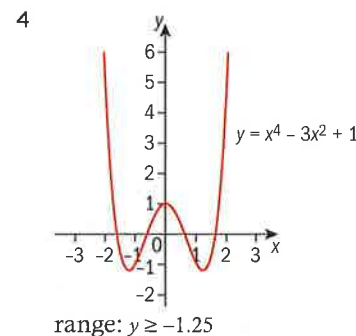
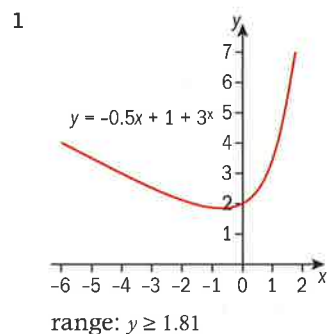
2 a, c



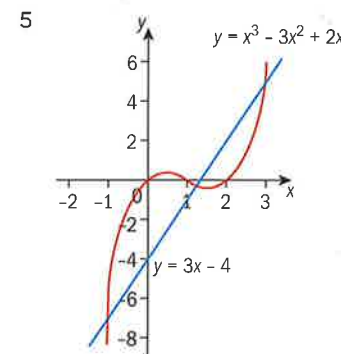
b $y=1, x=0$
 d $x=-1$ or 1.33
 e $\{y \mid y \in \mathbb{R}, y \neq 1\}$

3 a (-0.366, 0.669), (0.633, 2.01)
 b $y=0$

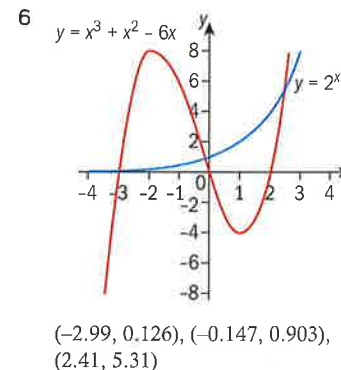
Exercise 4Z



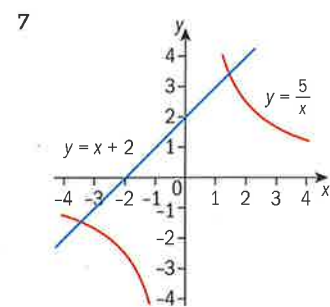
b two solutions
 c 1.32 or -1.32



(-1.11, -7.34), (1.25, -0.238), (2.86, 4.58)



(-2.99, 0.126), (-0.147, 0.903), (2.41, 5.31)



a $x = 1.45$ or -3.45
 b $y=0$ c $x=0$

Exercise 4AB

- 1 a time in hours, water consumption in litres
 b 07:00–20:00
 c 07:00–12:00, 14:00–16:00
 d 12:00–14:00, 16:00–20:00
 e 12:00 (local maximum at 16:00)
 f 07:00, 20:00 (local minimum at 14:00)
- 2 a time in minutes, temperature in °C
 b 100°C c 35°C
 d 1/2 minute e no
 f approximately 22°C

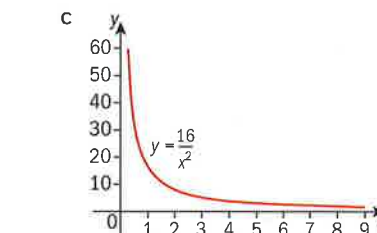
3 a

t	0	5	10	15	20
N	1	2	4	8	16

- b 13 s c 4096
- 4 a 45 m b 1.5 s and 5.5 s
 c 0–3.5 s d 3.5–7 s
 e 90 m, 3.5 s
 f ball returns to ground level
- 5 a i 3.8 m ii 2.2 m
 iii 02:00 and 06:00
 b $2 < t < 6$
- 6 a twice b 04:00–09:00
 c 16:00 d 5°C
 e 11:00–16:00
 f 13:00 and 19:30
 g no, the temperature at the start of the following day is 1°C whereas it was 3°C at the start of this day
- 7 a $y = \frac{16}{x^2}$

b

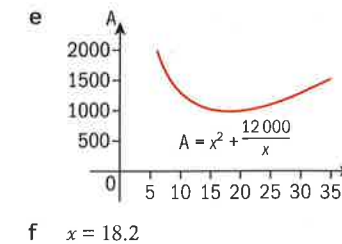
x	0.5	1	2	4	8	10
$y = f(x)$	64	16	4	1	0.25	0.16



- d height tends to 0
- 8 a 3000 cm³ b $y = \frac{3000}{x^2}$
 c $A = x^2 + \frac{12000}{x}$

d

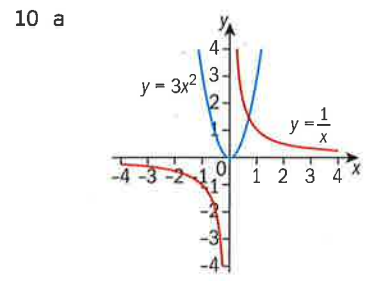
x (m)	5	10	15	20	25	30	35
$A(x)$ (cm ²)	2400	1300	1000	1000	1100	1300	1600



Review exercise

Paper 1 style questions

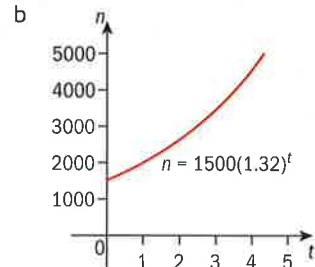
- 1 a 00:00–06:00
 b 11:30–17:00
 c 13°C
- 2 a 4500 SGD
 b 8000 SGD
- 3 a $x(x+5)$
 b
-
- 4 a 40 m
 b 45 m
 c 4 s
- 5 a $m=5$
 b $n = \frac{1}{5}$ $f(2) = \frac{4}{5}$
- 6 a $(x-5)(x+3)$
 b i (-3, 0) ii (1, -16)



b $x=0, y=0$ c $x=0.693$

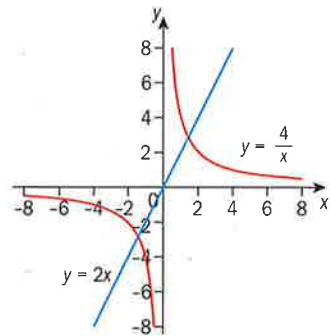
Paper 2 style questions

1 a 1980, 4554



c i 3000
ii 4 hours 20 minutes

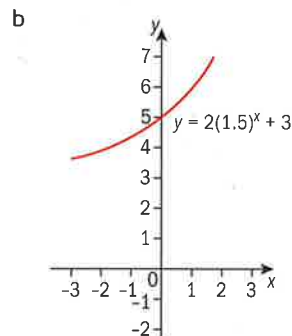
2 a, c



b $y=0, x=0$ d $x=\pm 1.41$

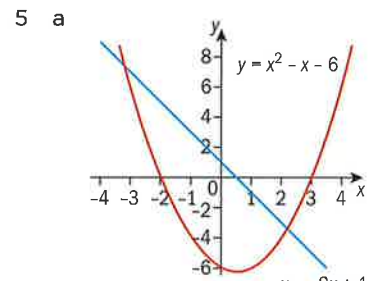
e $\{y \mid y \in \mathbb{R}, y \neq 0\}$

3 a $a=4.33, b=7.5$



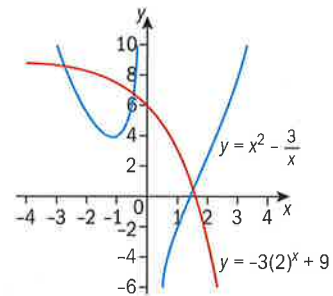
c $f(x) > 3$
d $x=3$ (approximately)
e $y=3$

4 a 98°C b $y=21$
c 21°C d 33.9°C



b $(0.5, -6.25)$
c -2 d $(0, 1)$
e $(2.19, -3.39), (-3.19, 7.39)$
f $x=2.19, -3.19$

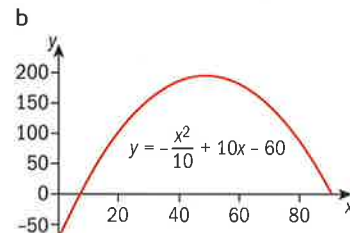
6 a, c



b $x=0$ d $y=9$
e $(-2.73, 8.55), (-0.454, 6.81), (1.53, 0.362)$

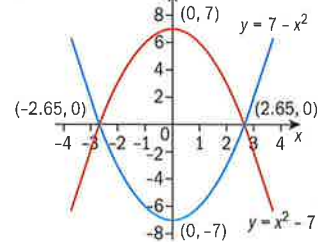
7 a

x	0	10	20	30	40	50	60	70	80	90
P	-60	30	100	150	180	190	180	150	100	30

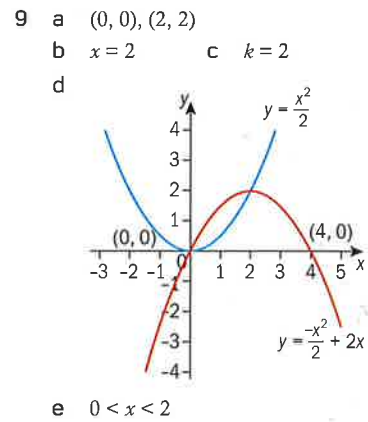


c i 190 euros
ii 50
iii 33 or 67
iv 60 euros

8 a, b



c $x=\pm 2.65$
d $c=1, 2, 3, 4, 5$



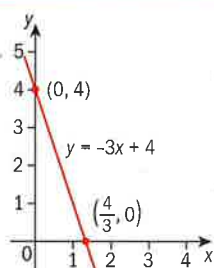
e $0 < x < 2$

Chapter 5

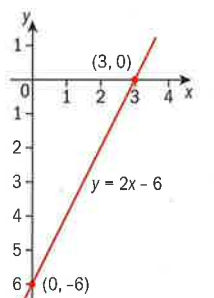
Skills check

1 a mean = 3.61 (3sf)
standard deviation = 1.21 (3sf)
The standard deviation implies that the data are close to the mean.
b mean = 4
standard deviation = 0.643 (3sf)
Mean = median, as the frequencies are symmetrical about the mean. The standard deviation implies that the data are very close to the mean.

2 a



b

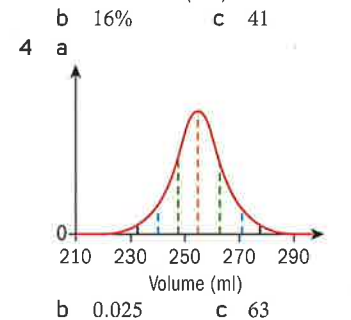
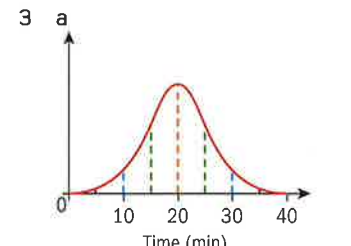
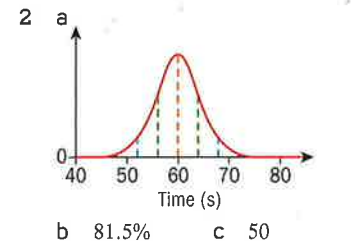
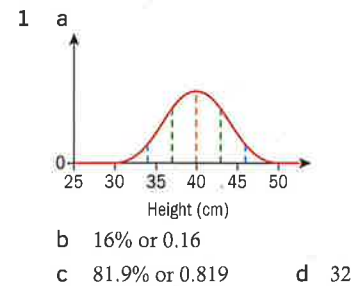


Investigation - related data?

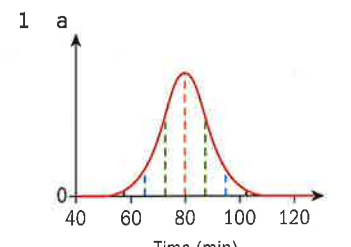
There is a positive correlation between height and shoe size. Inevitably the data will not fall on a straight line but as a

general trend, the taller the height of a person, the greater their shoe size.

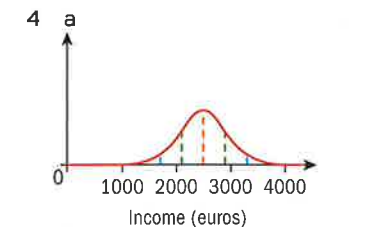
Exercise 5A



Exercise 5B



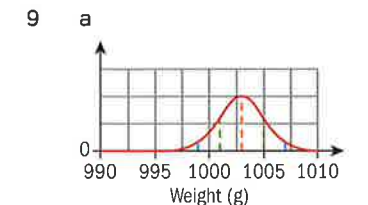
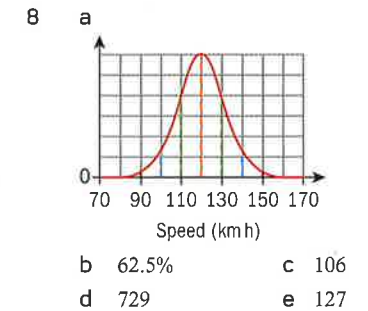
2 a i 0.159 ii 0.228
iii 0.819
b 134
3 a 0.0688 b 0.00621
c 278



5 a 78.9% b 0.00621 c 3
6 0.106
7 a 86.4% b 1
8 a 0.00621 b 3

Exercise 5C

1 4.93
2 179
3 20.8
4 222
5 a 3.47–4.99 kg
b 122 or 123 c 0.0685
d 87.7% e 5.48
6 a $a=29, b=30, c=31$
b 0.919 c 32.8
d 3124 (accept 3120 to 3125)
7 a 0.0004291
b 0.854 c 5885

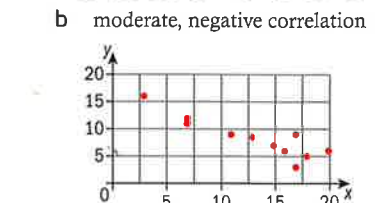
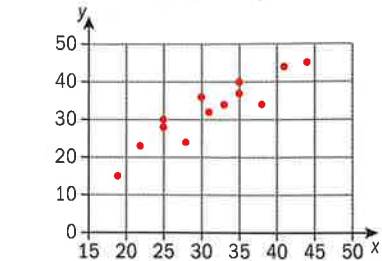


10 a 0.466%
b One weighing 2.34 kg is more likely
c 24 d 3.16

Exercise 5D

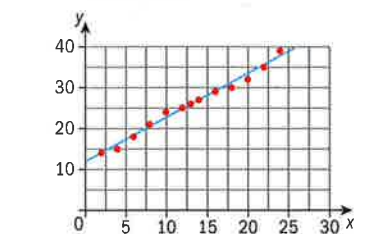
1 a strong, positive, linear
b strong, negative, linear
c moderate, positive, linear
d weak, positive, linear

e none
f perfect, negative, linear
g non-linear
h moderate, negative
2 a moderate, positive correlation



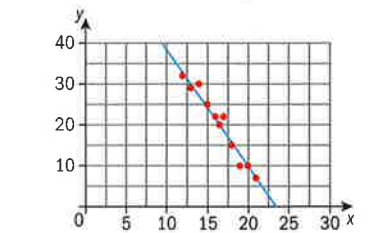
Exercise 5E

1 a i, iii very strong, positive, linear correlation



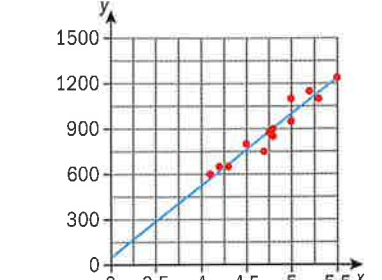
ii 13 and 25.75

b i, iii strong, negative, linear correlation



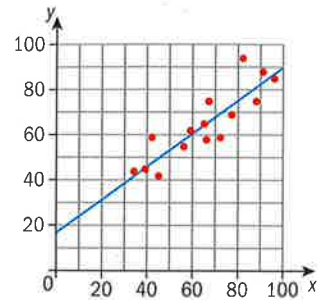
ii 16.5 and 20.2

2 a, c moderate, positive, linear correlation



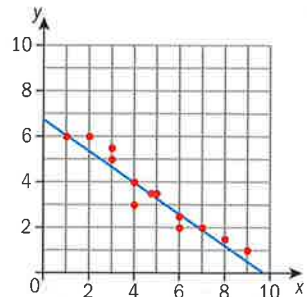
b 4.78 and 896 d 810kg

- 3 a, c moderate, positive, linear correlation



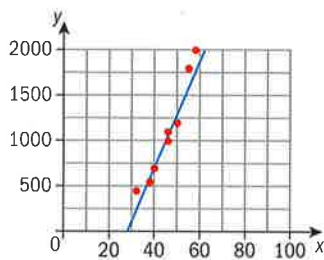
- b 65.3 and 65.1 d 54%

- 4 a, c moderately strong, negative, linear correlation



- b 4.83 and 3.5 d 4.5

- 5 a, c strong, positive, linear correlation



- b 45.6 and 1100
d 1500

Exercise 5F

- 1 0.931 very strong and positive
2 a 0.880
b strong, positive
3 -0.891 strong and negative
4 0.936 very strong and positive
5 0.990 very strong and positive
6 0.200 very weak and positive
7 0.985 very strong and positive
8 0.580 moderate and positive

Exercise 5G

- 1 a 0.994 very strong and positive
b $y = 1.47x + 116$
c 1586 rupees
2 a 0.974
b $y = 0.483x + 15.6$
c 19.5 cm
3 a mean of $x = 68.6$ and standard deviation of $x = 6.55$
mean of $y = 137.7$ and standard deviation of $y = 5.97$
b -0.860
c strong and negative
d $y = -0.784x + 191.5$
e 137 s
4 a 0.792
b $y = 0.193x + 1.22$ c 4
5 a $y = 0.0127x + 0.688$
b 1.58 AUD
6 a $y = 0.751x + 11.6$ b 49
7 a $y = 1.04x - 2.53$ b 60
8 a $y = 0.279x + 2.20$
b 13.4 hours

Exercise 5H

- 1 a H_0 : Genre of books is independent of age
 H_1 : Genre of books is not independent of age
b $130 \times \frac{97}{300} = 42.0$
c 4 d 26.9
e $26.9 > 9.488$ so reject null hypothesis
2 a H_0 : Hair color is independent of eye color
 H_1 : Hair color is not independent of eye color
b $85 \times \frac{90}{227} = 33.7$
c 4 d 44.3
3 a H_0 : Favorite flavor is independent of breed
 H_1 : Favorite flavor is not independent of breed
b $35 \times \frac{44}{140} = 11$
c $(3-1)(4-1) = 6$

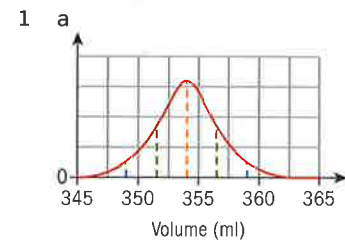
- d 0.675
e $0.675 < 12.59$ so do not reject the null hypothesis
4 a H_0 : Film genre is independent of gender
 H_1 : Film genre is not independent of gender
b $39 \times \frac{21}{80} = 10.2$
c 3 d 19.1
e $19.1 > 11.345$ so reject the null hypothesis
5 a H_0 : Grade is independent of number of hours spent playing computer games
 H_1 : Grade is not independent of number of hours spent playing computer games
b $90 \times \frac{96}{220} = 39.27 \approx 39.3$
c $(3-1)(3-1) = 4$ d 42.1
e $42.1 > 9.488$ so reject the null hypothesis
6 a H_0 : Employment grade is independent of gender
 H_1 : Employment grade is not independent of gender
b

11.5	71.5	539
20.5	127.5	960

c 2 d 180
e $180 > 4.605$ so reject the null hypothesis
7 a H_0 : Amount of sushi is independent of day of the week
 H_1 : Amount of sushi is not independent of day of the week
b $170 \times \frac{145}{470} = 52.4$
c 4 d 0.840
e $0.840 < 9.488$ so do not reject the null hypothesis.
8 a H_0 : Puppy's weight is independent of its parent's weight
 H_1 : Puppy's weight is not independent of its parent's weight
b $46 \times \frac{41}{141} = 13.38 \approx 13.4$
c 4
d 13.7
e $13.7 > 13.277$ so reject the null hypothesis

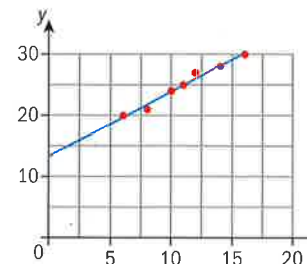
Review exercise

Paper 1 style questions



- b 0.0548 c 5
2 a 32.2% b 59
3 a 93.3%
b $p = 1.01$
4 a strong positive linear correlation
b none
c moderate negative linear correlation

- 5 diagram and c



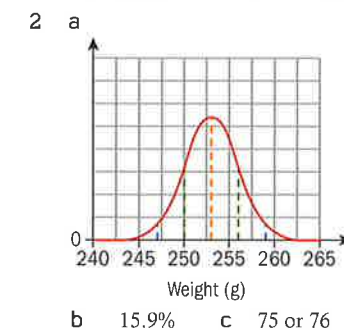
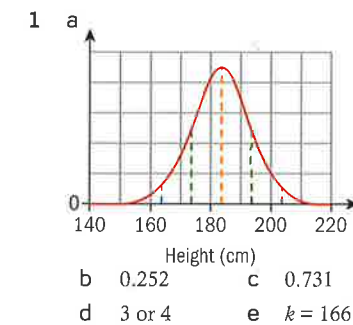
- a strong positive linear correlation
b mean of $x = 11$, mean of $y = 25$
d 23
6 a $r = 0.980$, strong and positive correlation
b $y = 0.801x - 77.4$
c 59 cm
7 a $r = 0.810$, strong, positive correlation
b $y = 0.215x + 14.3$
c 22.9 s
8 H_0 : Flavor is independent of age
 H_1 : Flavor is not independent of age
Expected values are:

14.1	11.8	11.1
10.6	9.0	8.4
13.3	11.2	10.5

dof = 4
 $\chi^2 = 0.604$
p-value = 0.963
 $0.963 > 0.05$ so do not reject H_0

- 9 a H_0 : The number of pins knocked down is independent of the hand used
b dof = 2
c $\frac{20 \times 60}{120} = 10$
d $0.422 > 0.10$ so do not reject H_0
10 a H_0 : The time to prepare for a test is independent of the outcome
b 2
c $0.069 > 0.05$ so do not reject H_0

Paper 2 style questions



- 3 a and d ii

b 166.9 cm c 67.3 kg
d i $y = 0.719x - 52.8$
e 69.4 kg
4 a $r = 0.823$
b strong, positive correlation
c $y = 0.219x + 3.85$
d 12 hours
5 a 0.9
b strong, positive correlation
c $y = 0.0666x - 2.36$

- 6 a $r = 0.89$
b strong, positive correlation
c $y = 0.0151x + 0.229$
d 1.44 euros
7 a $y = 0.163x - 15.0$
b 12.7 c 0.741
d moderate, positive correlation
8 H_0 : Game chosen is independent of gender
 H_1 : Game chosen is not independent of gender
dof = 2
expected values

39.4	14.8	26.8
29.6	11.2	20.2

- $\chi^2 = 0.667$
p-value = 0.717
 $0.717 > 0.05$, so do not reject H_0
9 a $p = 21.6$, $q = 14.4$, $r = 13.6$
b i H_0 : Extra-curricular activity is independent of gender
ii $(2-1)(3-1) = 2$
c $\chi^2 = 4.613$
d $4.163 > 4.605$ so reject H_0
10 a i $\frac{180 \times 300}{500} = 108$
ii $b = 72$, $c = 132$, $d = 88$
b H_0 : Position is independent of gender
 H_1 : Position is not independent of gender
c i $\chi^2 = 59.7$
ii dof = 2
iii $\chi^2 >$ critical value so reject H_0
11 a H_1 : Choice of candidate is not independent of where the voter lives
b $\frac{3680 \times 3720}{8000} = 1711.2 \approx 1711$
c i $\chi^2 = 58.4$ ii 2
d i reject H_0
ii $58.4 > 9.21$
12 a $\frac{90 \times 110}{200} = 49.5$
b i H_0 : Grade is independent of gender
ii 2
iii $\chi^2 = 0.400$
c $0.400 < 5.991$ so do not reject H_0

Chapter 6

Skills check

- 1 a $f(5) = 3 - 2(5) = -7$
 $f(-5) = 3 - 2(-5) = 13$
 b $f(2) = 3(2) + 5 = 11$
 $f(-3) = 3(-3) + 5 = -4$
 c $g(5) = 5^2 = 25$
 $g\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$
 d $g(2) = \frac{3}{2} = 1.5$
 $g(15) = \frac{3}{15} = 0.2$
 e $f(4) = \frac{4^2}{(4+1)} = 3.2$
 $f(-3) = \frac{(-3)^2}{(-3+1)} = -4.5$
- 2 a $\frac{C}{2\pi} = r$ b $\sqrt{\frac{A}{\pi}} = r$
 c $\sqrt{\frac{A}{4\pi}} = r$ d $\sqrt{\frac{3V}{\pi h}} = r$
 e $\sqrt[3]{\frac{3V}{2\pi}} = r$ f $r = \frac{2A}{C}$
- 3 a 16 b $\frac{1}{8}$ c $\frac{1}{16}$
- 4 a x^{-1} b x^{-4} c x^2
 d x^{-3} e x
- 5 a $y = 2x - 13$
 b $y = -3x + 14$

Investigation - tangent and the gradient function

5	x-coordinate	-3	-2	-1	0	1	2	3	4	x
	Gradient of Tangent	-6	-4	-2	0	2	4	6	8	2x

8	x-coordinate	-3	-2	-1	0	1	2	3	4	x
	Gradient of Tangent	-12	-8	-4	0	4	8	12	16	4x

Investigation - GDC and the gradient function

- 3 a -3.5
 b 2
 c 0
 d -1
 e 0
 f -0.5
- 4
- | Curve | Gradient function |
|------------|-------------------|
| $y = x^2$ | 2x |
| $y = 2x^2$ | 4x |
| $y = 3x^2$ | 6x |

5	Curve	$y = 4x$	$y = -3.5x$	$y = 2x + 4$	$y = 5$	$y = 3 - x$	$y = -3.5$	$y = 2 - \frac{1}{2}x$
	Gradient function	4	-3.5	2	0	-1	0	$-\frac{1}{2}$

	Curve	$y = x^2$	$y = 2x^2$	$y = 3x^2$	$y = 4x^2$	$y = -x^2$	$y = -2x^2$	$y = \frac{1}{2}x^2$
	Gradient function	2x	4x	6x	8x	-2x	-4x	x

6	x-coordinate	-3	-2	-1	0	1	2	3	4
	Gradient of Tangent	-3	-1	1	3	5	7	9	11

The algebraic rule: gradient of the tangent = $2x + 3$

- a $2x + 3$
 b $2x - 5$
 c $4x - 3$
 d $6x - 1$
 e $5 - 4x$
 f $2 - 2x$
 g $2x$
 h $2x$
 i $-2x$
 j $2x + 1$
 k $4x - 1$
 l $3 - 2x$

For the general curve: $ax^2 + bx + c$,
 gradient = $2ax + b$

- 1 $10x + 7$
 2 $5 + 14x$
 3 $x - 6$
 4 $-3x + 8$

Curve	$y = x^3 + 3x^2 + 2$	$y = x^3 + 4x^2 + 3x$	$y = x^3 + 5x^2 - 4x + 1$	$y = x^3 - x^2 - 5x - 4$
Gradient function	$3x^2 + 6x$	$3x^2 + 8x + 3$	$3x^2 + 10x - 4$	$3x^2 - 2x - 5$

Function	Formula	Gradient Function
Constant	$y = a$	0
Linear	$y = ax + b$	a
Quadratic	$y = ax^2 + bx + c$	$2ax + b$
Cubic	$y = ax^3 + bx^2 + cx + d$	$3ax^2 + 2bx + c$

Investigation - the gradient function of any curve

- 1 $4x^3$
 2 $5x^4$
 3 nx^{n-1}

Function	Gradient function
$y = ax^n$	nax^{n-1}

Exercise 6A

- 1 a 8x b $18x^2$ c $28x^3$
 d $15x^2$ e $4x^3$ f 5
 g 1 h 12 i 18x
- j $\frac{3x^2}{2}$ k x l $3x^3$
- 2 a 0 b $-9x^2$ c $-x^3$
 d $-2x^2$ e -1 f 0
 g $30x^5$ h $-63x^8$ i $4x^7$
 j $9x^{11}$ k $-6x^8$ l 0

Investigation - the gradient function of a cubic curve

Curve	$y = x^3$	$y = 2x^3$	$y = 3x^3$	$y = 4x^3$	$y = -x^3$	$y = -2x^3$	$y = \frac{1}{2}x^3$
Gradient function	$3x^2$	$6x^2$	$9x^2$	$12x^2$	$-3x^2$	$-6x^2$	$\frac{3}{2}x^2$

Curve	$y = x^3 - 4$	$y = 2x^3 - 3$	$y = x^3 + 5x$	$y = x^3 - 2x$
Gradient function	$3x^2$	$6x^2$	$3x^2 + 5$	$3x^2 - 2$

Curve	$y = x^3 + 2x^2$	$y = 2x^3 + \frac{1}{2}x^2$
Gradient function	$3x^2 + 4x$	$6x^2 + x$

- 3 a $6x + 15x^2$ b $20x^3 - 4$
 c $9 - 33x^2$ d $4x^3 + 3$
- 4 a $24x^5 - 5$ b $18x - 5$
 c $7 + 20x^4$ d $4x + 3$

Exercise 6B

- 1 a $36 - 12t^2$ b 12
 c $3t^2 - 10t$ d $4t + 1$
 e $7 - 4t$ f $36t - 9$
 g $3t^2 - 2t + 3$ h $6t - 3$
- 2 a 2r b $2r + 6$
 c $8r - 12$ d $8r - 20$
 e $6r + 30$ f $f'(r) = 70 - 10r$

Exercise 6C

- 1 $\frac{dy}{dx} = -\frac{6}{x^3}$ 2 $f'(x) = -\frac{8}{x^5}$
 3 $\frac{dy}{dx} = -\frac{7}{x^2}$ 4 $f'(x) = -\frac{16}{x^9}$
 5 $\frac{dy}{dx} = -\frac{35}{x^8}$ 6 $\frac{dy}{dx} = -\frac{2}{x^2}$
 7 $f'(x) = 14x - \frac{20}{x^6}$
 8 $\frac{dy}{dx} = -4 - \frac{5}{x^3}$
 9 $g'(x) = 3x^2 - \frac{6}{x^3}$
 10 $\frac{dy}{dx} = 4 + \frac{3}{x^2}$
 11 $g'(x) = 15x^2 + \frac{4}{x^5}$
 12 $\frac{dy}{dx} = 2x^3 + \frac{6}{x^9}$
 13 $\frac{dy}{dx} = \frac{x^3}{2} + 6x - \frac{10}{3x^5}$
 14 $g'(x) = 6x^2 - 2x + \frac{3}{x^3}$
 15 $A'(x) = 2x + \frac{5}{2x^2} - \frac{3}{2x^3}$

Exercise 6D

- 1 $\frac{dy}{dx} = 2x - 3 \Rightarrow \frac{dy}{dx} = 5$
 2 $\frac{dy}{dx} = 6 - 3x^2 \Rightarrow \frac{dy}{dx} = 6$
 3 $\frac{dy}{dx} = -8x^3 - 9x^2 \Rightarrow \frac{dy}{dx} = 135$
 4 $\frac{dy}{dx} = 20x + 8 \Rightarrow \frac{dy}{dx} = -12$
 5 $\frac{dy}{dx} = 3x^2 - 5 \Rightarrow \frac{dy}{dx} = 103$
 6 $\frac{dy}{dx} = -2x^3 \Rightarrow \frac{dy}{dx} = 16$
 7 $\frac{dy}{dx} = 21 - 36x^2 \Rightarrow \frac{dy}{dx} = -15$
- 8 $\frac{dy}{dx} = 6x - 5 \Rightarrow \frac{dy}{dx} = -17$
 9 $\frac{ds}{dt} = 40 - 10t \Rightarrow \frac{ds}{dt} = 10$
 10 $\frac{ds}{dt} = 35 + 12t \Rightarrow \frac{ds}{dt} = 35$
 11 $\frac{dv}{dt} = 80 \Rightarrow \frac{dv}{dt} = 80$
 12 $\frac{dv}{dt} = 0.7 \Rightarrow \frac{dv}{dt} = 0.7$
 13 $\frac{dA}{dh} = 42h^2 \Rightarrow \frac{dA}{dh} = 18\frac{2}{3}$
 14 $\frac{dW}{dp} = 21.75p^2 \Rightarrow \frac{dW}{dp} = 87$
 15 $\frac{dV}{dr} = 8r - \frac{18}{r^2}, \frac{dV}{dr} = 22$
 16 $\frac{dA}{dr} = 5 - \frac{16}{r^3}, \frac{dA}{dr} = 4.75$
 17 $\frac{dV}{dr} = 21r^2 + \frac{8}{r^2}, \frac{dV}{dr} = 86$
 18 $\frac{dA}{dr} = 2\pi r + \frac{2\pi}{r^2}, \frac{dA}{dr} = 4\pi$
 19 $\frac{dV}{dr} = 6 - \frac{15}{2r^2}, \frac{dV}{dr} = 5.7$
 20 $\frac{dC}{dr} = 45 - \frac{36}{r^4}, \frac{dC}{dr} = 9$

Exercise 6E

- 1 a $\frac{dy}{dx} = 2x + 3$ b $x = 2$
 c $y = 6$
- 2 a $\frac{dy}{dx} = 4x - 1$ b $x = -2$
 c $y = 11$
- 3 a $\frac{dy}{dx} = 3 - 2x$ b $a = 3, b = 4$

- 4 $\frac{dy}{dx} = 2x - 6, a = 6, b = 0$
 5 $\left(\frac{1}{2}, -3\frac{3}{4}\right)$
 6 $(-1, -10)$
 7 $(-1, 0), (1, 8)$
 8 $(-1, 6), (1, -4); y = -5x + 1$
 9 $(2, -11), (-2, 21); y + 8x - 5 = 0$
 10 a $b = -2$ b $\frac{dy}{dx} = 2x - 4$
 c at $x = 1, \frac{dy}{dx} = -2 = b$
 d $c = 1, d = -2$

- 11 a $b = 7$ b $\frac{dy}{dx} = 2x - 3$
 c at $x = 5, \frac{dy}{dx} = 2(5) - 3 = 7 = b$
 d at Q, $-3 = 2x - 3 \Rightarrow c = 0$
 $d = 0^2 - 3(0) - 3 = -3$
- 12 a $f'(x) = 4 - 2x$
 b $f(5) = -6, f'(5) = -6$
 c $(1, 2)$
- 13 a $f'(x) = 4x - 1$
 b $f'(2) = 7, f(2) = 7$
 c $(0.5, 1)$
- 14 a $f'(x) = 3 - 2x$
 b $f'(1) = 3 - 2(1) = 1$
 $f'(1) = 3(1) - 1^2 - 1 = 1$
 c $(4, -5)$
- 15 a $f'(x) = 4x - 1$ b $(0, -1), \left(\frac{5}{2}, 9\right)$
- 16 a $f'(x) = 2x + 5$ b $(-5, -5), (2, 9)$
- 17 $(-1, 2)$

Exercise 6F

- 1 a $\frac{dy}{dx} = 2x$ $y = 6x - 9$
 b $\frac{dy}{dx} = 6x^2$ $y = 6x - 4$
 c $\frac{dy}{dx} = 6 - 2x$ $y = 2x + 4$
 d $\frac{dy}{dx} = 6x$ $y = 6x - 13$
 e $\frac{dy}{dx} = 4x - 5$ $y = 7x - 14$
 f $\frac{dy}{dx} = 10 - 3x^2$ $y = -2x + 21$
 g $\frac{dy}{dx} = -4x$ $y = -12x + 29$
 h $\frac{dy}{dx} = 6 - 2x$ $y = 2x + 9$
 i $\frac{dy}{dx} = 8x - 3x^2$ $y = -16x + 24$
 j $\frac{dy}{dx} = 5 - 6x$ $y = 11x + 3$
 k $\frac{dy}{dx} = 12x - 6x^2$ $y = 8$
 l $\frac{dy}{dx} = 60 - 10x^2$ $y = 40x + 27$
 m $\frac{dy}{dx} = 2x^3$ $y = 128x - 391$
 n $\frac{dy}{dx} = 10x - 3$ $y = -3x + 17$

o $\frac{dy}{dx} = 10 - 4x$ $y = 10x$

p $\frac{dy}{dx} = \frac{3x^2}{4} - 4x$ $y = -x - 4$

q $\frac{dy}{dx} = \frac{3}{2}x$ $y = -3x$

r $\frac{dy}{dx} = 2x^2$ $y = 2x + \frac{5}{3}$

s $\frac{dy}{dx} = \frac{3}{4}x^2 - 14x$ $y = 31x + 37$

2 a $\frac{dy}{dx} = \frac{-24}{x^3}$ $3x + y - 9 = 0$

b $\frac{dy}{dx} = \frac{-18}{x^4}$ $18x + y - 29 = 0$

c $\frac{dy}{dx} = 6 + \frac{16}{x^3}$ $4x - y - 6 = 0$

d $\frac{dy}{dx} = 3x^2 - \frac{12}{x^3}$ $15x - y + 20 = 0$

e $\frac{dy}{dx} = 5 + \frac{8}{x^2}$ $11x - 2y - 8 = 0$

Exercise 6G

1 $\frac{dy}{dx} = 4x$ $x + 4y - 9 = 0$

2 $\frac{dy}{dx} = 12x^2$ $x + 3y - 11 = 0$

3 $\frac{dy}{dx} = 0.5 - 2x$ $2x - 7y - 25 = 0$

4 $\frac{dy}{dx} = 3x + 1$ $x - 5y + 22 = 0$

5 $\frac{dy}{dx} = 3 - 2x$ $x + 3y - 30 = 0$

6 $\frac{dy}{dx} = 2x + 4$ $x + 4y - 16 = 0$

7 $\frac{dy}{dx} = -\frac{4}{x^2}$ $x - y = 0$

8 $\frac{dy}{dx} = \frac{-12}{x^3}$ $x + 12y - 71 = 0$

9 $\frac{dy}{dx} = 6 - \frac{8}{x^2}$ $x - 2y + 27 = 0$

10 $\frac{dy}{dx} = 4x^3 + \frac{9}{x^4}$ $x + 5y - 19 = 0$

11 $\frac{dy}{dx} = -2 + \frac{1}{x^2}$ $2x + 4y - 5 = 0$

12 $\frac{dy}{dx} = 5 + \frac{9}{2x^2}$ $4x + 22y - 309 = 0$

Exercise 6H

1 $\frac{dy}{dx} = 2x - 8$ (5, 1)
 $y = 2x - 9$

2 $\frac{dy}{dx} = 3x^2 - 3$ (-2, -2)
 $y = 9x + 16$

3 $\frac{dy}{dx} = 1 - \frac{6}{x^2}$ (4, 5.5)

$m(\text{normal}) = -\frac{8}{5}$
 $16x + 10y - 119 = 0$

4 $\frac{dy}{dx} = 2x + \frac{2}{x^3}$ (-1, 0)

$m(\text{normal}) = \frac{1}{4}$
 $x - 4y + 1 = 0$

5 (2, 8) $y = 10x - 12$

$\left(\frac{-4}{3}, 8\right)$ $10x + y + 5\frac{1}{3} = 0$

6 (5, 20) $y = -14x + 50$

(-2, 20) $y = 14x + 8$

7 $11y - x + 1 = 0$

8 $y = \frac{-x}{6} - \frac{37}{6}$

9 a $x = \frac{3}{4}$ b $y = 0$

10 a $x = 2$ b $y = 12$

11 a $x = 4$ b $y = 5x - 11$

12 a $x = 0$ b $y = 3x - 3$

c $y = -\frac{1}{3}x - 3$ or
 $x + 3y + 9 = 0$

13 a $x = -1$ b $y = 16x + 15$
c $x + 16y + 17 = 0$

14 At $x = 2$, $y = 9$ tangent is
 $y = 36x - 63$

At $x = -5$, $y = 100$ tangent is
 $y = 36x + 280$

15 $k = 1$, $b = 12$

16 $k = 5$, $b = -6$

17 $k = \frac{1}{2}$, $b = 3$

18 $k = 7$, $b = -2$

19 $p = 2.25$, $q = -2$

20 $p = -4$, $q = -18$

Exercise 6I

1 a $V(0) = 100 \text{ cm}^3$
b $V(3) = 133 \text{ cm}^3$

c the rate of change of the volume of water in the container

d $\frac{dV}{dt} = 2 + 3t^2$. At $t = 3$,

$\frac{dV}{dt} = 2 + 3(3)^2 = 29 \text{ cm}^3\text{s}^{-1}$

e There is 133 cm^3 of water in the container and, at this time, water is flowing into the container at $29 \text{ cm}^3\text{s}^{-1}$

2 a $A(0) = 0$ b $A(5) = 45 \text{ cm}^2$

c the rate of change of the area of the pool of water

d $\frac{dA}{dt} = 4 + 2t$. At $t = 5$,

$\frac{dA}{dt} = 14 \text{ cm}^2\text{s}^{-1}$

e The area of the pool is 45 cm^2 and, at this time, the area is increasing at $14 \text{ cm}^2\text{s}^{-1}$

3 a $W(1) = 685 \text{ tonnes}$

b $\frac{dW}{dt} = 10t - \frac{640}{t^2}$

c i $\frac{dW}{dt}(3) = -41\frac{1}{9} \text{ tonnes}^{-1}$

ii $\frac{dW}{dt}(5) = 24.4 \text{ tonnes}^{-1}$

d At $t = 3$, oil is flowing from the tank, but at $t = 5$, oil is flowing into the tank.

e $t = 4$

f At $t = 4$, the weight of oil in the tank is at its maximum value. (This is 280 tonnes.)

4 a At $t = 1$, $\frac{dV}{dt} = 8 \text{ m}^3\text{min}^{-1}$

b $65 = 10 + 6t + t^2$
 $\Rightarrow t^2 + 6t - 55 = 0$
 $\Rightarrow t = 5 (>0)$
At $t = 5$

$\frac{dV}{dt} = 16 \text{ m}^3\text{min}^{-1}$

5 a $\frac{dy}{dt}(2) = -16 \text{ cm s}^{-1}$

$\frac{dy}{dt}(3) = -31 \text{ cm s}^{-1}$

b Use the GDC to solve $500 - 4t - t^3 = 0$, 7.77s

6 a $3.5 \text{ cm}^2\text{s}^{-1}$

b GDC gives $t = 6$. Hence, $9.5 \text{ cm}^2\text{s}^{-1}$

7 a $-23.75 \text{ tonnes/hour}$

b $t = 3 \text{ hours}$

8 a 44 degrees per second
b $t = \frac{1}{6} \text{ seconds}$

9 a -15 and -215 ; these are losses of USD 15 000 and 215 000.

b $\frac{dP}{dx} = -30x^2 + 80x + 10$

c i $P(2) = 85$, $\frac{dP}{dx}(2) = 50$

ii $P(3) = -10(3)^3 + 40(3)^2 + 10(3) - 15 = 105$,

$\frac{dP}{dx}(3) = -30(3)^2 + 80(3) + 10 = -20$

d i A profit is being made and if production is increased, the profit will increase also.

ii A profit is being made but if production is increased, the profit will decrease.

e $\frac{dP}{dx} = -30x^2 + 80x + 10 = 0$;

$x = 2.79$, $p = 107$.

GDC gives the answer. At this point the level of production yields a profit that is a maximum. Maximum profit = \$107000 when 2.79 tonnes are made.

Exercise 6J

1 $x = 3$ 2 $x = 3$

3 $x = -5$ 4 $x = -\frac{5}{2}$

5 $x = \pm 3$ 6 $x = \pm 2$

7 $x = \pm \frac{1}{2}$ 8 $x = \pm \frac{1}{4}$

9 $6x^2 - 18x + 12 = 0$ $x = 1, 2$

10 $9 + 12x + 3x^2 = 0$ $x = -1, -3$

11 $3x^2 - 6x - 45 = 0$
 $x^2 - 2x - 15 = 0$ $x = 5, -3$

12 $24x + 3x^2 + 36 = 0$
 $x^2 + 8x + 12 = 0$ $x = -2, -6$

13 $6x^2 - 12x = 0$ $x = 0, 2$

14 $60x - 15x^2 = 0$ $x = 0, 4$

15 $\frac{dy}{dx} = 1 - \frac{1}{x^2} = 0$ $1 = \frac{1}{x^2}$
 $x^2 = 1$ $x = \pm 1$

16 $\frac{dy}{dx} = 1 - \frac{4}{x^2} = 0$ $1 = \frac{4}{x^2}$
 $x^2 = 4$ $x = \pm 2$

17 $\frac{dy}{dx} = 4 - \frac{9}{x^2} = 0$ $4 = \frac{9}{x^2}$
 $4x^2 = 9$ $x = \pm 1.5$

18 $\frac{dy}{dx} = 8 - \frac{1}{2x^2} = 0$ $8 = \frac{1}{2x^2}$
 $x^2 = \frac{1}{16}$ $x = \pm \frac{1}{4}$

19 $\frac{dy}{dx} = 27 - \frac{8}{x^3}$ $27 = \frac{8}{x^3}$
 $x^3 = \frac{8}{27}$ $x = \frac{2}{3}$

20 $\frac{dy}{dx} = 1 - \frac{1}{x^3}$ $1 = \frac{1}{x^3}$
 $x^3 = 1$ $x = 1$

Exercise 6K

1 $3x^2 - 18x + 24 = 0$
 $3(x - 4)(x - 2) = 0$
 $x = 4$ -4 minimum
 $x = 2$ 0 maximum

2 $3x^2 + 12x + 9 = 0$
 $x = -1$ $(-1, 1)$ minimum
 $x = -3$ $(-3, 5)$ maximum

3 $9 + 6x - 3x^2 = 0$
 $x = -1$ $(-1, -5)$ minimum
 $x = 3$ $(3, 27)$ maximum

4 $3x^2 - 6x = 0$
 $x = 2$ $(2, 1)$ minimum
 $x = 0$ $(0, 5)$ maximum

5 $27 - 3x^2 = 0$ $(-3, 54)$ minimum
 $(3, 54)$ maximum

6 $18x - 3x^2 = 0$ $(0, 0)$ minimum
 $(6, 108)$ maximum

7 $\frac{dy}{dx} = 1 - \frac{1}{x^2}$ $(1, 2)$ minimum
 $(-1, -2)$ maximum

8 $\frac{dy}{dx} = 1 - \frac{9}{x^2}$ $(3, 6)$ minimum
 $(-3, -6)$ maximum

9 $\frac{dy}{dx} = \frac{1}{2} - \frac{8}{x^2}$ $(4, 4)$ minimum
 $(-4, -4)$ maximum

10 $\frac{dy}{dx} = -\frac{9}{x^2} + \frac{1}{4}$ $(6, 3)$ minimum
 $(-6, -3)$ maximum

11 $\frac{dy}{dx} = 2x + \frac{16}{x^2}$ $(-2, 12)$ minimum

12 $\frac{dy}{dx} = 9 - \frac{1}{3x^3}$ $(\frac{1}{3}, 4.5)$ minimum

Exercise 6L

1 $(2, 6)$ minimum

2 $(3, 29)$ maximum

3 $(-\frac{1}{2}, -3\frac{1}{4})$ minimum

4 $(\frac{5}{2}, 1\frac{3}{4})$ minimum

5 $(\frac{3}{2}, 13\frac{1}{4})$ maximum

6 $(-\frac{5}{4}, 23\frac{5}{8})$ maximum

7 $(5, -4)$ minimum

8 $(9, -81)$ minimum

9 $(-2, -4)$ minimum

Exercise 6M

1 a $b = 7 + h$ b $A = h(7 + h)$

2 a $x = 10 - t$ b $V = 3t(10 - t)$

3 a $y = 5 - 2x$ b $p = x^2(5 - 2x)$

4 a $R = \frac{1}{2}r^2(r + 25)$

b $R = \frac{1}{2}n(n - 25)^2$

5 a $L = 2m(m + 100 - 5m)$
 $= 2m(100 - 4m)$

b $L = 2\left(20 - \frac{x}{5}\right)\left(20 + \frac{4x}{5}\right)$

6 a $V = \pi r^2(17 - 2r)$

b $V = \frac{\pi h(17 - h)^2}{4}$

7 a $y = 5x^2 + 6x - \frac{3}{2}$

b $\frac{dy}{dx} = 10x + 6$

c $x = -0.6$, $y = -3.3$

d $c = -5.1$

8 a $N = 2n(5 - 1.5 + 1.2n)$
 $= 2n(3.5 + 1.2n)$

b $\frac{dN}{dn} = 7 + 4.8n$

c $N = -5.10$ (to 3 sf)

d $x = 3.25$

9 $A = \frac{1}{2}L\left(\frac{3L - 18}{5}\right) = \frac{L(3L - 18)}{10}$
 $L = 3$, $A = -2.7$, $B = -1.8$

10 $C = \pi r\left(\frac{30 - r}{3}\right)$, $r = 15$,
 $C = 75\pi$, $f = 5$

11 a $b = 10$
 $X = 2b(b + 10)$

b $b = -5$, $X = -50$

12 $A = t(12 - 2t)$
 $t = 3$, $A = 18$, max.

13 $A = 2y(30 - 3y)$
 $y = 5$, $A = 150$, max.

14 $A = 3M(2M - 28)$
 $M = +7$, $A = -294$, min.

15 $A = g^2 + (8 - g^2)$; $A' = 4g - 16 = 0$
 $\Rightarrow g = 4$, $A = 32$, min

16 $S = x^2 + (6 - x)^2$
 $S' = 4x - 12 \Rightarrow x = 3$, $y = 3$

17 $V = r^2 h = r^2(6 - r)$
 $V' = 12r - 3r^2 = 0$
 $r = 4$, $V = 32$

18 $V = m^2(9 - m)$
 $V' = 18m - 3m^2 = 0$

$m = 6 \Rightarrow V = 108$, max.
 $m = 0 \Rightarrow V = 0$, min.

Exercise 6N

- 1 $w = 10\text{ m}, l = 20\text{ m} \quad 2 \quad x = 12$
- 3 $2x^2 + 6xh = 150$
 $V = 2x^2h = 2x^2 \left[\frac{150 - 2x^2}{6x} \right]$
 $\Rightarrow V = \frac{500}{3} w = 5\text{ cm}, l = 10\text{ cm}$
 $h = \frac{10}{3}\text{ cm}$
- 4 $w = 4\text{ cm}, l = 6\text{ cm}$
- 5 $w = 60\text{ cm}$
- 6 $V = 268(3\text{ sf})\text{ cm}^3$
 $r = 8, h = 4$
- 7 $V_{\text{max}} = 1000\text{ cm}^3$
- 8 $2\pi r^2 + 6\pi rh = 600$
 $V = (600 - 2\pi r^2)$
 $V = 300r - \pi r^3$
 $V' = 300 - 3\pi r^2$
 $h = \frac{20}{\sqrt{\pi}} \quad r = \frac{10}{\sqrt{\pi}}\text{ cm}$
- 9 $V' = 576 - 192x + 12x^2 = 0$
 $\Rightarrow x^2 - 16x + 48 = 0$
 $(x - 4)(x - 12) = 0$
 $x \neq 12, x = 4 \Rightarrow V = 1024\text{ cm}^3$
- 10 $V = 160x - 52x^2 + 4x^3$
 $V' = 160 - 104x + 12x^2$
 $40 - 26x + 3x^2 = 0$
 $(20 - 3x)(2 - x) = 0$
 $x = 2$
 $x \neq \frac{20}{3}$
 $\Rightarrow V = 144\text{ cm}^3$
- 11 a i $\frac{14}{\pi} = 4.46\text{ cm (3 sf)}$
 b i $\frac{350}{4\pi} = 27.9\text{ cm (3 sf)}$
 c i $\pi r^2 h = 350$
 ii $h = \frac{350}{\pi r^2}$
 iii $A = 2\pi r^2 + \frac{700}{r}$
 iv $r = 3.82\text{ cm (3 sf)}$,
 $h = 7.64\text{ cm (3 sf)}$
 v $A = 275\text{ cm}^2(3\text{ sf})$
- 12 a $W = 250\text{ m}$
 b 1150 m
 c $LW = 50\,000$
 d $W = 182.6\text{ m}, L = 273.9\text{ m}$,
 perimeter = 913 m (all 3 sf)
- 13 a $\$3950$
 b $LW = 50\,000$
 c $W = 165\text{ m (3 sf)}$,
 $L = 303\text{ m (3 sf)}$,
 cost = $\$3633.18$

- 14 a $h = 16\text{ cm}$
 Page area = $13 \times 22 = 286\text{ cm}^2$
 b $293\text{ cm}^2(3\text{ sf})$ c $A = wh$
 d $P = (w + 4)(h + 6)$
 f Width 9.8 cm (3 sf)
 Height 14.7 cm (3 sf)
- 15 a i Width = 50 cm
 iii Frame = 480 cm
 b $225\,000 = 2x^2h$
 d $L = 6x + \frac{450\,000}{x^2}$
 $\Rightarrow \frac{dL}{dx} = 6 - \frac{900\,000}{x^3}$
 Setting this equal to zero
 $\Rightarrow x = \sqrt[3]{150\,000} = 53.1(3\text{ sf})$
 Width 53.1 cm (3 sf)
 Length 106.2 cm (3 sf)
 Height 39.8 cm (3 sf)
 Length of frame 478 cm (3 sf)

Chapter 7

Skills check

- 1 a $88.0(3\text{ sf})$
 b $s = \frac{A - \pi r^2}{\pi r}$
- 2 a 655.20 GBP
 b 730.24
 c 96 euros
- 3 $x = 1, y = -5$

Investigation - number sequences

Triangle Numbers:

1	3	3	1	
1	4	6	4	1

Natural Numbers:	1	2	3	4	5	6	7	8	9
Cube Numbers:	1	8	27	64	125	216	343	512	729

Investigation - allowances

- A: Total allowance = $10\,400\text{ euros}$
 B: Total allowance = $9\,693\text{ euros}$
 Option A the best

Exercise 7A

- 1 a 31 b 599
 2 a $u_1 + 2d = 8$
 $u_1 + 8d = 26$
 b $u_1 = 2, d = 3$
 3 3.5
 4 a $4n - 1$ b 199

- 5 a $39, 36$ b 17 c 8
 6 a 4 b 53
 7 56
 8 a 4 b 43 c 21
 9 a 2.5 b 35.5 c 73
 10 a $19, 26$ b 7 c 187

Exercise 7B

- 1 a 26 b 246 c 6175
 2 a $5k + 2 - (k + 4) = 10k - 2 - (5k + 2) = 4k - 2 = 5k - 4$
 $k = 2$
 b $6, 12, 18$ c 6
 d 150 e 1950
- 3 a i 6 ii -10 b $28\,700$
 4 a $16 - 4n$ b $-11\,680$
 5 a i -3 ii 5
 b -5010
 6 5775 7 127.5
 8 a $3k + 4 - (4k - 2) = 6k - (3k + 4) = -k + 6 = 3k - 4$
 $4k = 10$
 $k = 2.5$
 b $8, 11.5, 15$ c 3.5
 d 57 e 487.5

Exercise 7C

- 1 a $\$475$ b $\$4725$
 2 a $2\text{ m } 50\text{ s}$ b $32\text{ m } 30\text{ s}$
 3 $p = a = 400$
 4 a $\$2400$ b $\$12\,750$
 c Option 2 has $\$750$ more.
 5 a $\$190$ b $\$2550$
 6 a 36 b 1050

Exercise 7D

- 1 a 2 b 2097152
 2 a $\frac{1}{3}$ b $\frac{2}{6561} = 0.000305$
 3 a -0.5 b -10
 4 a 2 b 320
 5 a 1.5 b 205.03125
 6 a -8 b -0.125
 7 $\frac{2}{3}$
 8 a 2 b 24576

Investigation - grains of rice

Grains of rice is $2^{64} - 1 = 1.84 \times 10^{19}$

Investigation - becoming a millionaire

After 27 months you would become a millionaire.

Exercise 7E

- 1 a 4 b 0.25 c 32.0
 2 a 4 or -4
 b $r = 4$ then
 sum = $11\,184\,810$
 $r = -4$ then
 sum = $-6\,710\,886$
 3 a -3 b $29\,524$
 4 a 0.5 or -0.5
 b $r = 0.5$ then sum = 83.9
 $r = -0.5$ then sum = -28.0
 5 16382
 6 -64.125

Exercise 7F

- 1 0.975 m 2 $49\,431.11\text{ GBP}$
 3 $10\,230\text{ BGN}$
 4 a 112.57 Dinar
 b 1273.37 Dinar
 5 236221
 6 a $142\,800$ b $157\,663$
 7 a 1.05 b $\$40\,811$
 8 a Common ratio = $\frac{24}{8} = 3$
 and $\frac{72}{24} = 3$
 b 648 c 8744

Exercise 7G

- 1 10815.82 ringgits
 2 a 391.50 euros b 54.18 GBP
 3 a 606.40 euros b 726.23 CAD
 c 73.77 CAD
 4 a 888 euros b 7338.84 SEK
 c 661.16 SEK
 5 a 1992.00 ZAR b 125.50 BRL
 6 a 288.56 euros b 19.18 GBP
 7 a 3297.50 USD b 939.38 EUR
 c $\text{lost } 43.98\text{ EUR}$
 8 a 206 yuan b $174\,655\text{ yen}$
 c 0.85 GBP
 9 a 45 euros b 2518.84 GBP
 c 486.27 euros
 10 a $22\,475\text{ IDR}$ b $229\,761\text{ CLP}$
 11 a $p = 1.3175, q = 107.99$
 b i 176.06 EUR
 ii 146.40 GBP
 12 a 1907.10 GBP b 16.95 GBP

Exercise 7H

- 1 a 7715.52 JPY b 11 years
 2 a A has 3105.94 euros ;
 B has 3090.64 euros and
 C has 3067.47 euros
 b 9.21 or 10 years
 c 16.2 or 17 years
 3 a $\$6110.73$ b $r = 3.79$
 4 a $23\,348.49\text{ EGP}$
 b 22.4 or 23 years
 5 a $61\,252.15\text{ SGD}$
 b $75\,070.16\text{ SGD}$
 6 Mr Lin has $11\,698.59\text{ CNY}$ and Mr Lee has $11\,707.24\text{ CNY}$ so Mr Lee has the most interest.
 7 a 1348.85 GBP
 b 2965 GBP c 11.6 or 12 years
 8 a $a \left(1 + \frac{6}{100} \right) + (8000 - a)$
 $\left(1 + \frac{5}{100} \right) = 8430$
 b $3000\text{ euros in Bank A and } 5000\text{ euros in Bank B}$

Exercise 7I

- 1 3.69 euros 2 3745833 MXN
 3 8811.63 USD 4 50.77 CAD
 5 13.69 KRW 6 $28\,687.26\text{ GBP}$
 7 $60\,303.57\text{ USD}$
 8 $119\,985.99\text{ euros}$

Review exercise

Paper 1 style questions

- 1 a 11.8% b 6.21 or 7 years
 2 a $\text{USD } 256\,944$
 b 2.32%
 3 a $\text{GBP } 220.10$ b 4.49 or 5 years
 c 16.5 or 17 years
 4 a 94.13 EUR b 0.99 AUD
 5 a $\text{£}1607$ b $\text{£}8073.70$
 6 a $\text{€}35\,220$
 b $\text{€}26.4$ or 27 months
 7 a first term = 6 , common difference = 3
 b 153 c 3975
 8 132
 9 a first term = ± 15 , common ratio = ± 2
 b 480 c 3825 or 1275
 10 a $\frac{1}{3}$ b $\frac{2}{27}$
 c 81.0 or $\frac{59078}{729}$

- 11 a first term = ± 8 , common ratio = $\pm \frac{1}{2}$
 b -0.25 c -15.75 or 5.25
 12 a $\$450$ b $\$1009.11$
 c one
 13 a 288 b $3r^5 = 96$
 c 2

Paper 2 style questions

- 1 a i $\$2750$ ii $\$1920$
 b $\$21\,250$
 c option two (by $\$1250$)
 2 a A: $\$1800$, B: $\$1767.54$, C: $\$1920$, D: $\$1910.06$
 b C—largest total amount
 c 6.27%
 3 a i $\$2250, \2500
 ii $\$6750$
 iii $\frac{20}{2}(2000 + 6750) = \$87\,500$
 b i $\$2940$
 ii $2800 \times 1.05^4 = 3403.42$
 c $\$5085$ (option 1)
 4 a $(6k + 4) - 5k = 5k - (3k + 1) \Rightarrow k + 4 = 2k - 1 \Rightarrow k = 5$
 b $16, 25, 34$ c 9
 d 142 e 2030
 5 a $31\,496.19\text{ GBP}$
 b i 18 years
 ii 467.23 GBP
 6 a 12 b $\frac{1}{5}$ c 2.50

Chapter 8

Skills check

- 1 a 5 is an integer, real and rational (since it can be written as $\frac{5}{1}$)
 b $1.875 = 1\frac{7}{8}$ is not an integer, but is both real and rational, since it can be written as $\frac{15}{8}$
 c $0.333 = \frac{333}{1000}$ is not an integer, but is both real and rational. Note that $0.333 \neq \frac{1}{3}$
 d $0.303\,003\,000\,3\dots$ is real, but not rational.
 e $\sqrt{0.5625} = \frac{3}{4}$ is both real and rational.
 f $\sqrt[3]{2.744} = 1.4 = \frac{7}{5}$ is both real and rational.
 g π^2 is real, but not rational.

2 For a-d: -2, -1, 0, 1, 2, 3

3 a i 1, 2, 3, 4, 6, 12
 ii 1, 2, 4, 8 iii 1, 17
 iv 1, 5, 25
 v 1, 2, 3, 4, 6, 8, 12, 24

b i 2, 3 ii 2 iii 17
 iv 5 v 2, 3

c 17 is prime.

d Zero has an infinite number of factors. Zero is an integer, it is rational and it is real, but it is not prime.

Investigation - a contradiction?

There is no contradiction, because some pupils study **both** Chemistry and Biology.

The question, 'How many?' cannot be resolved; there is not enough information.

But, **at least 2** study both subjects and there might be as many as 13.

Investigation - intuition

- 1 a Not fair, and not practical
 b Not fair
 c Fair, but not practical
 2 a Fair b Not fair
 c Not fair. Or is it?
 d Not fair e Fair

Exercise 8A

- 1 a & b
 $M = \{2, 3, 4\}$, $n(M) = 3$
 $N = \{1, 2, 3, 4, 5\}$, $n(N) = 5$
 $P = \{1, 2, 3, 4, 5\}$, $n(P) = 5$
 $S = \{(1, 4), (2, 3), (3, 2), (4, 1)\}$, $n(S) = 4$
 $T = \{(0, 5), (1, 4), (2, 3), (3, 2), (4, 1), (5, 0)\}$, $n(T) = 6$
 $V = \{\}$ or \emptyset , $n(V) = 0$
 $W = \{1, 2, 4, 5, 10, 20\}$, $n(W) = 6$
 X is an infinite set and so elements cannot be listed, $n(X) = \infty$
- 2 a $\{4, 5, 6\}$ b $\{2, 4, 6\}$
 c $\{7, 9, 11\}$ d $\{5, 9, 13, 17, 21\}$
 e $\{(2, 2), (4, 4), (6, 6), (8, 8), (10, 10)\}$
 f $\{(6, 3), (10, 5)\}$
- 3 a $\{x | x = 2y, y \in \mathbb{Z}^+\}$
 b $\{p | p \text{ is prime}\}$
 c $\{x | -2 \leq x \leq 2, x \in \mathbb{Z}\}$
 d $\{x | 2 \leq x \leq 8, x \in \mathbb{Z}\}$
 e $\{x | -2 \leq x \leq 8, x \text{ is even}\}$
 f $\{x | x = 3y, 1 \leq y \leq 6, y \in \mathbb{Z}\}$

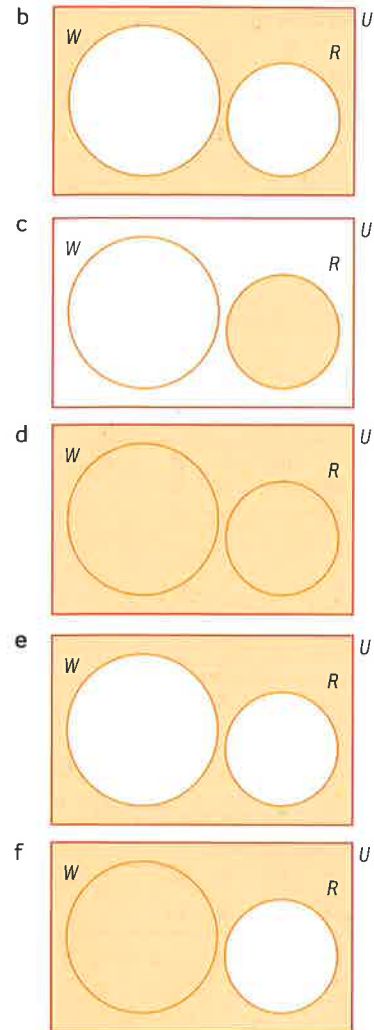
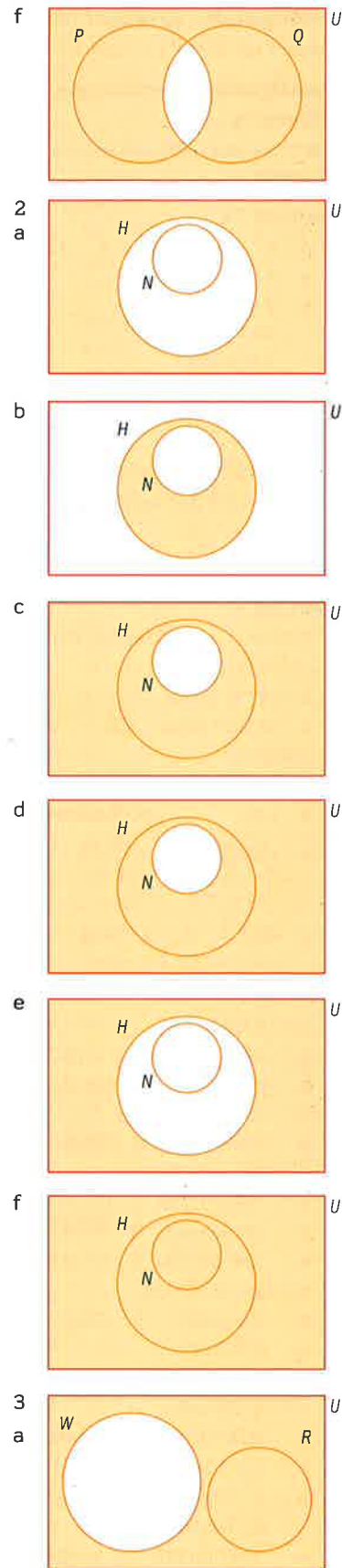
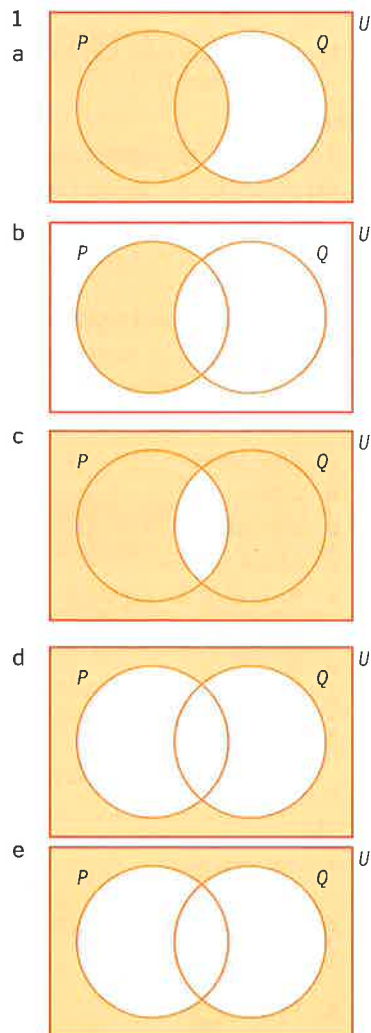
Exercise 8B

- 1 False 2 True 3 False
 4 True 5 True 6 True
 7 True 8 True

Exercise 8C

- 1 a False b True c False
 d True e True f False
 g False h False
- 2 a i $\emptyset, \{a\}$
 ii $\emptyset, \{a\}, \{b\}, \{a, b\}$
 iii $\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}$
 iv There are 16 of these!
- b 2^n c 64 d 7
- 3 a i There are none.
 ii $\{a\}, \{b\}$
 iii $\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}$
 iv There are 14 of these!
- b $2^n - 2$ c 62 d 8

Exercise 8D

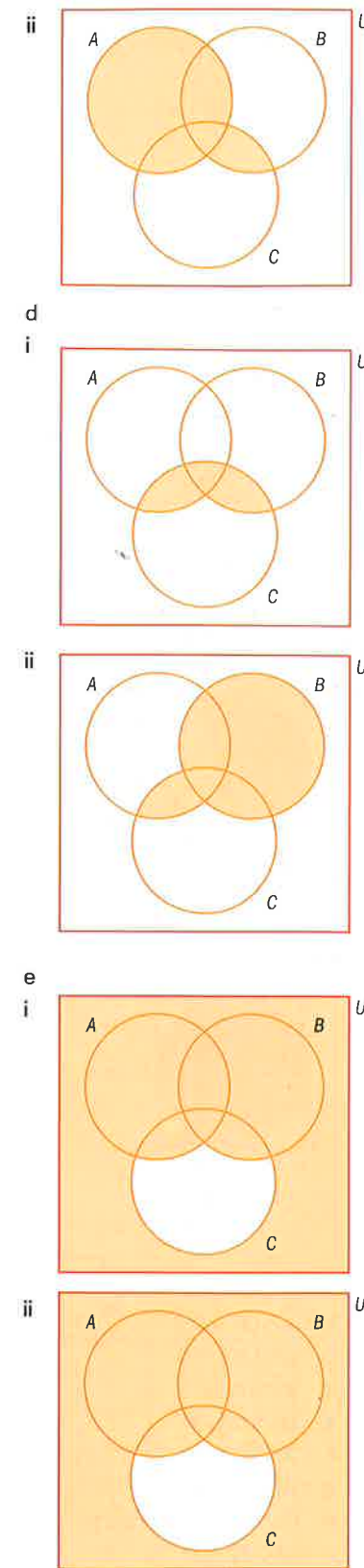
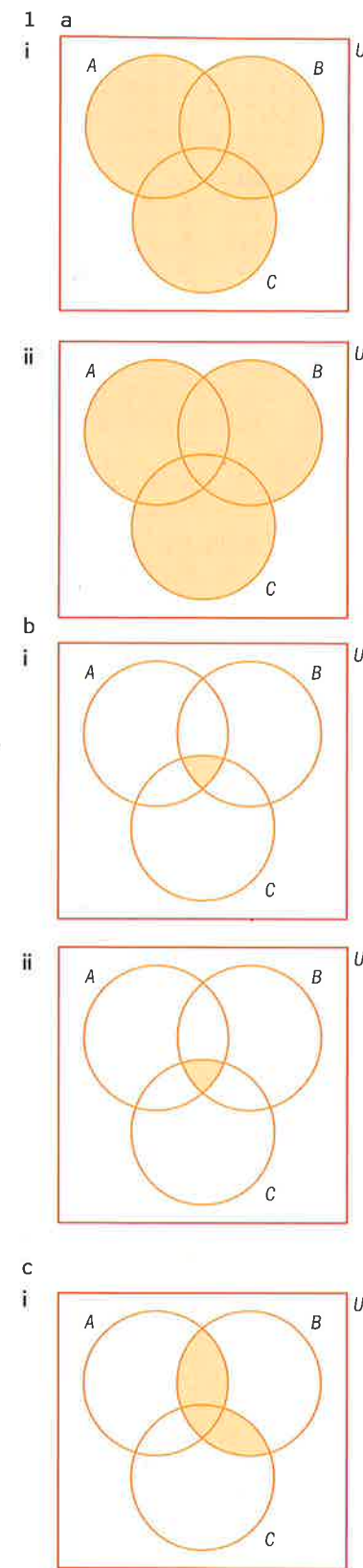


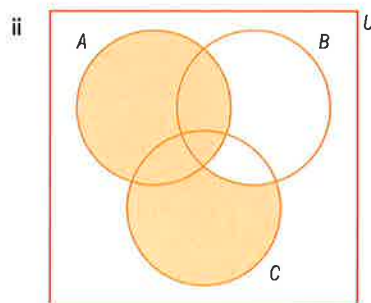
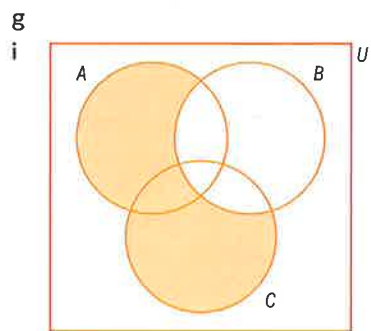
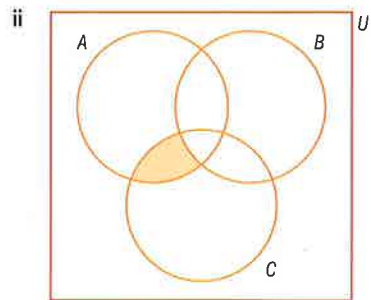
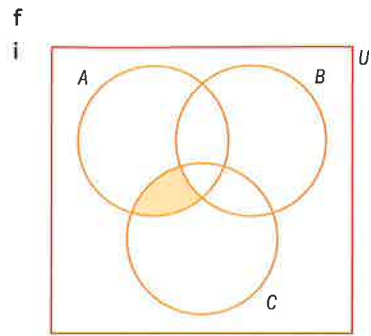
- 4 a $\{1, 2, 3, 4\}$ b $\{1, 4\}$
 c \emptyset d \emptyset e $\{4\}$
 f $\{0, 1, 2, 3, 4, 5\}$
 g $\{0, 1, 2, 3, 4, 5\}$
 h $\{2, 4, 5, 6, 7\}$ i $\{1, 2, 3, 4, 5\}$
 j $\{0, 1, 2, 3, 4, 5, 6, 7\}$
 k False l False
 m True n False o False

Exercise 8E

- 1 a False b True
 c False d False
 e False f False
 g False h False
- 2 a b, c, d, e, f, g, h, k
 b b, d, e, f c c, g, h, k
 d c, d, e, k e b, f, g, h
- 3 a q, t, x, w b p, r
 c p, q, r, t, x, w d q, x, w
 e p, q, r, x, w

Exercise 8F





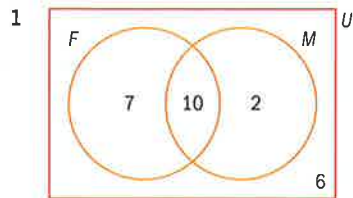
- 2 a $(A' \cup B') \cap C$
 b $A \cap (B' \cup C')$
 c $(A' \cap B') \cap C$
 d $A' \cap (B \cap C')$
 e $(A' \cap C) \cup B$
 f $A \cap (C' \cup B)'$
 g $A \cap (B \cup C)'$
 h $(A \cap C) \cup (A \cup (B \cup C))'$
 i $(A \cup B)' \cap C$
 j $A' \cap (B \cup C)'$

- 3 a 1 b 3 c 4
 d 2 e 7 f 6
 g 5 h 8
 4 a 1, 2, 4 b 3, 6, 7
 c 1, 4, 7 d 2, 5, 6
 e 3, 4, 7 f 2, 6, 8
 g 2, 3, 6 h 4, 7, 8

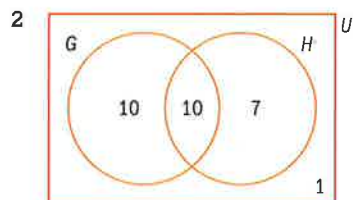
Exercise 8G

- 1 6 2 14 3 21
 4 21 5 13 6 11
 7 7 8 6 9 14

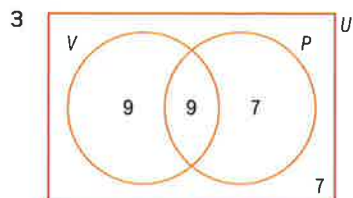
Exercise 8H



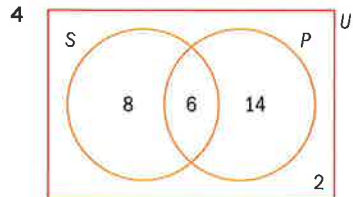
- a 7 b 19 c 6 d 15



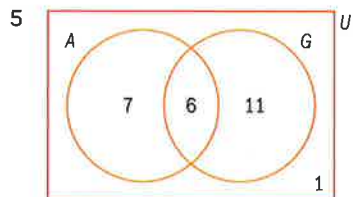
- a 28 b 11 c 10 d 17



- a 9 b 14 c 7 d 16



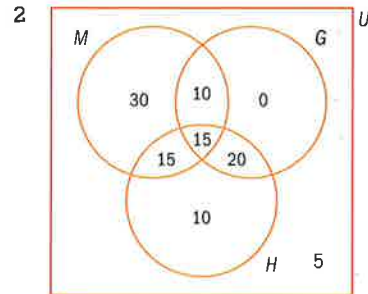
- a 6 b 22 c 8



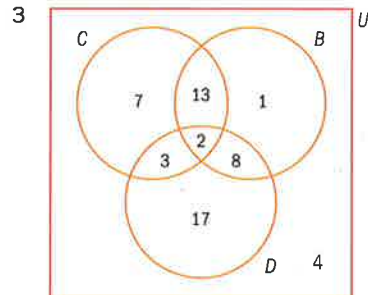
- a 6 b 11 c 24

Exercise 8I

- 1 a 70 b 70 c 55
 d 25 e 25 f 75
 g 70 h 35 i 55
 j 25 k 45

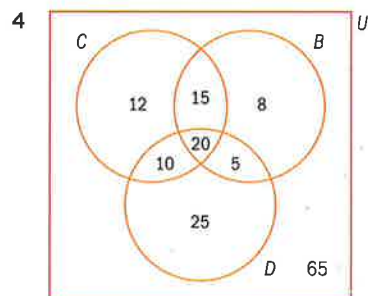


- a 100 b 45 c 20
 d 15 e 30

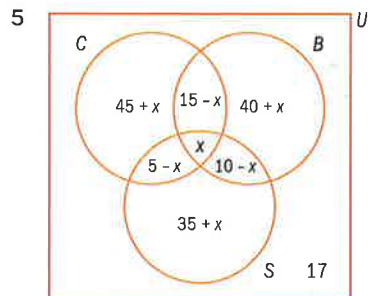


4 are not fulfilling their responsibilities.

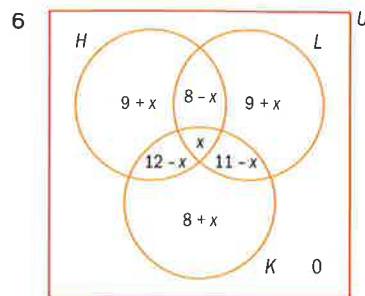
- a 25 b 24 c 29
 d 5 e 14



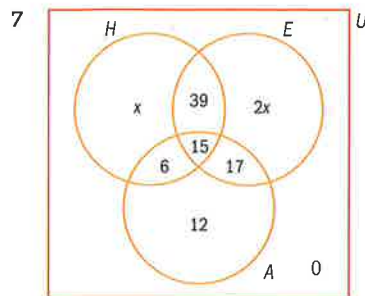
- a 50 b 65
 c 103 d 15



- x = 3
 a 129 b 24 c 146
 d 15 e 9



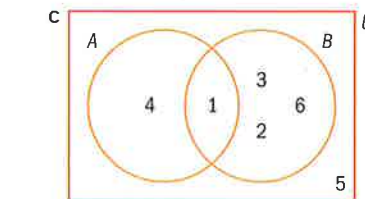
- a 8 b 15 c 3
 d 0 e 17



- a x = 9 b 89

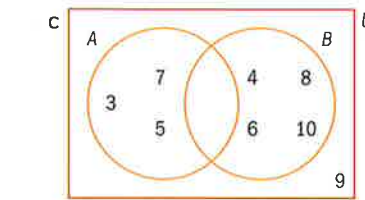
Exercise 8J

- 1 a {1, 4}
 b {1, 2, 3, 6}



- d $\frac{2}{6}$ e $\frac{4}{6}$ f $\frac{4}{6}$
 g $\frac{1}{6}$ h $\frac{5}{6}$

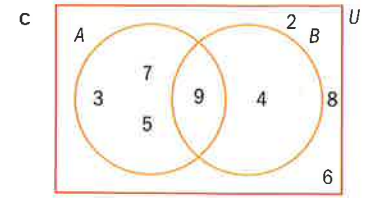
- 2 a {3, 5, 7} b {4, 6, 8, 10}



- d $\frac{3}{8}$ e $\frac{4}{8}$ f $\frac{5}{8}$
 g $\frac{4}{8}$ h 0 i $\frac{7}{8}$

- l $\frac{1}{8}$ m 1

- 3 a {3, 5, 7, 9}
 b {4, 9}



- d $\frac{4}{8}$ e $\frac{2}{8}$ f $\frac{1}{8}$ g $\frac{5}{8}$

- 4 a {HH, HT, TH, TT}

- b $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$

- 5 a {HHHH, HHHT, HHTH, THHH, HHTT, THTT, THTH, THTH, HTTT, THHH, THHT, THHT, TTHH, THTT, TTHT, TTTH, TTTT}

- b $\frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{1}{8}$

- 6 a $\frac{1}{16}$ b $\frac{1}{16}$ c $\frac{4}{16}$ d $\frac{4}{16}$

e $1 - \left(\frac{1}{16} + \frac{4}{16} + \frac{4}{16} + \frac{1}{16} \right) = \frac{6}{16}$

- f {HHHH, HHHT, HHTH, HTHH, HHTT, HTHT, HTTH, HTTT, THHH, THHT, THHT, TTHH, THTT, TTHT, TTTH, TTTT}

Exercise 8K

- 1 a $\frac{23}{40}$ b $\frac{5}{40}$ c $\frac{5}{40}$
 d $\frac{15}{20}$ e $\frac{8}{23}$ f $\frac{8}{23}$

- 2 a $\frac{14}{30}$ b $\frac{8}{30}$ c $\frac{6}{10}$
 d $\frac{8}{20}$ e $\frac{4}{16}$ f 0

- 3 a $\frac{8}{17}$ b $\frac{2}{17}$ c $\frac{8}{17}$
 d $\frac{7}{9}$ e 0 f 1

- 4 a $\frac{12}{34}$ b $\frac{16}{34}$ c $\frac{28}{34}$
 d $\frac{12}{22}$ e $\frac{6}{18}$ f $\frac{10}{22}$

- 5 a $\frac{13}{24}$ b $\frac{4}{24}$ c $\frac{8}{24}$
 d $\frac{17}{24}$ e $\frac{7}{24}$ f $\frac{12}{24}$

- g $\frac{9}{24}$
 6 a $\frac{5}{22}$ b $\frac{18}{22}$
 c $\frac{10}{15}$ d $\frac{3}{8}$

- 7 a $\frac{12}{28}$ b $\frac{4}{13}$ c $\frac{4}{16}$

- d $\frac{3}{28}$ e $\frac{12}{21}$

- 8 a $\frac{12}{27}$ b $\frac{12}{20}$ c $\frac{7}{19}$

- d $\frac{2}{7}$ e $\frac{12}{17}$

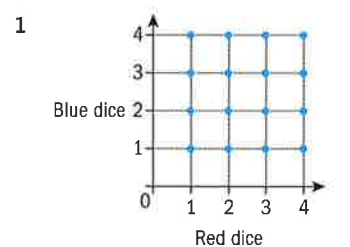
Exercise 8L

- 1 $A \cap B = \{1\}$
 2 $A \cap B = \emptyset$, so A and B are mutually exclusive
 3 $A \cap B = \{2\}$
 4 $A \cap B = \emptyset$, so A and B are mutually exclusive
 5 $A \cap B = \{9\}$
 6 $A \cap B = \emptyset$, so A and B are mutually exclusive
 7 $A \cap B = \{6\}$
 8 $A \cap B = \emptyset$, so A and B are mutually exclusive

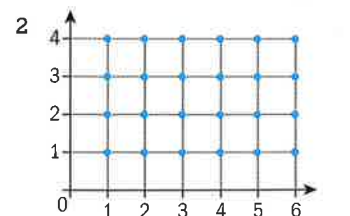
Exercise 8M

- 1 Not independent
 2 Independent
 3 Not independent
 4 Independent events
 5 Not independent events
 6 Not independent events

Exercise 8N

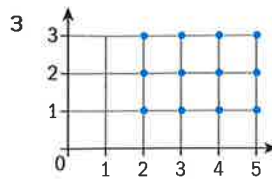


- a $\frac{3}{8}$ b $\frac{3}{8}$ c $\frac{1}{4}$ d $\frac{9}{16}$

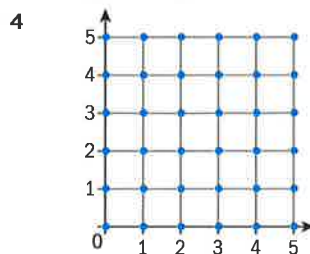


- a $\frac{6}{24}$ b $\frac{13}{24}$ c $\frac{6}{24}$

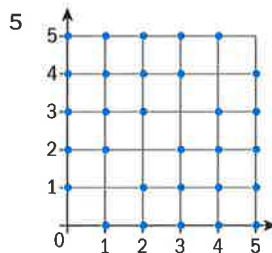
- d $\frac{11}{24}$ e $\frac{4}{24}$



a $\frac{2}{12}$ b $\frac{4}{12}$ c $\frac{9}{12}$
 d $\frac{5}{12}$ e $\frac{8}{12}$



a $\frac{6}{36}$ b $\frac{23}{36}$ c $\frac{26}{36}$
 d $\frac{13}{36}$ e $\frac{27}{36}$

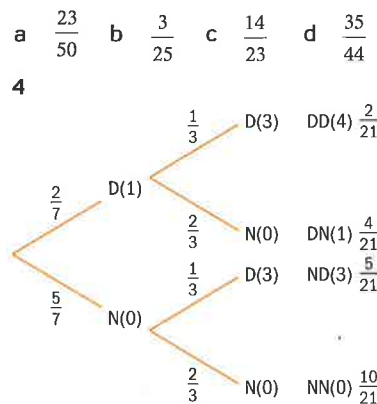
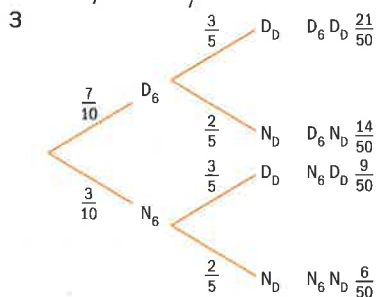


a 0 b $\frac{20}{30}$ c $\frac{22}{30}$
 d $\frac{10}{30}$ e $\frac{24}{30}$

Exercise 80

1 a $\frac{60}{121}$ b $\frac{85}{121}$ c $\frac{60}{121}$
 d $\frac{1}{2}$ e $\frac{55}{85}$

2 a $\frac{12}{25}$ b $\frac{21}{25}$
 c $\frac{3}{7}$ d $\frac{5}{7}$

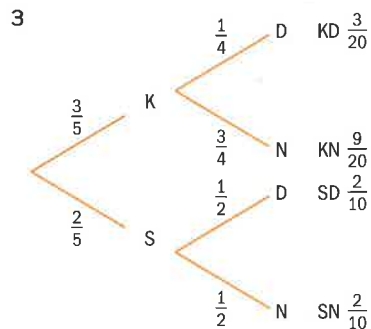


a $\frac{10}{21}$ b $\frac{3}{7}$ c $\frac{6}{11}$ d $\frac{1}{3}$
 5 a 0.97 b $\frac{80}{97}$ c $\frac{72}{97}$

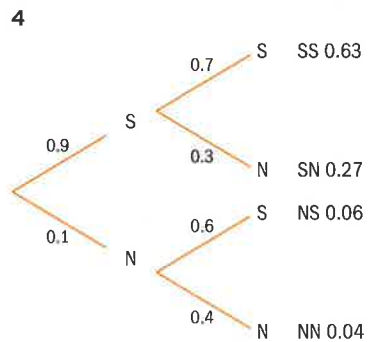
Exercise 8P

1 a $\frac{60}{110}$ b $\frac{80}{110}$ c $\frac{60}{110}$
 d $\frac{1}{2}$ e $\frac{5}{8}$

2 a $\frac{20}{132}$ b $\frac{90}{132}$ c $\frac{1}{2}$



a $\frac{3}{20} + \frac{4}{20} = \frac{7}{20}$ b $\frac{2}{10}$
 c $\frac{3}{20} = \frac{3}{7}$ d $\frac{4}{1-7/20} = \frac{4}{13}$



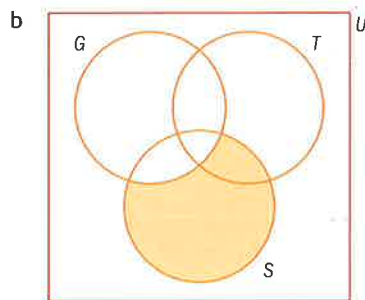
a 0.63
 b 0.33
 c $\frac{0.27}{0.33} = \frac{9}{11}$
 d $\frac{0.9}{0.96} = \frac{15}{16}$

5 $\frac{3}{8}$

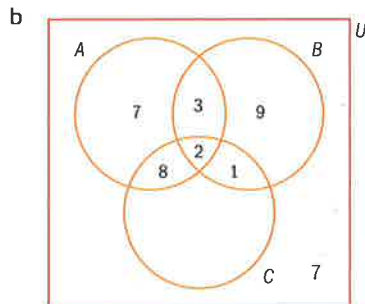
Review exercise

Paper 1 style questions

1 a i 6 ii 5
 iii 10 iv 24

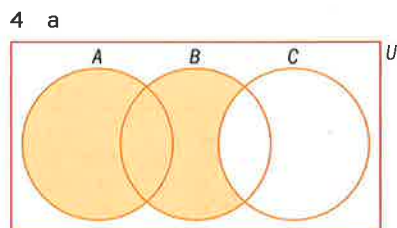


2 a Diagram should show a rectangle containing 3 intersecting **labelled** circles



c $40 - (7 + 3 + 2 + 1 + 8 + 9 + 7) = 3$

- 3 a False
 b True
 c False
 d True
 e True
 f True



- b 15
 c i 5, 10, 15, 20 ii 10, 20, 30
 5 a For example 2, -3 etc.
 b For example $\frac{3}{5}$ (not $\frac{6}{1}$)
 c For example $\frac{3}{5}$ or $\frac{6}{1}$
 d For example $\frac{3}{5}, \sqrt{2}, \pi$
 e For example $\sqrt{2}, \pi$
 f For example $\sqrt{2}, \pi$

6 a $\frac{4}{60}$ or 6.67% or 0.0667

b $\frac{56}{60}$ or 93.3% or 0.933

c $\frac{16}{20}$ or 80% or 0.8

7 a $\frac{3}{15}$ or 20% or 0.2

b $\frac{3}{14}$ or 21.4% or 0.214

c $\frac{4}{15} \times \frac{3}{14} = \frac{2}{35}$ or 5.71% or 0.0571

8 a 12

b $\frac{3}{12} = \frac{1}{4}$ or 25%

c $\frac{4}{12} = \frac{1}{3}$ or 33.3% (3 sf)

9 a $3400 \leq w < 3700$

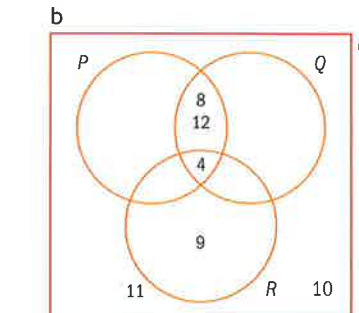
b $\frac{5}{50} = \frac{1}{10}$ or 10% or 0.1

c $1 - \frac{5}{50} = \frac{45}{50} = \frac{9}{10}$ or 90% or 0.9

d $\frac{20}{45} = \frac{4}{9}$ or 44.4% or 0.444

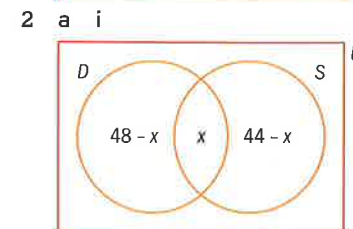
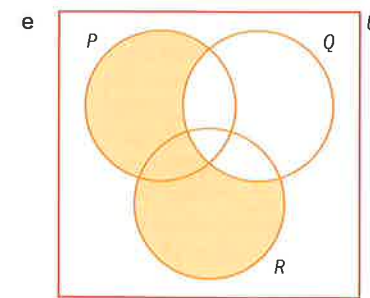
Paper 2 style questions

1 a $U = \{8, 9, 10, 11, 12\}$



c i none ii none

d $P \cup Q$: the set of numbers that are either multiples of 4 or factors of 24, or both

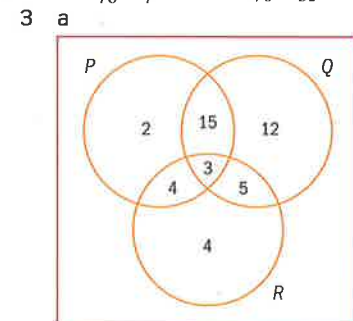


ii $48 - x + x + 44 - x = 70$
 $\Rightarrow x = 22$

iii Those members who **did not** attend for **both** Drama and Sports (or equivalent)

iv $P(D \text{ or } S) = \left[\frac{48-22}{70} + \frac{44-22}{70} \right]$
 $= \frac{48}{70} \text{ or } \frac{24}{35}$

b i $\frac{30}{70} = \frac{3}{7}$ ii $\frac{12}{70} = \frac{6}{35}$



b $50 - (2 + 3 + 4 + 15 + 5 + 4 + 12) = 5$

c i $P(\text{fruit juice}) = \frac{35}{50}$

ii $P([P \cup Q] \cap R') = \frac{29}{50}$

iii $P(Q|P) = \frac{6}{24}$

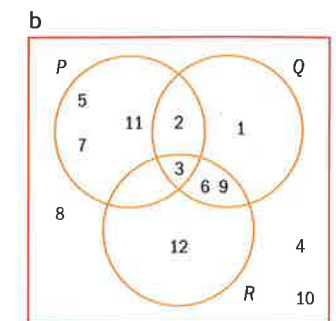
d $P(\text{both drank all three}) = \frac{3}{50} \times \frac{2}{49} = \frac{6}{2450}$

4 a i $P = \{2, 3, 5, 7, 11\}$

ii $Q = \{1, 2, 3, 6, 9\}$

iii $R = \{3, 6, 9, 12\}$

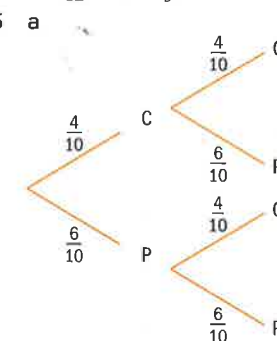
iv $P \cap Q \cap R = \{3\}$



c i $\{2, 3, 5, 6, 7, 9, 11\}$
 ii $\{1, 4, 8, 10\}$
 iii $\{4, 8, 10\}$

d i $\frac{5}{12}$ ii $\frac{3}{12}$

iii $\frac{4}{12}$ iv $\frac{2}{5}$



b i $P(\text{chocolate, chocolate}) = \frac{4}{10} \times \frac{4}{10} = 0.16$

ii $P(\text{one is plain}) = P(\text{chocolate, plain}) + P(\text{plain, chocolate})$
 $= \frac{4}{10} \times \frac{6}{10} + \frac{6}{10} \times \frac{4}{10} = 0.48$

c i $a = 8, b = 9$ ii

iii $P(\text{at least one plain}) = 1 - P(\text{two chocolate}) = 1$

d $P(\text{tin, chocolate}) = \frac{1}{2} \times \frac{4}{10} + \frac{1}{2} \times \frac{1}{10} = 0.25$

6 a i $P(\text{desert}) = \frac{13}{60}$

ii $P(\text{waterlogged and low growth rate}) = \frac{16}{60}$

iii $P(\text{not temperate}) = 1 - \frac{18}{60} = \frac{42}{60}$

b i $P(\text{high growth rate or waterlogged environment, but not both}) = \frac{4}{60} + \frac{7}{60} + \frac{16}{60} = \frac{27}{60}$

ii P (low, given desert)

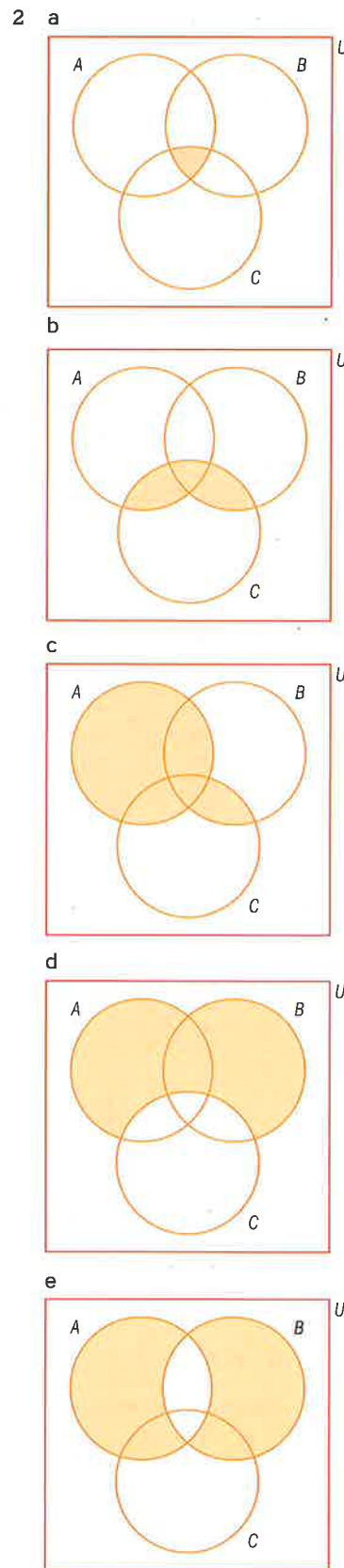
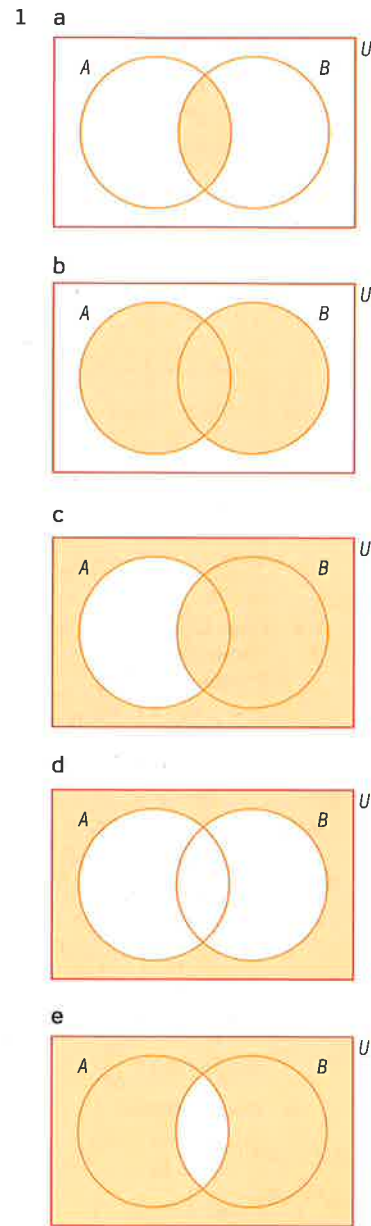
$$= \frac{9}{13}$$

c i $\frac{36}{60} \times \frac{35}{59} = \frac{21}{59}$

ii $\frac{45}{60} \times \frac{44}{59} = \frac{33}{59}$

Chapter 9

Skills check



Investigation - logical thinking

- That food must not be taken into the sports hall by anyone and that drinks must not be taken into the sports hall by anyone either.
 - According to the sign, yes!
 - Again, yes!
 - Unfortunately, it does not.
 - 'No food **or** drink allowed in the sports hall.'
- 'No food **and** no drink allowed in the sports hall.'

Exercise 9A

- | | |
|------|------|
| 1 Y | 2 N |
| 3 N | 4 Y |
| 5 Y | 6 Y |
| 7 Y | 8 N |
| 9 Y | 10 Y |
| 11 Y | 12 Y |
| 13 Y | 14 N |

Exercise 9B

- | | |
|--------------|--------------|
| 1 Exclusive | 2 Exclusive |
| 3 Inclusive | 4 Inclusive |
| 5 Inclusive | 6 Inclusive |
| 7 Exclusive | 8 Exclusive |
| 9 Exclusive | 10 Exclusive |
| 11 Exclusive | 12 Exclusive |

Note that there are some cases (notably 7) where there is some ambiguity about the version of 'or'. Such ambiguity **must** be removed.

Exercise 9C

- The student is a not council member.
 - She does not own a mobile phone.
 - n is a composite number.
 - ABCD is not a parallelogram.
 - Surabaya is not the capital of Indonesia.
- This word starts with a consonant.
 - There is an odd number of pages in this book.
 - This price is exclusive of sales tax.
 - This shape is something other than quadrilateral.
 - He walked at a variable speed.
- There are other marks in between the highest and the lowest.

- There are degrees of difficulty.
 - She may have scored exactly 50%
 - Richard may have one foot in the classroom and one foot out
 - Unless the average (mean) was an exact integer, the negation is correctly stated.
- Clearly not.
 - This method works, but leads to some awkwardly constructed sentences.
- x is less than or equal to five.
 - y is greater than or equal to seven.
 - z is less than ten.
 - b is greater than 19.
 - Neither
 - ' x is non-negative.'
 - Courtney was at school on Friday.
 - This chair is not broken.
 - The hockey team either won or drew (tied) their match.
 - The soccer team did not win the tournament.
 - The hotel has running water.
 - His signature is legible.
 - James is my age or younger.
 - The class contains at least eight boys.
 - Her family name begins with a letter other than P.
 - He has at most one sister.
 - X is a male doctor.
 - X is a female whose profession is something other than a doctor.
 - X is a married woman.
 - X is a single (unmarried) man.
 - R is a positive rotation between 0° and 90° inclusive.
 - R is a rotation of less than or equal to 90° .

Exercise 9D

- Susan speaks French and Susan speaks Spanish.
 - Susan does not speak French and Susan speaks Spanish.
 - Susan speaks French and Susan

- There are degrees of difficulty.
 - She may have scored exactly 50%
 - Richard may have one foot in the classroom and one foot out
 - Unless the average (mean) was an exact integer, the negation is correctly stated.
- Clearly not.
 - This method works, but leads to some awkwardly constructed sentences.
- x is less than or equal to five.
 - y is greater than or equal to seven.
 - z is less than ten.
 - b is greater than 19.
 - Neither
 - ' x is non-negative.'
 - Courtney was at school on Friday.
 - This chair is not broken.
 - The hockey team either won or drew (tied) their match.
 - The soccer team did not win the tournament.
 - The hotel has running water.
 - His signature is legible.
 - James is my age or younger.
 - The class contains at least eight boys.
 - Her family name begins with a letter other than P.
 - He has at most one sister.
 - X is a male doctor.
 - X is a female whose profession is something other than a doctor.
 - X is a married woman.
 - X is a single (unmarried) man.
 - R is a positive rotation between 0° and 90° inclusive.
 - R is a rotation of less than or equal to 90° .

- Susan does not speak French and Susan does not speak Spanish.
 - Susan does not speak French and Susan speaks Spanish.
 - Jorge speaks Portuguese and Mei Ling speaks Malay.
 - Jorge does not speak Portuguese and Mei Ling speaks Malay.
 - Jorge speaks Portuguese and Mei Ling does not speak Malay.
 - Jorge does not speak Portuguese and Mei Ling does not speak Malay.
 - It is not the case that Jorge speaks Portuguese and Mei Ling speaks Malay.
 - All dogs bark and All flowers are yellow.
 - Not all dogs bark and All flowers are yellow.
 - All dogs bark and Not all flowers are yellow.
 - Not all dogs bark and Not all flowers are yellow.
 - It is not the case that all dogs bark and all flowers are yellow.
 - China is in Africa and Rwanda is in Asia.
 - China is not in Africa and Rwanda is in Asia.
 - China is in Africa and Rwanda is not in Asia.
 - China is not in Africa and Rwanda is not in Asia.
 - It is not the case that China is in Africa and Rwanda is in Asia.
 - Chicago is the largest city in Canada and Jakarta is the largest city in Indonesia.
 - Chicago is not the largest city in Canada and Jakarta is the largest city in Indonesia.
 - Chicago is the largest city in Canada and Jakarta is not the largest city in Indonesia.
 - Chicago is not the largest city in Canada and Jakarta is not the largest city in Indonesia.
- It is not the case that Chicago is the largest city in Canada and Jakarta is the largest city in Indonesia.
 - $x \leq 5$ and $x \geq 5$
 - $x > 5$ and $x \geq 5$
 - $x \leq 5$ and $x < 5$
 - $x > 5$ and $x < 5$
 - It is not the case that $x \leq 5$ and $x \geq 5$.

Yes, if $x = 5$. Hence **e** is better expressed as $x \neq 5$.
 - ABCD is a parallelogram and ABCD is a rectangle.
 - ABCD is a not a parallelogram and ABCD is a rectangle.
 - ABCD is a parallelogram and ABCD is not a rectangle.
 - ABCD is not a parallelogram and ABCD is not a rectangle.
 - It is not the case that is a parallelogram and ABCD is a rectangle.

Statement **b** cannot possibly be true in this case.
 - Triangle ABC is right-angled at C and $AB^2 = AC^2 + BC^2 + 1$
 - Triangle ABC is not right-angled at C and $AB^2 = AC^2 + BC^2 + 1$
 - Triangle ABC is right-angled at C and $AB^2 \neq AC^2 + BC^2 + 1$
 - Triangle ABC is not right-angled at C and $AB^2 \neq AC^2 + BC^2 + 1$
 - It is not the case that triangle ABC is (both) right-angled at C and $AB^2 = AC^2 + BC^2 + 1$

a, **b** and **c** cannot possibly be true in this case.
e, **d** must be true in this case.
 - n is an odd integer and n is an even integer.
 - n is not an odd integer and n is an even integer.
 - n is an odd integer and n is not an even integer.
 - n is not an odd integer and n is not an even integer.
 - It is not the case that n is an odd integer and n is an even integer.

a cannot possibly be true. d cannot possibly be true, but only if the universal set is the set of integers.
b and c are necessarily true, but only if the universal set is the set of integers.
e must be true

10

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

$p \wedge \neg p$ is a logical contradiction because all the entries in its column are False.

11 $p \wedge \neg q$

p	q	$\neg q$	$p \wedge \neg q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

12 $p \wedge q$

p	q	$p \wedge q$	n
T	T	T	20
T	F	F	18
F	T	F	15
F	F	F	7

Exercise 9E

- x is less than or equal to 36.
 - x is less than or equal to 36, but not both.
 - Neither
- $p \vee r$
 - $p \vee r$
 - $q \vee r$
 - $(q \vee r) \wedge \neg p$
 - No
- $p \vee q$
 - $p \vee q$
 - $p \vee r$
 - $q \vee r$
 - $p \vee q \vee r$
 - $(p \vee q) \wedge \neg r$

(Note that the brackets are required.)
 - 1, 2, 3, 4, 6, 9, 12, 18, 24, 30, 36
 - 1, 2, 3, 4, 9, 24, 30

- 1, 4, 6, 9, 12, 16, 18, 24, 25, 30, 36
- 2, 3, 6, 12, 16, 18, 25
- 1, 2, 3, 4, 6, 9, 12, 16, 18, 24, 25, 30, 36
- 2, 3, 6, 12, 18, 24, 30

- 4 a $p \vee q$ b $q \vee r$
c $p \vee r$ d $r \wedge q$ or $q \wedge r$

- 5 a $p \vee \neg q$ b $\neg p \wedge \neg q$
6 a x ends in zero or x is not divisible by 5; 7

- b x ends in zero or x is not divisible by 5 but not both; 7

- c x ends in zero and x is not divisible by 5; FALSE

- d x ends in zero and x is divisible by 5; 10

- e x does not end in zero and x is divisible by 5; 15

- 7 a i $p \wedge q$ ii $p \vee q$
iii $p \vee q$ iv $\neg p \vee \neg q$

- v $\neg(p \vee q)$ vi $\neg(p \wedge q)$
vii $\neg p \wedge \neg q$

- b i i ii iii
iii v and vii
iv iv and vi

Exercise 9F

1 a Final columns only given:

p	q	$p \wedge q$	$p \vee q$	$\neg p$	$\neg q$	$\neg(p \vee q)$	$\neg(p \wedge q)$	$\neg p \wedge \neg q$
T	T	T	T	F	F	F	F	F
T	F	F	T	F	T	F	T	F
F	T	F	T	T	F	T	T	F
F	F	F	F	T	T	T	T	T

- b vi I am not studying both French and Chinese. (Equivalents in red in a.)

The columns in blue are the same; hence, the statements are equivalent.

2 a

p	$\neg p$	$\neg(\neg p)$
T	F	T
F	T	F

b

p	$p \wedge p$
T	T
F	F

c

p	q	$p \wedge q$	$p \vee (p \wedge q)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

d

p	q	$\neg p$	$\neg p \wedge q$	$p \vee (\neg p \wedge q)$	$p \vee q$
T	T	F	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	F	T	F	F	F

3 $(p \wedge \neg q) \vee (\neg p \wedge q)$

p	q	$p \wedge \neg q$	$\neg p \wedge q$	$(p \wedge \neg q) \vee (\neg p \wedge q)$	$p \vee q$
T	T	F	F	F	F
T	F	T	F	T	T
F	T	F	T	T	T
F	F	F	F	F	F

$(p \wedge \neg q) \vee (\neg p \wedge q) \Leftrightarrow p \vee q$

4 a Tautology since:

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

b Contradiction since:

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

- c Neither
d Tautology since:

p	q	$p \vee q$	$\neg p \wedge \neg q$	$(p \vee q) \vee (\neg p \wedge \neg q)$
T	T	T	F	T
T	F	T	F	T
F	T	T	F	T
F	F	F	T	T

- e Tautology
f Neither
g Neither
h Contradiction

Exercise 9G

1 $p \vee (q \wedge r)$: Neither

p	q	r	$(q \wedge r)$	$p \vee (q \wedge r)$
T	T	T	T	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

2 $(p \vee \neg q) \vee r$: Neither

p	q	r	$\neg q$	$(p \vee \neg q)$	$(p \vee \neg q) \vee r$
T	T	T	F	T	T
T	T	F	F	T	T
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	F	T
F	T	F	F	F	F
F	F	T	T	T	T
F	F	F	T	T	T

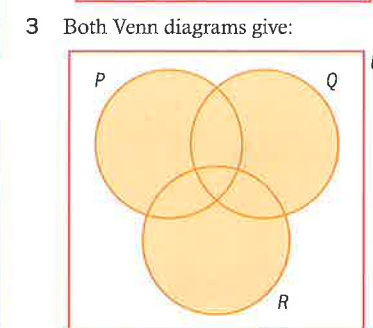
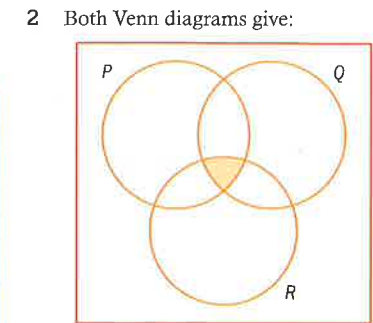
3 $(p \wedge q) \vee (p \wedge \neg r)$: Neither

p	q	r	$\neg r$	$p \wedge q$	$p \wedge \neg r$	$(p \wedge q) \vee (p \wedge \neg r)$
T	T	T	F	T	F	T
T	T	F	T	T	T	T
T	F	T	F	F	F	F
T	F	F	T	F	T	T
F	T	T	F	F	F	F
F	T	F	T	F	F	F
F	F	T	F	F	F	F
F	F	F	T	F	F	F

- 4 Neither
5 Contradiction
6 Neither
7 Neither
8 Neither; Question 1.

Exercise 9H

- 1 The two statements are equivalent. There is no need to use brackets.



- 4 The final column of $p \wedge (q \vee r)$ is T T T F F F F F. The final column of $(p \wedge q) \vee r$ is T T T F T F T F. There is a need to use brackets.
- 5 $P \cap (Q \cup R)$ and $(P \cap Q) \cup R$ are **not** equivalent.
- 6 The final column of $p \vee (q \wedge r)$ is T T T T T F F F. The final column of $(p \vee q) \wedge r$ is T F T F T F F F. There is a need to use brackets.
- 7 $P \cup (Q \cap R)$ and $(P \cup Q) \cap R$ are **not** equivalent.
- 8 The final column of $(\neg p \wedge q) \vee (\neg q \wedge r) \vee (\neg r \wedge p)$ is F T T T T T T T F. The final column of $(\neg p \vee q) \wedge (\neg q \vee r) \wedge (\neg r \vee p)$ is T F F F F F F T. They are not equivalent.

Exercise 9I

1

p	q	$p \wedge q$	$p \Rightarrow q$	$p \vee q$	$p \Rightarrow p \vee q$
T	T	T	T	T	T
T	F	F	F	T	T
F	T	F	T	T	T
F	F	F	T	F	T

$p \Rightarrow p \wedge q$: invalid argument;
 $p \Rightarrow p \vee q$: tautology

2

p	q	$p \wedge q$	$p \wedge q \Rightarrow p$	$p \vee q$	$p \vee q \Rightarrow p$
T	T	T	T	T	T
T	F	F	T	T	T
F	T	F	T	T	F
F	F	F	T	F	T

$p \wedge q \Rightarrow p$: tautology;
 $p \vee q \Rightarrow p$: invalid argument

3

p	q	$p \wedge q$	$p \vee q$	$p \Rightarrow p \vee q$	$p \Rightarrow p \wedge q$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	F	T	T	F
F	F	F	F	T	T

Invalid argument

4

p	q	$p \wedge q$	$p \vee q$	$p \wedge q \Rightarrow p \vee q$	$(p \wedge q) \Rightarrow p$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	F	T	T	T
F	F	F	F	T	T

Invalid argument

- 5 $(p \wedge q \Rightarrow p) \vee (p \Rightarrow p \wedge q)$. Final column TTTT. Tautology.

- 6 $\neg(p \wedge q) \Rightarrow \neg p \vee \neg q$. Final column TTTT. Tautology.

- 7 $\neg(p \vee q) \Rightarrow \neg p \vee \neg q$. Final column TTTT. Tautology.

- 8 $\neg p \vee \neg q \Rightarrow \neg(p \wedge q)$. Final column TTTT. Tautology.

- 9 $\neg(p \vee q) \Rightarrow \neg p \wedge \neg q$. Final column TTTT. Tautology.

Exercise 9J

1 $[(p \Rightarrow q) \wedge \neg p] \Rightarrow \neg q$; invalid

p	q	$\neg p$	$p \Rightarrow q$	$(p \Rightarrow q) \wedge \neg p$	$\neg q$	$[(p \Rightarrow q) \wedge \neg p] \Rightarrow \neg q$
T	T	F	T	F	F	T
T	F	F	F	F	T	T
F	T	T	T	T	F	F
F	F	T	T	T	T	T

2 $[(p \Rightarrow q) \wedge q] \Rightarrow p$; invalid

p	q	$p \Rightarrow q$	$(p \Rightarrow q) \wedge q$	$[(p \Rightarrow q) \wedge q] \Rightarrow p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

3 $[(p \Rightarrow q) \wedge \neg q] \Rightarrow p$; valid

p	q	$\neg q$	$p \Rightarrow q$	$(p \Rightarrow q) \wedge \neg q$	$[(p \Rightarrow q) \wedge \neg q] \Rightarrow p$
T	T	F	T	F	T
T	F	T	F	F	T
F	T	F	T	F	T
F	F	T	T	T	T

4 $[(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge \neg r] \Rightarrow \neg p$; valid

p	q	r	$\neg p$	$\neg r$	$p \Rightarrow q$	$q \Rightarrow r$	$[(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge \neg r]$	$[(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge \neg r] \Rightarrow \neg p$
T	T	T	F	F	T	T	F	T
T	T	F	F	T	T	F	F	T
T	F	T	F	F	F	T	F	T
T	F	F	F	T	F	T	F	T
F	T	T	T	F	T	T	F	T
F	T	F	T	T	T	F	F	T
F	F	T	T	F	T	T	F	T
F	F	F	T	T	T	T	T	T

5 $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (\neg p \Rightarrow \neg r)$; invalid

p	q	r	$\neg p$	$\neg r$	$\neg p \Rightarrow \neg r$	$p \Rightarrow q$	$q \Rightarrow r$	$[(p \Rightarrow q) \wedge (q \Rightarrow r)]$	$[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (\neg p \Rightarrow \neg r)$
T	T	T	F	F	T	T	T	T	T
T	T	F	F	T	T	T	F	F	T
T	F	T	F	F	T	F	T	F	T
T	F	F	F	T	T	F	T	F	T
F	T	T	T	F	F	T	T	T	F
F	T	F	T	T	T	T	F	F	T
F	F	T	T	F	F	T	T	T	F
F	F	F	T	T	T	T	T	T	T

6 $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (\neg p \Rightarrow \neg r)$; invalid

p	q	r	$\neg p$	$\neg r$	$\neg p \Rightarrow \neg r$	$p \Rightarrow q$	$q \Rightarrow r$	$[(p \Rightarrow q) \wedge (q \Rightarrow r)]$	$[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (\neg p \Rightarrow \neg r)$
T	T	T	F	F	T	T	T	T	T
T	T	F	F	T	T	T	F	F	T
T	F	T	F	F	T	F	T	F	T
T	F	F	F	T	T	F	T	F	T
F	T	T	T	F	F	T	T	T	F
F	T	F	T	T	T	T	F	F	T
F	F	T	T	F	F	T	T	T	F
F	F	F	T	T	T	T	T	T	T

7 $[(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge (r \Rightarrow s)] \Rightarrow (\neg s \Rightarrow \neg p)$; valid

p	q	r	s	$\neg s$	$\neg p$	$p \Rightarrow q$	$q \Rightarrow r$	$r \Rightarrow s$	Triple conjunction	$\neg s \Rightarrow \neg p$	Final statement
T	T	T	T	F	F	T	T	T	T	T	T
T	T	T	F	T	F	T	T	F	F	F	T
T	T	F	T	F	F	T	F	T	F	T	T
T	T	F	F	T	F	T	F	T	F	F	T
T	F	T	T	F	F	F	T	T	F	T	T
T	F	T	F	T	F	F	T	F	F	F	T
T	F	F	T	F	F	F	T	T	F	T	T
T	F	F	F	T	F	F	T	T	F	F	T
F	T	T	T	F	T	T	T	T	T	T	T
F	T	T	F	T	T	T	T	F	F	T	T
F	T	F	T	F	T	T	F	T	F	T	T
F	T	F	F	T	T	T	F	T	F	T	T
F	F	T	T	F	T	T	T	T	T	T	T
F	F	T	F	T	T	T	T	F	F	T	T
F	F	F	T	F	T	T	T	T	T	T	T
F	F	F	F	T	T	T	T	T	T	T	T

Exercise 9K

- $\neg(p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$ is a tautology.
- $\neg(p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$ is a tautology.
- $(p \wedge q) \Leftrightarrow p$ has final column TFTT, $(p \vee q) \Leftrightarrow p$ has final column TTFT. The statements are not equivalent.
- The statements $\neg(p \wedge \neg q)$ and $\neg p \vee q$ are equivalent.
- The statements $\neg(p \vee \neg q)$ and $\neg p \wedge q$ are equivalent.
- The statement $(p \vee \neg q) \Leftrightarrow (\neg p \wedge q)$ is a contradiction.
- The statement $\neg(p \vee q) \Leftrightarrow (p \wedge q)$ is neither a contradiction nor a tautology.
- The statement $(p \wedge \neg q) \Leftrightarrow (\neg p \vee q)$ is a contradiction.

Exercise 9L

p	q	Statement $p \Rightarrow q$	Converse $q \Rightarrow p$	Inverse $\neg p \Rightarrow \neg q$	Contrapositive $\neg q \Rightarrow \neg p$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

Exercise 9M

- a Valid b Valid c Valid
d Invalid; counterexample 18
e Valid f Valid
g Invalid; counterexample 12
h Invalid; counterexample 3 and 7
i Invalid; counterexample 2 and 5
j Valid k Valid

- l Valid m Valid
- n Valid
- o Invalid; counterexample rhombus
- p Invalid; counterexample $x = -5$
- q Valid
- r Invalid; counterexample $x = -10$
- s Valid

2 & 3

- a Converse: If ABCD is a quadrilateral, then ABCD is a square. Invalid; Counterexample rectangle.
- Inverse: If ABCD is not a square, then ABCD is not a quadrilateral. Invalid; Counterexample rectangle.

- Contrapositive: If ABCD is a not a quadrilateral, then ABCD is not a square. Valid.
- b Converse: If ABCD is a parallelogram, then ABCD is a rectangle. Invalid: Counterexample any parallelogram with internal angles not equal to 90° .
Inverse: If ABCD is not a rectangle, then ABCD is not a parallelogram. Invalid: Counterexample rhombus.
Contrapositive: If ABCD is not a parallelogram, then ABCD is a not rectangle. Valid.
- c Converse: If an integer is divisible by two then it is divisible by four. Invalid: Counterexample 10.
Inverse: If an integer is not divisible by four then it is not divisible by two. Invalid: Counterexample 10.
Contrapositive: If an integer is not divisible by two then it is not divisible by four. Valid.
- d Converse: If an integer is odd then it is divisible by three. Invalid: Counterexample 25.
Inverse: If an integer is not divisible by three then it is an even integer. Invalid: Counterexample 25.
Contrapositive: If an integer is an even integer then it is not divisible by three. Invalid: Counterexample 18.
- e Converse: If an integer is even then it is divisible by two. Valid.
Inverse: If an integer is not divisible by two then it is not an even integer. Valid.
Contrapositive: If an integer is not an even integer then it is not divisible by two. Valid.
- f Converse: If an integer is divisible by twelve then it is divisible by both four and by three. Valid.
Inverse: If an integer is not divisible by both four and by three then it is not divisible by twelve. Valid.
Contrapositive: If an integer is not divisible by twelve then it is not divisible by both four and by three. Valid.

- g Converse: If an integer is divisible by eight then it is divisible by both four and by two. Valid.
Inverse: If an integer is not divisible by both four and by two then it is not divisible by eight. Valid.
Contrapositive: If an integer is not divisible by eight then it is not divisible by both four and by two. Invalid: Counterexample 12.
- h Converse: If two integers are both even, then their sum is even. Valid.
Inverse: If the sum of two integers is not even, then the two integers are not both even. Valid.
Contrapositive: If two integers are not both even, then their sum is not even. Invalid: Counterexample 3 and 7.
- i Converse: If two integers are both even, then their product is even. Valid.
Inverse: If the product of two integers is not even, then the two integers are not both even. Valid.
Contrapositive: If two integers are not both even, then their product is not even. Invalid: Counterexample 2 and 5.
- j Converse: If one integer is odd and the other is even, then their sum is odd. Valid.
Inverse: If the sum of two integers is not odd, then either the two integers are both odd or the two integers are both even. Valid.
Contrapositive: If either the two integers are both odd or the two integers are both even, then their sum is not odd. Valid.
- k Converse: If two integers are both odd, then their product is odd. Valid.
Inverse: If the product of two integers is not odd, then the two integers are not both odd. Valid.
Contrapositive: If two integers are not both odd, then their product is not odd. Valid.
- l Converse: If $a^2 + b^2 = c^2$, then triangle ABC is right angled. Valid.

- Inverse: If triangle ABC is not right angled, then $a^2 + b^2 \neq c^2$. Valid.
Contrapositive: If $a^2 + b^2 \neq c^2$, then triangle ABC is not right angled. Valid.
- m Converse: If the square of an integer is odd, then the integer is odd. Valid.
Inverse: The square of an even integer is even. Valid.
Contrapositive: If the square of an integer is even, then the integer is even. Valid.
- n Converse: If triangle ABC has three equal sides, then triangle ABC has three equal angles. Valid.
Inverse: If triangle ABC does not have three equal angles, then triangle ABC does not have three equal sides. Valid.
Contrapositive: If triangle ABC does not have three equal sides, then triangle ABC does not have three equal angles. Valid.
- o Converse: If quadrilateral ABCD has four equal angles, then ABCD has four equal sides. Invalid: Counterexample rectangle.
Inverse: If quadrilateral ABCD does not have four equal sides, then ABCD does not have four equal angles. Invalid: Counterexample rectangle.
Contrapositive: If quadrilateral ABCD does not have four equal angles, then ABCD does not have four equal sides. Invalid: Counterexample rhombus.
- p Converse: If $x = 5$, then $x^2 = 25$. Valid.
Inverse: If $x^2 \neq 25$, then $x \neq 5$. Valid.
Contrapositive: If $x \neq 5$, then $x^2 \neq 25$. Invalid: Counterexample $x = -5$.
- q Converse: If $x = 3$, then $x^3 = 27$. Valid.
Inverse: If $x^3 \neq 27$, then $x \neq 3$. Valid.
Contrapositive: If $x \neq 3$, then $x^3 \neq 27$. Valid.
- r Converse: If $x > 5$, then $x^2 > 25$. Valid.
Inverse: If $x^2 \leq 25$, then $x \leq 5$. Valid.

- Contrapositive: If $x \leq 5$, then $x^2 \leq 25$. Invalid: Counterexample $x = -10$.
- s Converse: If $x < 3$, then $x^3 < 27$. Valid.
Inverse: If $x^3 \geq 27$, then $x \geq 3$. Valid.
Contrapositive: If $x \geq 3$, then $x^3 \geq 27$. Valid.

- ii Picasso did not paint picture A and van Gogh painted picture A.

b

p	q	$\neg p$	$\neg q$	$p \vee \neg q$	$\neg p \wedge q$
T	T	F	F	T	F
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	F

Review exercise

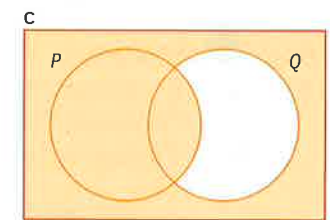
Paper 1 style questions

1 a

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$\neg(p \vee q) \Rightarrow \neg p \wedge \neg q$
T	T	T	F	F	F	F	T
T	F	T	F	F	T	F	T
F	T	T	F	T	F	F	T
F	F	F	T	T	T	T	T

- b She does not dance well and she does not sing beautifully.

- 2 a If the train leaves from gate z, then it leaves today and not from gate 8.
b $\neg r \Leftrightarrow (p \vee q)$



3 a

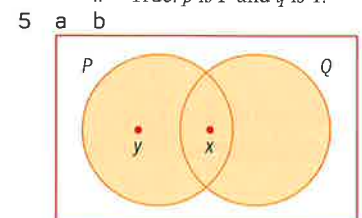
p	q	$p \Rightarrow q$	$\neg p$	$\neg q$	$\neg q \vee \neg p$	$\neg p \vee q$
T	T	T	F	F	T	T
T	F	F	F	T	T	F
F	T	T	T	F	F	T
F	F	T	T	T	T	T

- b $(p \Rightarrow q) \Leftrightarrow (\neg p \vee q)$

4 a

p	q	$\neg p$	$\neg p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

- b i False: p is T and q is F.
ii True: p is F and q is T.



- c i $\neg q \Rightarrow \neg p$ ii $\neg p \vee q$
iii $\neg q \Rightarrow p$ iv $p \wedge \neg q$
- d i Since it is the contrapositive

- 6 a i Picasso painted picture A or van Gogh did not paint picture A.

c

p	q	r	q/r	$\neg p$	$(q/r) - \neg p$
T	T	T	T	F	F
T	F	F	T	F	F
T	F	T	T	F	F
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	F	T	F

p	q	r	$\neg q$	$p/\neg q$	$r \Rightarrow (p/\neg q)$
T	T	T	F	T	T
T	T	F	F	T	T
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	F	F
F	T	F	F	F	T
F	F	T	T	T	T
F	F	F	T	T	T

d

p	q	r	x
F	T	T	3
F	T	F	12
F	F	T	2

e

p	q	r	$(q/\neg r) - \neg p$	$r \Rightarrow (p/\neg q)$
T	T	T	F	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	F
F	T	F	T	T
F	F	T	T	T
F	F	F	F	T

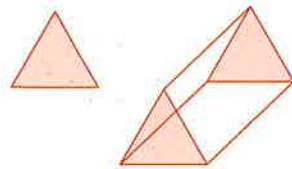
x is not a multiple of 5 and is either a multiple of 3 or a factor of 90, but not both.

Chapter 10

Skills check

- 1 a $x = 5.85\text{m}$ b $y = 51.3^\circ$
2 a $x = 51.2^\circ$ b 2740m^2

Investigation - drawing a prism



Relationships between volumes:

Square based pyramid = $\frac{1}{3}abh$

Cuboid = abh

The volume of a pyramid is $\frac{1}{3}$ the volume of a cuboid with the same base and height.

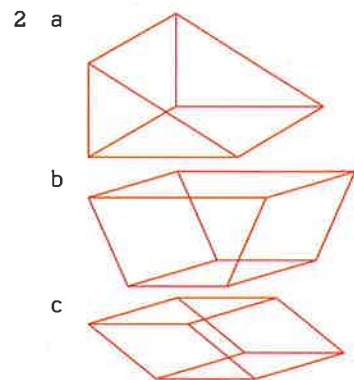
Cone = $\frac{1}{3}\pi r^2 h$

Cylinder = $\pi r^2 h$

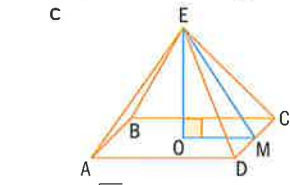
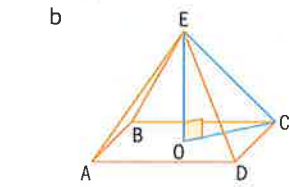
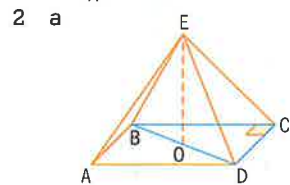
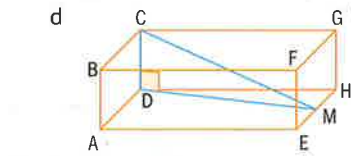
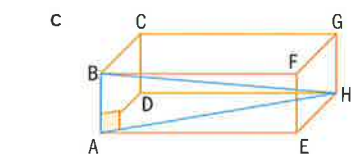
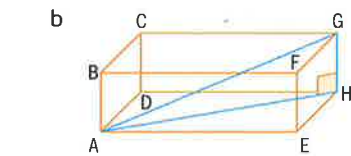
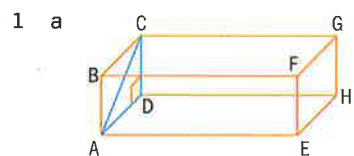
The volume of a cone is $\frac{1}{3}$ the volume of a cylinder with the same base and height.

Exercise 10A

- triangular prism
 - 5 faces, 9 edges, 6 vertices
 - 5 plane faces
 - rectangular-based pyramid
 - 5 faces, 8 edges, 5 vertices
 - 5 plane faces
 - hemisphere
 - 2 faces, 1 edge, no vertices
 - 1 plane face, 1 curved face



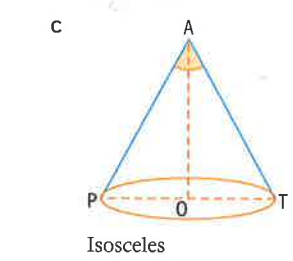
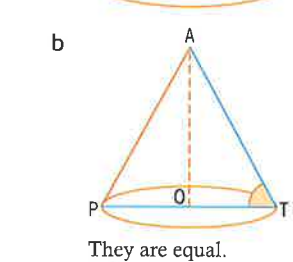
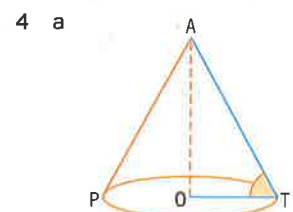
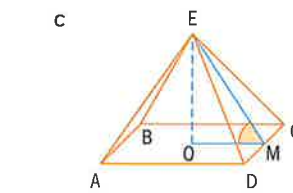
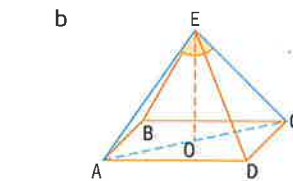
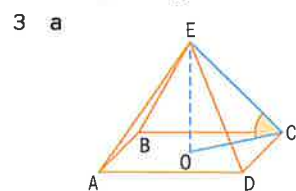
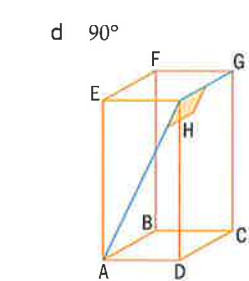
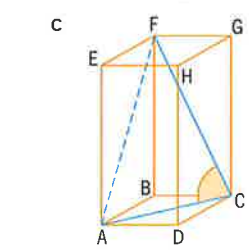
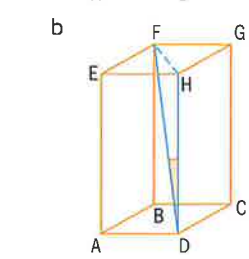
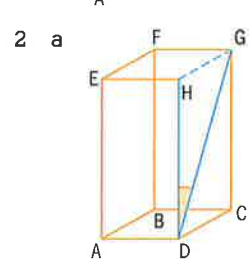
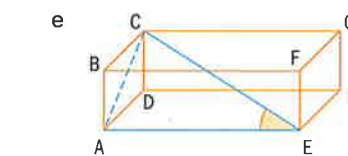
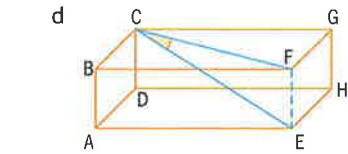
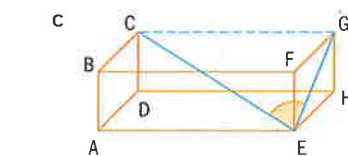
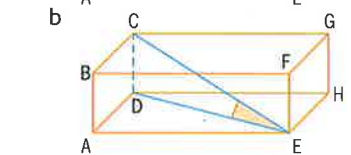
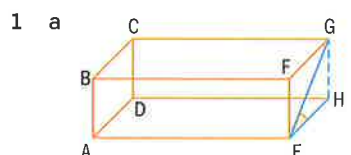
Exercise 10B



- $\sqrt{52}$ cm or 7.21 cm (3sf)
 - $\sqrt{97}$ cm or 9.85 cm (3sf)
 - $\sqrt{117}$ cm or 10.8 cm (3sf)
 - $\sqrt{133}$ cm or 11.5 cm (3sf)

- a 0.849 m b 1.56 m
- c 1.53 m
- $\sqrt{65}$ or 8.06 cm (3sf)
- a 1.14 m b 1.88 m
- a 326 m b 180 m
- c 214 m

Exercise 10C



Exercise 10D

- $\sqrt{116}$ cm or 10.8 cm (3sf)
 - 15.6°
 - 5 cm
 - 63.4°
- $\sqrt{8}$ m or 2.83 m (3sf)
 - 35.3° c 19.5°
- $\sqrt{52}$ cm or 7.21 cm (3sf)
 - 33.7°
 - 56.3°
 - $\sqrt{117}$ cm or 10.8 cm (3sf)
 - $\sqrt{97}$ cm or 9.85 cm (3sf)
 - 40.5°
- a 5 cm b 7.43 cm
 - c 39.3° d 70.3° e 74.1°
- $\sqrt{34}$ cm or 5.83 cm (3sf)
 - 59.0° c 61.9°
 - a 1.26 m b 64.5°

Exercise 10E

- 24 cm² b 23.5 m²
 - c 73.9 cm²
- 3.90 cm² b 5.20 cm
 - c 52.6 cm²
- 6.5 cm b 16.25 cm²
 - c 90 cm²
- 1000 cm
- 175 m² b 1.75×10^2 m
- 43.4 m² b 53 litres
 - c US\$243.80 (2dp)

Exercise 10F

- 30π cm² or 94.2 cm² (3sf)
 - 4π cm² or 12.6 cm² (3sf)
 - 6.75π cm² or 21.2 cm² (3sf)
 - 4.125π m² or 13.0 m² (3sf)
- 38π cm² or 119 cm² (3sf)
 - 22.08π cm² or 69.4 cm² (3sf)
- 8.92 cm (3sf)
- 120π cm³ or 377 cm³ (3sf)
 - 12 cm (2sf)

Exercise 10G

- 23.4 dm³ b 90 m³
 - c 8000 cm³ d 160 cm³
 - e 12 m³ f 210 cm³
- 5.03 m b 15.1 m²
 - c 151 m³
- 60° b 10.8 cm²
 - c 65.0 cm² d 877 cm³
- x^3 b $3x^3$
 - c $\frac{3x^3}{8}$ or 0.375 x^3 d $10x^2$
- $25x^2$ b $11025 = 25x^2$
 - c 21
- 21 cm b 2205 cm²

Exercise 10H

- $75\,140\pi$ mm³ or 236 000 mm³ (3sf)
 - $\frac{1}{6}\pi = 0.524$ cm³ (3sf)
 - 32.7 cm³ d 1130 cm³
 - 32.7 cm³ f 8 dm³
- 13.6 m³ b 13 600 dm³
 - c 13 600 litres
- $V = \frac{x^2 h}{3}$ b $V = 2\pi x^3$
 - $V = 9\pi x^3$ d $V = 4.5\pi x^3$
- 36 cm² b 6 cm²
 - c 60° d 3.72 cm

- 3.63 cm
 - 36 mm
- 6750π cm³ or 21 200 cm³ (3sf)
 - No. The second container has a volume (20 400 cm³) smaller than the first
- 1.2 cm
 - 1.25 cm
 - 28.8 cm²
 - 4.89 cm³
 - number of pencils in one layer = $5.6 \div 0.7 = 8$
number of layers = $1.4 \div 0.7 = 2$
total number of pencils = $8 \times 2 = 16$
 - 27.6 cm³
 - 26%

Review exercise

Paper 1 style questions

- 5896 cm²
 - 28.56 dm³
- $\sqrt{116}$ cm or 10.8 cm (3sf)
 - 24.9°
- $\sqrt{41}$ cm or 6.40 cm (3sf)
 - 8.62 cm
 - 43.6°
- $\sqrt{90}$ cm or 9.49 cm (3sf)
 - 28.5 cm²
 - 150 cm²
- 8 cm
 - 11.3 cm
 - Yes, as the greatest distance between two points in this cube is 13.9 cm (3sf) which is bigger than 13.5 cm

Paper 2 style questions

- 27.0 m b 93.7 m
 - c 61.3° d US\$677 502
- 9 cm c 9.49 cm
 - d 71.6° e 1.53 kg
- 58.3 cm³ b 508 g

- c 7.842cm d 63.2°
 e 37.2° f 99.3cm²
 4 a 8.58cm
 b i 9.46cm ii 45.8°
 c 215cm² d 183cm³

Chapter 13

Exercise 1A

- 1 a 11 b 10 c 8
 d 4 e 5 f 3
 g 20 h 3
 2 a 5 b 1.5
 c 1.25 d 24
 3 a 12 b 540
 c 16 d 5
 4 a 5 b 8
 c 8 d 2
 5 a 2 b 4 c 34

Exercise 1B

- 1 a 1, 2, 3, 6, 9, 18
 b 1, 3, 9, 27
 c 1, 2, 3, 5, 6, 10, 15, 30
 d 1, 2, 4, 7, 14, 28
 e 1, 2, 3, 6, 13, 26, 39, 78
 2 a 2² × 3² b 2² × 3 × 5
 c 2 × 3³ d 2⁵ e 2⁴ × 7
 3 a 40 b 240
 4 a 8 b 18

Exercise 1C

- 1 a $\frac{11}{12}$ b $1\frac{1}{15}$ c 1
 d $2\frac{49}{81}$
 2 a $\frac{4}{9}$ b $\frac{7}{20}$
 c $\frac{2}{3}$ d $\frac{5}{8}$
 3 a $\frac{18}{5}$ b $\frac{22}{7}$
 c $\frac{93}{4}$ d $\frac{167}{72}$
 4 a $4\frac{4}{7}$ b $33\frac{1}{3}$
 c $4\frac{1}{4}$ d $14\frac{8}{11}$
 5 a 0.32 b 0.714
 c 3.8 d 2.65

Exercise 1D

- 1 a 52% b 70%
 2 a 2.24 CHF b 0.54 GBP
 c 187.57 EUR d 10400 JPY

Exercise 1E

- 1 576 GBP
 2 14875 JPY
 3 7%
 4 26.5%
 5 26500000
 6 32 USD
 7 0.60 GBP
 8 No. 1% decrease
 9 5 : 4
 10 95.1 : 100
 11 21 : 160
 12 11.2m
 13 200000 : 1; 0.4 cm
 14 45 USD, 27 USD
 15 75, 45 and 30

Exercise 1F

- 1 7500 USD, 10500 USD, 6000 USD
 2 18 min, 27 min, 30 min.

Exercise 1G

- 1 a $3x^2 - 6x$ b $x^3 - xy + \frac{x^2}{y}$
 c $b^2 + 3ab - 2ac$
 2 a $3pq(1 - 2pq^2r)$
 b $3c(4ac + 5b - c)$
 c $abc(2a + 3b - 5c)$

Exercise 1H

- 1 $t = \frac{u-v}{g}$ 2 $c = \sqrt{a^2 - b^2}$
 3 $r = \frac{C}{2\pi}$ 4 $b = \frac{a \sin B}{\sin A}$
 5 $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

Exercise 1I

- 1 2.49 2 3.73 3 40.1

Exercise 1J

- 1 4 2 4 3 -3
 4 3 5 5 6 9
 7 2 8 -2 9 3

- 10 1.5 11 1 12 2


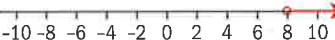
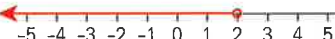
Exercise 2A

- 1 a $x = 1, y = 1$ b $x = 1, y = -2$
 c $x = -3, y = 4$
 2 a $x = 6, y = -1$ b $x = 2, y = -1$
 c $x = -2, y = 2$
 d $x = 2, y = 1$
 e $x = 3, y = -1$

Exercise 2B

- 1 a 17 b 144 c 64
 2 a 1 b $\frac{1}{9}$ c $\frac{1}{16}$
 3 a 525.21875 b 4.08
 c 1.667

Exercise 2C

- 1 a $x \leq 3$

 b $x > 8$

 c $x < 2$

 2 a $x \leq 5$ b $x > -1\frac{1}{2}$
 c $x \geq -1$

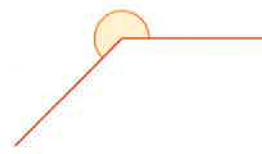

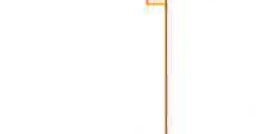
Exercise 2D


- 1 a 3.25 b 6.18 c 0
 2 2, 3
 3 a 2 b 2 c 2

Exercise 2E

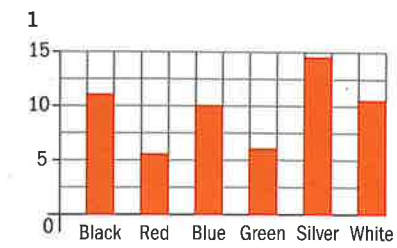
- 1 27.6cm 2 2.24m
 3 5.03cm

Exercise 2F

- 1 a 
 b 
 c 

- d 
 2 a reflex b obtuse c acute
 3 a obtuse b acute
 c reflex d acute
 e reflex f reflex

Exercise 3A

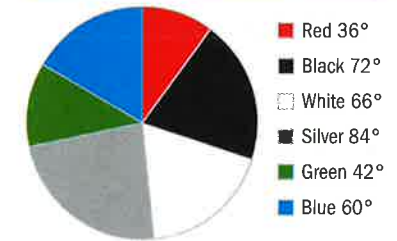


- No. of visits
 1 ****
 2 *****
 3 ****
 4 ***
 8 *
 12 *
 Key: * = 1 student

Exercise 3B

Diagonals	Irregular	Rectangle	Parallelogram	Rhombus	Square	Trapezium	Kite
Perpendicular	X	X	X	✓	✓	X	✓
Equal	X	✓	X	X	✓	X	X
Bisect	X	✓	✓	✓	✓	X	(one)
Bisect angles	X	X	X	✓	✓	X	(two)

- 2 a Kite, isosceles triangle, parallelogram, right angled triangles (2), scalene triangle, rhombus, arrowhead.
 b Square, isosceles triangle, right angled triangle, trapeziums (2), rhombus.



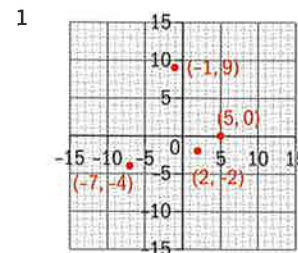
Exercise 3C

- 1 a 10.7cm b 16.4cm
 c 20.4cm d 62.8cm
 e 14.6cm f 17.5cm

Exercise 3D

- 1 63.6cm² 2 23.0cm²
 3 37.7cm² 4 10.3cm²
 5 6.48m² 6 42.3cm²

Exercise 3E



- 2 A (4, 9), B (-4, 2), C (-8, -6) and D (8, -8)

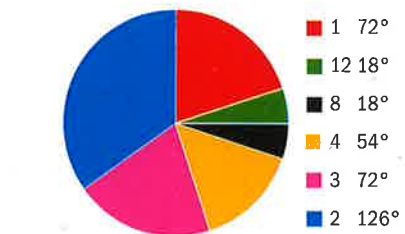
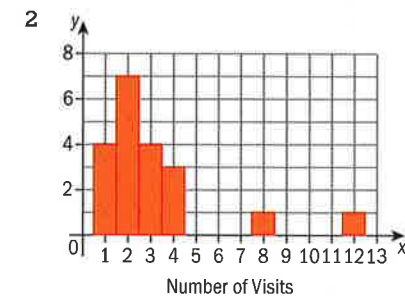
Exercise 3F

- 1 (5, 5) 2 (-1, -1)
 3 $(\frac{1}{2}, 2\frac{1}{2})$

Exercise 3G

- 1 5 2 9.43 3 14.8

- Color
 Black *****
 Red ***
 Blue *****
 Green ****
 Silver *****
 White *****
 Key: * = 2 cars



Mark scheme

Practice paper 1

1 a $\frac{\sqrt{7^2 - (6.4)(-5)}}{3125}$ M1
 = 0.00288 A1
 b i 0.003 A1
 ii 0.00029 A1
 iii 2.88×10^{-3} A1 A1

2 a 24 A1
 b $\frac{66}{24}$ M1
 = 2.75 A1

c Standard deviation = $\sqrt{\frac{915}{30}} = 1.13$ A1
 d Median is the middle value A1
 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5,
 Median = 3

3 a $\frac{0-4}{6-0}$ M1
 = $-\frac{2}{3}$ A1

b $y = -\frac{2}{3}x + 4$ A1

c $m(L_2) = \frac{3}{2}$ M1
 $(y-2) = \frac{3}{2}(x-3)$ A1
 $c = -2.5$ A1

4 a

p	q	$\neg p$	$\neg p \Rightarrow q$	Inverse
T	T	F	T	F
T	F	F	T	T
F	T	T	T	T
F	F	T	F	T

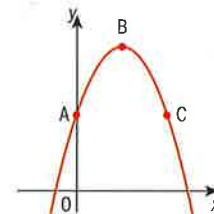
b $p \Rightarrow \neg q$ A1 A1 A1 ft
 A1 A1
 Award A1 for correct negations, A1 for correct order
 c Above A1
 d The final two columns are not the same. A1

5 a $u_1 r = 162, u_1 r^4 = -6$ A1 A1
 $\frac{u_1 r^4}{u_1 r} = \frac{-6}{162}$ M1
 $r = -\frac{1}{3}$ A1

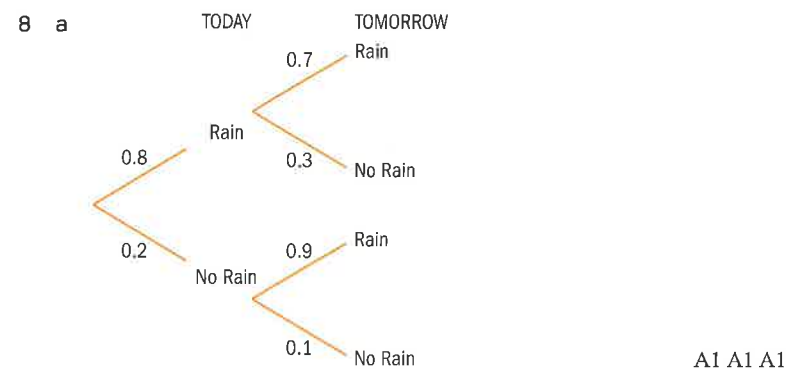
b $u_1 \left(\frac{-1}{3}\right) = 162$ M1
 $u_1 = -486$ A1 ft

6 a $BD = \sqrt{3^2 + 4^2}$ M1
 = 5 m A1
 b $BE = \sqrt{5^2 + 2.5^2}$ M1
 = 5.59 m A1 ft
 c $\tan(\theta) = \frac{2.5}{5}$ M1
 Award M1 for correct trigonometric ratio
 $\theta = 26.6^\circ$ A1 ft

7 a When $x = 0, f(x) = 5, (0, 5)$ A1
 b $f'(x) = 6 - 4x, f'(x) = 0$ when $6 - 4x = 0$, so $x = \frac{3}{2}$ M1
 When $x = \frac{3}{2}, f(x) = 5 + 9 - \frac{9}{2} = 9.5. B = (1.5, 9.5)$ A1
 c A1



d $5 = 5 + 6x - 2x^2$ A1 A1
 $0 = x(6 - 2x)$
 $x = 0$ or $x = 3$, so $C = (3, 5)$



b $0.8 \times 0.3 + 0.2 \times 0.1$ A1 M1
 Award A1 for two correct products, M1 for adding their products
 = 0.26 A1

9 a $f(0) = 10 - (8) a^{-0}$ M1
 $y = 2$ A1
 b $y = 10$ A1 A1
 Award A1 for $y = a$ constant, A1 the constant being 10
 c $10 - (8) a^{-1} = 8$ M1
 $a = 4$ A1

- 10 a 58 kg A1
 b $66 - 52$ M1
 Award M1 for correct quartiles seen
 $= 14$ A1
 c $\frac{10}{40} \times \frac{9}{39}$ A1M1
 Award A1 for two correct fractions, M1 for multiplying their fractions
 $= \frac{3}{52}$ A1

- 11 a $FV = 4000 \left(1 + \frac{3}{1200}\right)^{5 \times 12}$ M1A1
 Award M1 for substituted compound interest formula, A1 for correct substitutions

OR

- $N = 5$
 $I\% = 3$
 $PV = -4000$ M1 A1
 $P/Y = 1$
 $C/Y = 12$ Award (A1) for $C/Y = 12$ seen, M1 for other correct entries.

OR

- $N = 60$
 $I\% = 3$
 $PV = -4000$ M1 A1
 $P/Y = 12$
 $C/Y = 12$ Award (A1) for $C/Y = 12$ seen, M1 for other correct entries.
 $= 4646.47$ A1 C3

- b $FV = 4000 \left(1 + \frac{3}{400}\right)^{5 \times 4}$ M1
 Award M1 for correctly substituted compound interest formula,

OR

- $N = 5$
 $I\% = 3$
 $PV = -400$ M1
 $P/Y = 1$
 $C/Y = 4$

M1 for all correct entries seen.

OR

- $N = 20$
 $I\% = 3$
 $PV = -4000$ M1
 $P/Y = 4$
 $C/Y = 4$

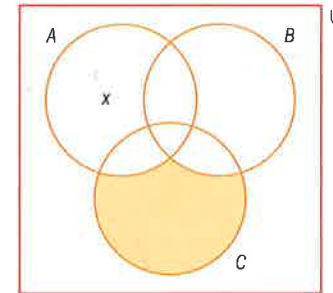
M1 for all correct entries seen.

- $FV = 4644.74$ A1
 Difference = €1.73 A1 C3

Illustrating use of GDC notation acceptable in this case only. However on P2 an answer given with no working would receive G2.

- 12 a -15 A1
 b $S_{50} = \frac{50}{2}(2(437) + 49(-15))$ $S_{50} = 3475$ M1
 c $437 - 15(k - 1) < 0$ M1
 Award M1 for correct substitution in correct formula
 $k > 30.13\dots$ A1
 $k = 31$ A1

- 13 a $(A \cap C) \cup B$ A1 A1
 Award A1 for $A \cap C$ seen



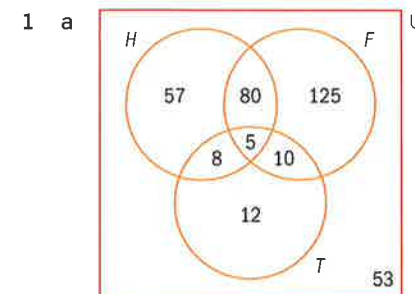
- b x in correct position on the Venn diagram A2
 c $(A \cup B)' \cap C$ A2

Award A1 for all of C shaded

- 14 a $f'(x) = 2x - k$ A1 A1
 $y = f(x)$ has a minimum point with coordinates $(3, p)$.
 b $2x - k = 0$ M1
 $k = 6$ A1
 c $f(3) = 3^2 - 6 \times 3$ M1
 $p = -9$ A1
- 15 a If the four sides of a quadrilateral are not equal then the quadrilateral is not a rhombus. A1 A1
 b If the four sides of a quadrilateral are equal then the quadrilateral is a rhombus. A1 A1
 c The converse of **this** statement p **is** always true. A square is also a rhombus. A1 A1

Mark scheme

Practice paper 2



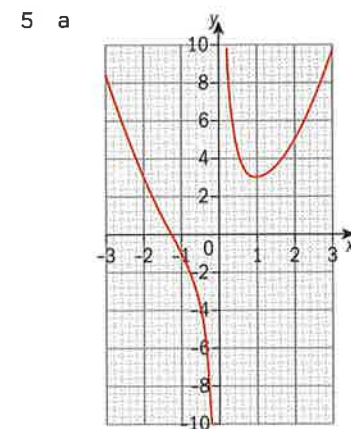
(A1) (A1) (A1) (A1) [4 marks]

- b 12 (A1) [1 mark]
- c $350 - (57 + 80 + 125 + 8 + 5 + 10 + 12) = 53$ (M1) (A1) (G2) [2 marks]
- d $\frac{200}{350} \left(= \frac{4}{7}, 0.571, 57.1\% \right)$ (A1) (A1) (G2) [2 marks]
- e $\frac{13}{350} (= 0.371, 37.1\%)$ (A1) (A1) (G2) [2 marks]
- f $\frac{15}{350} \times \frac{14}{349} = \frac{3}{1745} (0.00172, 0.172\%)$ (A1) (M1) (A1) (G2) [3 marks]
- Total [14 marks]
- 2 a i $r = 0.982$ (G2) (A1) (A1) [4 marks]
- ii (very) strong, positive (A1) (A1) (G2) [2 marks]
- b $y = 1.60x + 67.3$ (M1) (A1) (G2) [2 marks]
- c $y = 1.60(6) + 67.3 = 77$ (Accept 76.9 or 76) (M1) (A1) (G2) [2 marks]
- d H_0 : the time of the lesson and score (in the examination) are independent. (A1) [1 mark]
- e 2 (A1) [1 mark]
- f $\frac{40 \times 71}{146} = 19.4... = 19$ (M1) (A1) (AG) [2 marks]
- g $\chi^2_{calc} = 3.42$ (G2) [2 marks]
- h $\chi^2_{calc} < \chi^2_{crit} (5.991)$ (R1) [1 mark]
- or $0.18122 > 0.05$
- Total [15 marks]
- 3 a i $x(2x - t)$ (A1)(A1) (A1) (A1) (A1) [6 marks]
- ii $x = 0; x = 4$ (M1) (A1) (G2) [6 marks]
- iii $2 \times 4 - t = 0$ (A1) [1 mark]
- $t = 8$ (A1) [1 mark]
- b $a = 2$ (A1) [1 mark]
- c i -6 (G1) (G1) [2 marks]
- ii 10 (A1) (A1) [2 marks]
- d $5m + c = 10$ (A1) (A1) [2 marks]
- $m + c = -6$ (A1) [2 marks]
- e Subtracting the terms in the second equation from the terms in the first equation gives $4m = 16$ $m = 4$ (A1) (A1) [2 marks]
- $c = -10$ (A1) [2 marks]
- f $g(x) = 0$ (may be implied) (M1) (A1) (G2) [2 marks]
- $4x - 10 = 0$ (A1) (A1) [2 marks]
- $x = 2.5$ (A1) (A1) [2 marks]
- g $1 < x < 5$ (A1) (A1) [2 marks]
- Total [17 marks]
- 4 a area PVR = $\frac{1}{2} \times 45 \times 60 \times \sin 75 = 1303.99... = 1304$ (M1) (A1) (A1) (G3) [4 marks]
- b $x^2 = 45^2 + 60^2 - 2 \times 45 \times 60 \times \cos 75$ (M1) (A1) (A1) (G2) [3 marks]
- $x = 65.0$ km

- c $\frac{\sin 75}{65.01...} = \frac{\sin \alpha}{60}$ (M1) (A1) (A1) (G2) [3 marks]
- $\alpha = 63.0$ (allow 63.1 if 65 is used)
- d $MR = \frac{60 \sin 75}{2} = 28.97... = 29$ km (M1) (A1) (M1) (A1) (AG) [4 marks]
- e volume = $150^2 \times 2.85 = 64125 \text{ m}^3 (= 64100 \text{ m}^3)$ (M1) (A1) (G2) [2 marks]
- f $64125 \times 1.25 = 80156.25$ (M1) (A1)(G1) [2 marks]
- g $\frac{64125 \times 1000}{3} = 21375000 (= 21400000)$ (M1) (A1) (G2) [2 marks]
- h $2.1375 \times 10^7 (2.14 \times 10^7)$ (A1)(A1) [2 marks]
- Total [22 marks]

The answer includes the units.

Answer should be consistent with part g.



- b -1.26 (A1) (A1) (A1) [3 marks]
- c $f'(x) = 2x - \frac{2}{x^2}$ (G1) [1 mark]
- d $f'(x) = 0$ (M1) $2x - \frac{2}{x^2} = 0$ (M1) $2x = \frac{2}{x^2}$ (A1) $x^3 = 1$ (AG) [3 marks]
- $x = 1$ (G1) [1 mark]
- e 3 (A2) [2 marks]
- f the curve is increasing (or equivalent) (M1) (A1) (G2) (A1) (A1) (A1) (G3) [5 marks]
- g i $2(-2) - \frac{2}{(-2)^2} = -4.5$ (A1) (G2) (A1) (A1) (A1) (G3) [5 marks]
- ii $y = -4.5x - 6$ (A1) (A1) (A1) (G3) [5 marks]
- $4.5x + y + 6 = 0$ (or any multiple)
- h use of their (0, -6) (M1) $\sqrt{(1-0)^2 + (3+6)^2} = 9.06$ (or $\sqrt{82}$) (M1) (A1) (G3) [3 marks]
- Total [22 marks]