Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 1

Specimen paper

Candidate session number

1 hour 30 minutes

Instructions to candidates

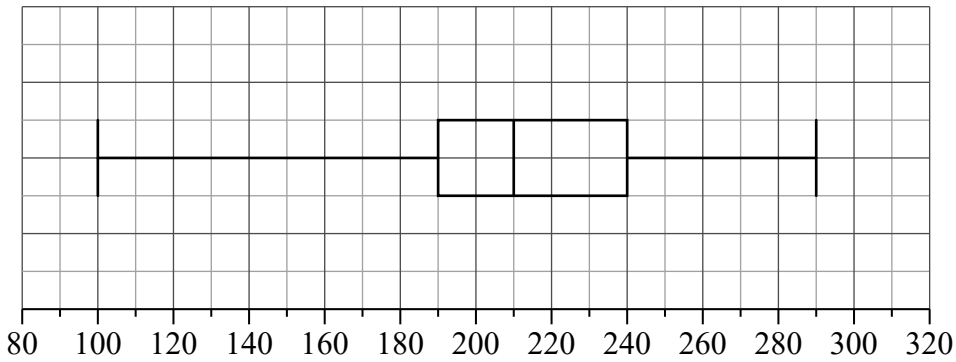
- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 6]

Palvinder breeds Labrador puppies at his farm. Over many years he recorded the weight (g) of the puppies.

The data is illustrated in the following box and whisker diagram.



Weight of Labrador puppies (g)

(a) Write down the median weight of the puppies. [1]

(b) Write down the upper quartile. [1]

(c) Find the interquartile range. [2]

The weights of these Labrador puppies are normally distributed.

(d) Find the weight of the heaviest possible puppy that is not an outlier. [2]

(This question continues on the following page)

(Question 1 continued)

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

2. [Maximum mark: 6]

The Osaka Tigers basketball team play in a multilevel stadium.

The most expensive tickets are in the first row. The ticket price, in Yen (¥), for each row forms an arithmetic sequence. Prices for the first three rows are shown in the following table.

Ticket pricing per game	
1st row	6800 Yen
2nd row	6550 Yen
3rd row	6300 Yen

(a) Write down the value of the common difference, d [1]

(b) Calculate the price of a ticket in the 16th row. [2]

(c) Find the total cost of buying 2 tickets in each of the first 16 rows. [3]

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

3. [Maximum mark: 6]

At the end of a school day, the Headmaster conducted a survey asking students in how many classes they had used the internet.

The data is shown in the following table.

Number of classes in which the students used the internet	0	1	2	3	4	5	6
Number of students	20	24	30	k	10	3	1

(a) State whether the data is discrete or continuous. [1]

The mean number of classes in which a student used the internet is 2.

(b) Find the value of k. [4]

It was not possible to ask every person in the school, so the Headmaster arranged the student names in alphabetical order and then asked every 10th person on the list.

(c) Identify the sampling technique used in the survey. [1]

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

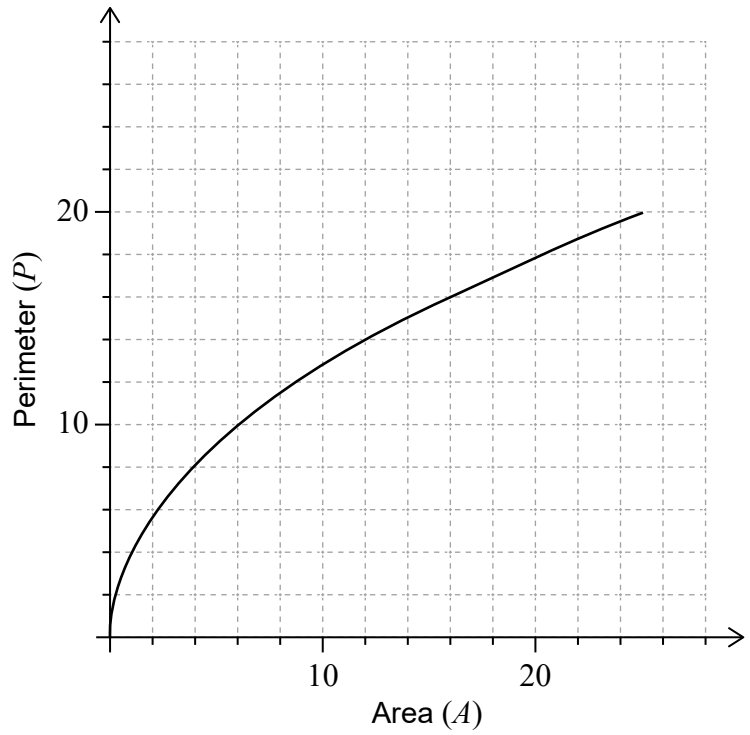
. . . . . .

. . . . . .

. . . . . .

4. [Maximum mark: 6]

The perimeter of a given square P can be represented by the function $P(A) = 4\sqrt{A}$, $A \geq 0$, where A is the area of the square. The graph of the function P is shown for $0 \leq A \leq 25$.



(a) Write down the value of $P(25)$. [1]

The range of $P(A)$ is $0 \leq P(A) \leq n$.

(b) Hence write down the value of n. [1]

(c) On the axes above, draw the graph of the inverse function, P^{-1}. [3]

(d) In the context of the question, explain the meaning of $P^{-1}(8) = 4$. [1]

(This question continues on the following page)

(Question 4 continued)

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

5. [Maximum mark: 6]

Professor Rincewind investigated the growth of a population of quantum weather butterflies (Papilio tempestae) in the vicinity of a lake in Llamedos.

He found that their population P could be modelled by the function $P = 1000 - 1.02^{(340 - 1.8t)}$, $t \geq 0$, where t is the number of months since the start of the study.

(a) Find the population of quantum weather butterflies: (round to the nearest butterfly)

(i) at the start of the study

(ii) after 15 months [3]

(b) Calculate the time taken for the population to increase above 800 . [2]

(c) According to this model, find the highest possible population of quantum weather butterflies in this location. [1]

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

6. [Maximum mark: 5]

As part of a study into healthy lifestyles, Jing visited Surrey Hills University. Jing recorded a person's position in the university and how frequently they ate a salad. Results are shown in the table.

	Salad meals per week			
	0	**1 – 2**	**3 – 4**	**> 4**
Students	45	26	18	6
Professors	15	8	5	12
Staff and Administration	16	13	10	6

Jing conducted a χ^2 test for independence at a 5% level of significance.

(a) State the null hypothesis. [1]

(b) Calculate the p-value for this test. [2]

(c) State, giving a reason, whether the null hypothesis should be accepted. [2]

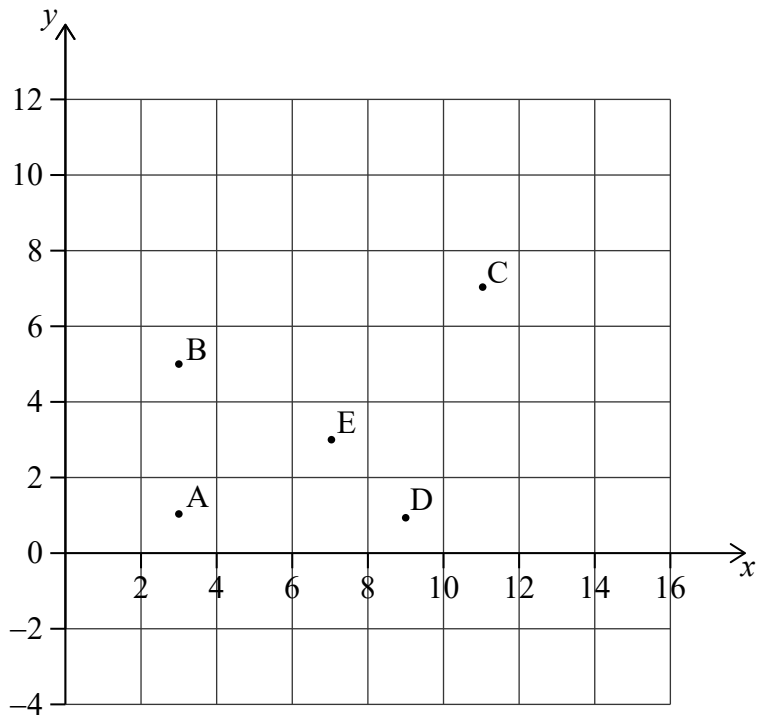
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

7. [Maximum mark: 6]

Points $A(3, 1)$, $B(3, 5)$, $C(11, 7)$, $D(9, 1)$ and $E(7, 3)$ represent snow shelters in the Blackburn National Forest. These snow shelters are illustrated in the following coordinate axes.

Horizontal scale: 1 unit represents $1\,\text{km}$.

Vertical scale: 1 unit represents $1\,\text{km}$.

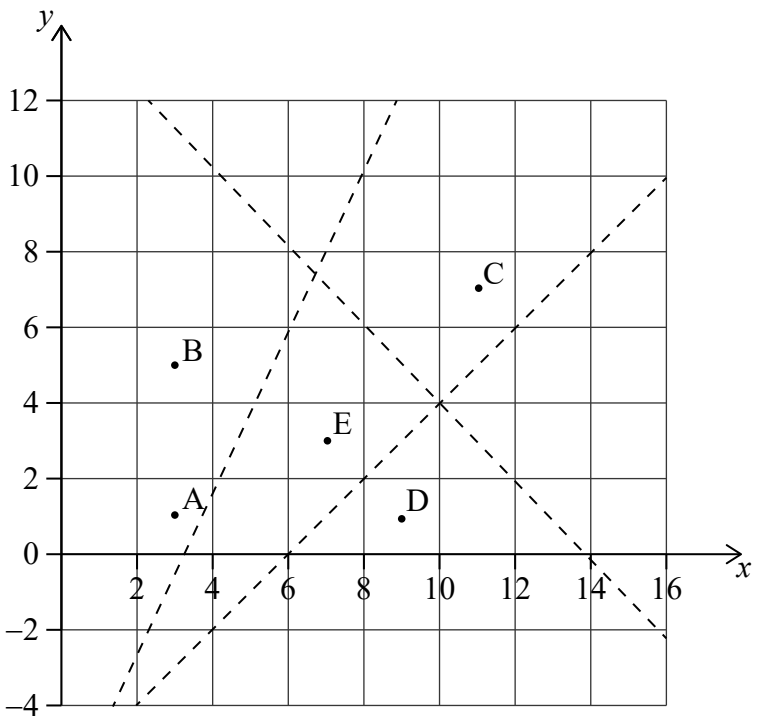


(a) Calculate the gradient of the line segment AE. [2]

(This question continues on the following page)

(Question 7 continued)

The Park Ranger draws three straight lines to form an incomplete Voronoi diagram.



(b) Find the equation of the line which would complete the Voronoi cell containing site E.
Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$. [3]

(c) In the context of the question, explain the significance of the Voronoi cell containing site E. [1]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

8. [Maximum mark: 4]

The intensity level of sound, L measured in decibels (dB), is a function of the sound intensity, S watts per square metre ($\mathrm{W\,m^{-2}}$). The intensity level is given by the following formula.

$$L = 10\log_{10}(S \times 10^{12}),\ S \geq 0$$

(a) An orchestra has a sound intensity of $6.4 \times 10^{-3}\,\mathrm{W\,m^{-2}}$. Calculate the intensity level, L of the orchestra. [2]

(b) A rock concert has an intensity level of $112\,\mathrm{dB}$. Find the sound intensity, S. [2]

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

9. [Maximum mark: 6]

Ms Calhoun measures the heights of students in her mathematics class. She is interested to see if the mean height of male students, μ_1, is the same as the mean height of female students, μ_2. The information is recorded in the table.

Male height (cm)	150	148	143	152	151	149	147	
Female height (cm)	148	152	154	147	146	153	152	150

At the 10% level of significance, a t-test was used to compare the means of the two groups. The data is assumed to be normally distributed and the standard deviations are equal between the two groups.

(a) (i) State the null hypothesis.

(ii) State the alternative hypothesis. [2]

(b) Calculate the p-value for this test. [2]

(c) State, giving a reason, whether Ms Calhoun should accept the null hypothesis. [2]

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

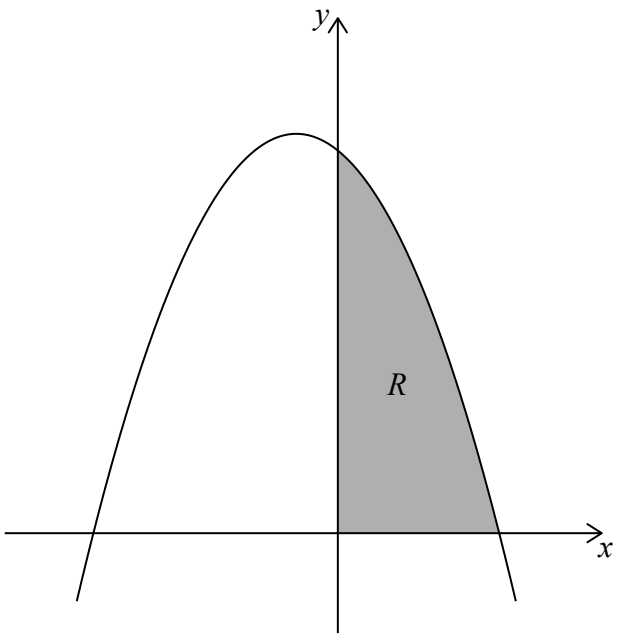
. . . . . .

. . . . . .

. . . . . .

10. [Maximum mark: 5]

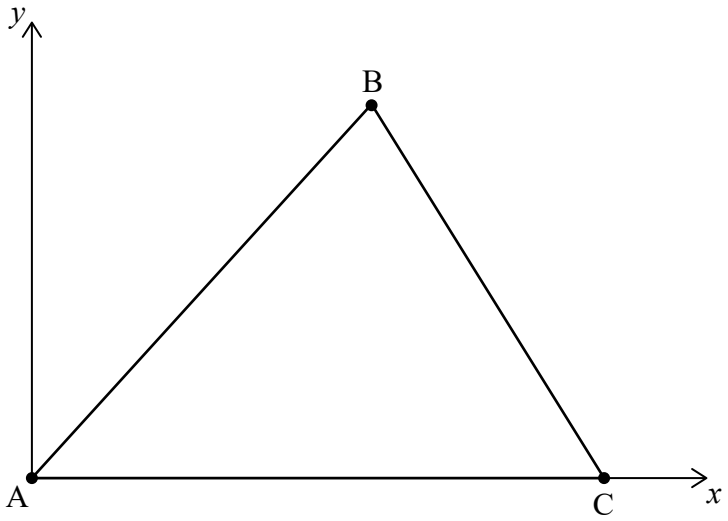
The following diagram shows part of the graph of $f(x) = (6 - 3x)(4 + x)$, $x \in \mathbb{R}$. The shaded region R is bounded by the x-axis, y-axis and the graph of f.



(a) Write down an integral for the area of region R. [2]

(b) Find the area of region R. [1]

The three points $\mathrm{A}(0, 0)$, $\mathrm{B}(3, 10)$ and $\mathrm{C}(a, 0)$ define the vertices of a triangle.



(c) Find the value of a, the x-coordinate of C, such that the area of the triangle is equal to the area of region R. [2]

(This question continues on the following page)

(Question 10 continued)

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

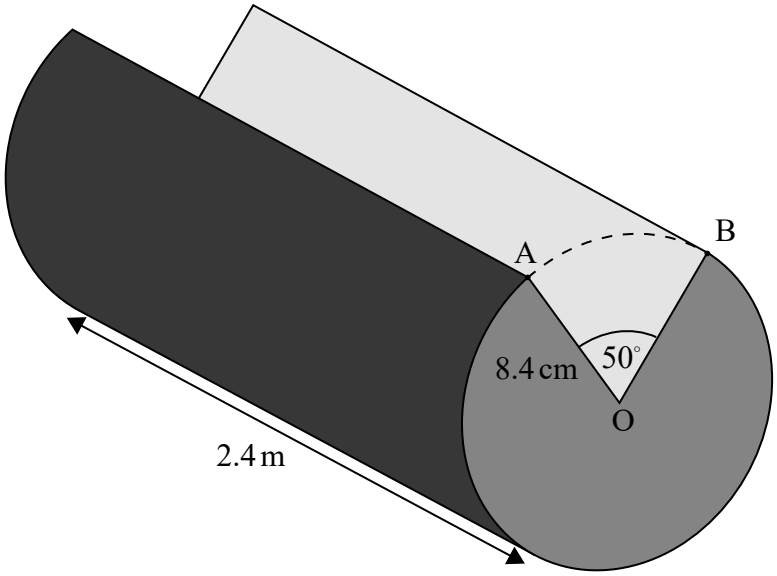
. . . . . .

. . . . . .

11. [Maximum mark: 4]

Helen is building a cabin using cylindrical logs of length $2.4\,\text{m}$ and radius $8.4\,\text{cm}$. A wedge is cut from one log and the cross-section of this log is illustrated in the following diagram.

diagram not to scale



Find the volume of this log.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

12. [Maximum mark: 6]

Jae Hee plays a game involving a biased six-sided die.
The faces of the die are labelled -3, -1, 0, 1, 2 and 5.
The score for the game, X, is the number which lands face up after the die is rolled.
The following table shows the probability distribution for X.

Score x	-3	-1	0	1	2	5
$\mathbf{P}(X = x)$	$\frac{1}{18}$	p	$\frac{3}{18}$	$\frac{1}{18}$	$\frac{2}{18}$	$\frac{7}{18}$

(a) Find the exact value of p. [1]

Jae Hee plays the game once.

(b) Calculate the expected score. [2]

Jae Hee plays the game twice and adds the two scores together.

(c) Find the probability Jae Hee has a **total** score of -3. [3]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

13. [Maximum mark: 6]

Mr Burke teaches a mathematics class with 15 students. In this class there are 6 female students and 9 male students.

Each day Mr Burke randomly chooses one student to answer a homework question.

(a) Find the probability that on any given day Mr Burke chooses a female student to answer a question. [1]

In the first month, Mr Burke will teach his class 20 times.

(b) Find the probability he will choose a female student 8 times. [2]

(c) Find the probability he will choose a male student at most 9 times. [3]

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

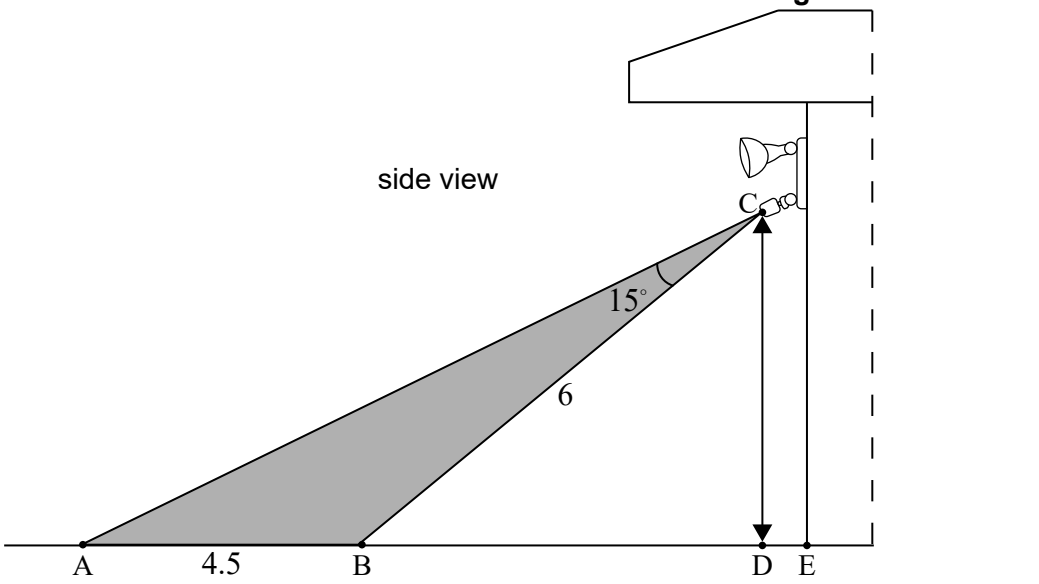
. . . . . .

. . . . . .

14. [Maximum mark: 8]

Ollie has installed security lights on the side of his house that are activated by a sensor. The sensor is located at point C directly above point D. The area covered by the sensor is shown by the shaded region enclosed by triangle ABC. The distance from A to B is $4.5\,\text{m}$ and the distance from B to C is $6\,\text{m}$. Angle $A\hat{C}B$ is 15°.

diagram not to scale



(a) Find $C\hat{A}B$. [3]

Point B on the ground is $5\,\text{m}$ from point E at the entrance to Ollie's house. He is $1.8\,\text{m}$ tall and is standing at point D, below the sensor. He walks towards point B.

(b) Find the distance Ollie is **from the entrance to his house** when he first activates the sensor. [5]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

Please **do not** write on this page.

Answers written on this page
will not be marked.