Geometry challenges
(surfaces and volumes)

1.
Twoflower is sitting at the top of the temple of Bel-Shamharoth, which has the shape of a large octogonal pyramid. The side of the base is 15m and the height is 25m. He wants to go down as quickly as possible, so he decides to slide down, but he doesn't know whether to slide via an edge or via the middle of one side.

a) What is the angle of his trajectory if he goes down an edge?
b) What is the angle of his trajectory if he goes down the middle of a side?
c) Calculate the overall surface area of the temple.

2.
In a swordsmanship demonstration a participant cuts a spherical watermelon into four equal pieces.
a) Given that the diameter of the watermelon was 44cm, calculate the surface area of each of those pieces.
A cantaloupe can be considered to be spherical and hollow. Its radius is 12cm and the hollow has a radius of 7cm.
b) What is its volume?
The swordmaster cuts the cantaloupe it in half, then cuts one half into two equal parts and then one of those parts into two equal bits.
c) Calculate the surface area of one of those bits.
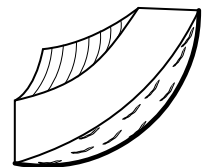
3.
A cube is cut by a plane P that passes through three vertices as shown in the picture (a).
This results in a part of the cube being separated as shown in b, and said part is placed so that it forms a pyramid as shown in c.
Assuming that the edges of the cube are 1m long, calculate the height of said pyramid.