Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 2

Specimen paper

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

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Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1.   [Maximum mark: 17]

In this question, give all answers to two decimal places.

Bryan decides to purchase a new car with a price of €14 000, but cannot afford the full amount. The car dealership offers two options to finance a loan.

Finance option A:

A 6 year loan at a nominal annual interest rate of 14 % **compounded quarterly**. No deposit required and repayments are made each quarter.

(a)   (i)   Find the repayment made each quarter.

(ii)   Find the total amount paid for the car.

(iii)   Find the interest paid on the loan.   [7]

Finance option B:

A 6 year loan at a nominal annual interest rate of r % **compounded monthly**. Terms of the loan require a 10 % deposit and monthly repayments of €250.

(b)   (i)   Find the amount to be borrowed for this option.

(ii)   Find the annual interest rate, r.   [5]

(c)   State which option Bryan should choose. Justify your answer.   [2]

Bryan's car depreciates at an annual rate of 25 % per year.

(d)   Find the value of Bryan's car six years after it is purchased.   [3]

Turn over

2.   [Maximum mark: 14]

Slugworth Candy Company sell a variety pack of colourful, shaped sweets.

The sweets are produced such that 80% are star shaped and 20% are shaped like a crescent moon. It is known that 10% of the stars and 30% of the crescent moons are coloured yellow.

(a)   Using the given information, **copy** and complete the following tree diagram.   [2]
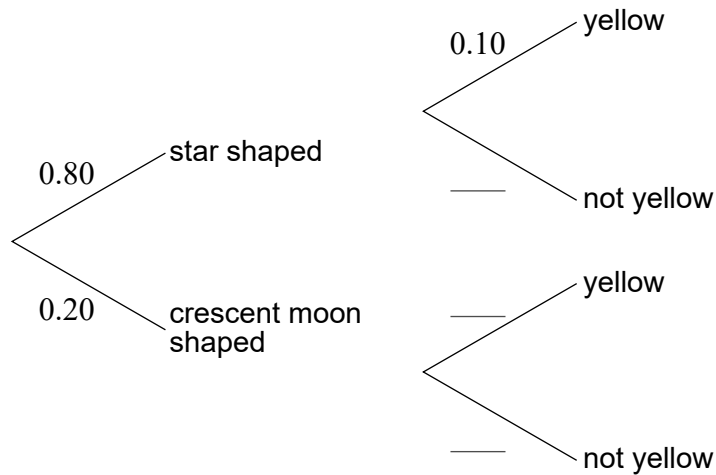
(b)   A sweet is selected at random.

   (i)   Find the probability that the sweet is yellow.

   (ii)   Given that the sweet is yellow, find the probability it is star shaped.   [4]

(This question continues on the following page)

(Question 2 continued)

According to manufacturer specifications, the colours in each variety pack should be distributed as follows.

Colour	Brown	Red	Green	Orange	Yellow	Purple
Percentage (%)	15	25	20	20	10	10

Mr Slugworth opens a pack of 80 sweets and records the frequency of each colour.

Colour	Brown	Red	Green	Orange	Yellow	Purple
Observed Frequency	10	20	16	18	12	4

To investigate if the sample is consistent with manufacturer specifications, Mr Slugworth conducts a χ^2 goodness of fit test. The test is carried out at a 5% significance level.

(c)   Write down the null hypothesis for this test.   [1]

(d)   **Copy** and complete the following table in your answer booklet.   [2]

Colour	Brown	Red	Green	Orange	Yellow	Purple
Expected Frequency						

(e)   Write down the number of degrees of freedom.   [1]

(f)   Find the p-value for the test.   [2]

(g)   State the conclusion of the test. Give a reason for your answer.   [2]

3.   [Maximum mark: 17]

The Malvern Aquatic Center hosted a 3 metre spring board diving event. The judges, Stan and Minsun awarded 8 competitors a score out of 10. The raw data is collated in the following table.

Competitors	A	B	C	D	E	F	G	H
Stan's score (x)	4.1	3	4.3	6	7.1	6	7.5	6
Minsun's score (y)	4.7	4.6	4.8	7.2	7.8	9	9.5	7.2

(a)   (i)   Write down the value of the Pearson's product–moment correlation coefficient, r.

(ii)   Using the value of r, interpret the relationship between Stan's score and Minsun's score.   [4]

(b)   Write down the equation of the regression line y on x.   [2]

(c)   (i)   Use your regression equation from part (b) to estimate Minsun's score when Stan awards a perfect 10.

(ii)   State whether this estimate is reliable. Justify your answer.   [4]

The Commissioner for the event would like to find the Spearman's rank correlation coefficient.

(d)   **Copy** and complete the information in the following table.   [2]

Competitors	A	B	C	D	E	F	G	H
Stan's Rank		8					1	4
Minsun's Rank		8					1	4.5

(e)   (i)   Find the value of the Spearman's rank correlation coefficient, r_s.

(ii)   Comment on the result obtained for r_s.   [4]

The Commissioner believes Minsun's score for competitor G is too high and so decreases the score from 9.5 to 9.1.

(f)   Explain why the value of the Spearman's rank correlation coefficient r_s does not change.   [1]

4. [Maximum mark: 15]

The Happy Straw Company manufactures drinking straws.

The straws are packaged in small closed rectangular boxes, each with length $8\,\text{cm}$, width $4\,\text{cm}$ and height $3\,\text{cm}$. The information is shown in the diagram.
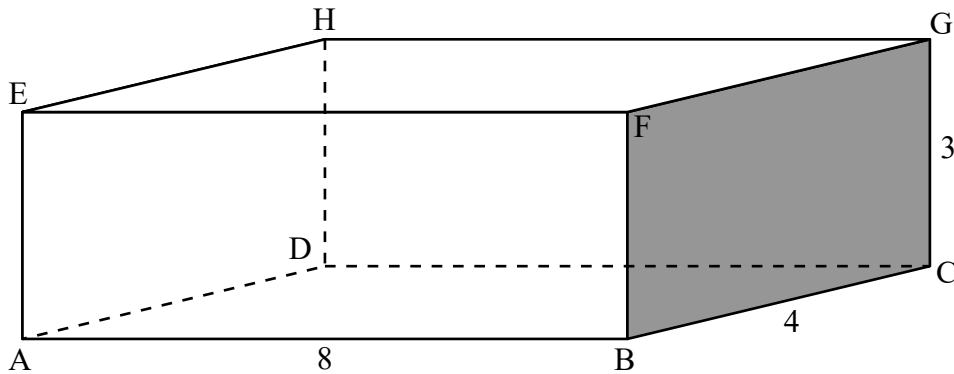
(a) Calculate the surface area of the box in cm^2. [2]

(b) Calculate the length AG. [2]

Each week, the Happy Straw Company sells x boxes of straws. It is known that $\dfrac{dP}{dx} = -2x + 220$, $x \geq 0$, where P is the weekly profit, in dollars, from the sale of x thousand boxes.

(c) Find the number of boxes that should be sold each week to maximize the profit. [3]

The profit from the sale of $20\,000$ boxes is $\$1700$.

(d) Find $P(x)$. [5]

(e) Find the least number of boxes which must be sold each week in order to make a profit. [3]

Turn over

5. [Maximum mark: 17]

The braking distance of a vehicle is defined as the distance travelled from where the brakes are applied to the point where the vehicle comes to a complete stop.

The speed, $s \text{ ms}^{-1}$, and braking distance, d m, of a truck were recorded. This information is summarized in the following table.

Speed, $s \text{ ms}^{-1}$	0	6	10
Braking distance, d m	0	12	60

This information was used to create Model A, where d is a function of s, $s \geq 0$.

$$\text{Model A: } d(s) = ps^2 + qs, \text{ where } p, q \in \mathbb{Z}$$

At a speed of 6 ms^{-1}, Model A can be represented by the equation $6p + q = 2$.

(a) (i) Write down a second equation to represent Model A, when the speed is 10 ms^{-1}.

(ii) Find the values of p and q [4]

(b) Find the coordinates of the vertex of the graph of $y = d(s)$. [2]

(c) Using the values in the table and your answer to part (b), sketch the graph of $y = d(s)$ for $0 \leq s \leq 10$ and $-10 \leq d \leq 60$, clearly showing the vertex. [3]

(d) Hence, identify why Model A may not be appropriate at lower speeds. [1]

Additional data was used to create Model B, **a revised model** for the braking distance of a truck.

$$\text{Model B: } d(s) = 0.95 s^2 - 3.92 s$$

(e) Use Model B to calculate an estimate for the braking distance at a speed of 20 ms^{-1}. [2]

The actual braking distance at 20 ms^{-1} is 320 m.

(f) Calculate the percentage error in the estimate in part (e). [2]

(This question continues on the following page)

(Question 5 continued)

It is found that once a driver realizes the need to stop their vehicle, 1.6 seconds will elapse, on average, before the brakes are engaged. During this reaction time, the vehicle will continue to travel at its original speed.

A truck approaches an intersection with speed $s \, \mathrm{m\,s}^{-1}$. The driver notices the intersection's traffic lights are red and they must stop the vehicle within a distance of $330 \, \mathrm{m}$.

diagram not to scale
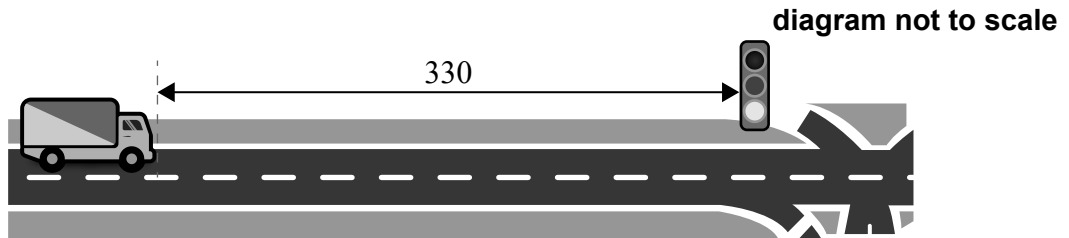
(g)   Using model B and taking reaction time into account, calculate the maximum possible speed of the truck if it is to stop before the intersection.   [3]