

Math 3 study guide.

Algebra

Knowing and understanding.

The student is required to know:

- what a polynomial is, how many terms it is composed of and how many factors any term is composed of.
- what a polynomial equation is, what the degree of a polynomial equation is and how many solutions any given equation has.
- what the absolute value of a number or an expression is.
- what an inequality is and how the solution to one of them is properly written.

Note: only the definition of polynomial might be asked in the text.

Skills.

The student is required to be capable of:

- Identifying the terms and factors of any polynomial.
- Arranging any given polynomial in order from highest power to lowest.
- Simplifying an expression in order to turn it into a polynomial.
- Factorising any second degree polynomial with real solutions, including those that have non-whole solutions.
- Solving any second degree equation with real solutions, including those that have non-whole solutions.
- Factorizing any third degree polynomial that has whole solutions.
- Solving any third degree equation that has whole solutions.
- Applying Ruffini's rule to find the solutions of any third degree polynomial that has three whole solutions.
- Simplifying rational expressions, including numerical and algebraic expressions.
- Calculating the absolute value of a number.
- Solving first degree equations involving absolute value using the "split strategy".
- Solving any first degree inequality, presenting the solutions in the correct format.
- Being able to represent, in the real number line, the solutions to any first degree inequality.

Functions.

Knowledge

- Of the definition of function and types of functions, names of the input variable, names of the output variable, domain and range.
- Of the basic functions and their properties (straight lines, parabolas, third and fourth degree polynomials, exponential, square root and logarithm).
- Of the four possibilities that make a function not to respond.
- Of what a graph represents.
- Of what a limit is and what it is used for.
- Of what an asymptote is and why it happens.

Skills

- To plot a function identifying vertical and horizontal asymptotes.
- To build a function to model a given simple mathematical behaviour.
- To obtain the domain of a function.

- To read a graph and identify the values of a function, the values of x for which the function has a given value, where the function is positive, where it is negative, where the function increases and decreases and where the function has positive or negative curvature.
- To calculate the limit of a function in a value where the function responds and to calculate the limit of a function in a value where the function doesn't respond but the limit is determined.
- To obtain a function that passes through two or three given points.
- To calculate approximately the value of a limit using the calculator.

Derivatives

Knowledge

- Of the concept of rate of change of a function.
- Of what the rate of change of a function means in the case of a linear function and between two given points in a curve.
- Of the relationship between the derivative and the rate of change.
- Of the definition of the derivative of a function in a point.
- Of the meaning of the derivative in functions that express the progress of a phenomenon over time or any property over space or time.
- Of the relationship between the physical magnitudes of position, speed and acceleration.
- Of the concept of exponential function and the relationship between the number e and the concept of derivative.

Skills

- Calculate the rate of change of a function given two points in a readable graph or two pairs of input-output values.
- Calculate the rate of change of any function given the function and the two values of x within which it must be calculated.
- Use the definition of the derivative to differentiate any given function.
- Identify, in a plotted curve, where the derivative is positive, negative, zero or can't be calculated.
- Derive polynomial functions, trigonometric, logarithmic, exponential as well as products, quotients or compositions of these functions.
- Use the derivative to calculate the slope of the function in any given point.

Using the derivative

Knowledge

- Of the concept of sign analysis of a function and what it is used for.
- Of the concept of sign analysis of the derivative of a function and what it is used for.
- Of the concept of sign analysis of the second derivative of a function and what it is used for.
- Of the meaning of the derivative of a function in a point and what information it provides.
- Of the concepts of local maximum, absolute maximum, local minimum, absolute minimum, terrace point and inflection point.

Skills

- Obtaining the equation of the tangent line to any function given said function and the x value of the point where said tangent touches the function.
- Use the derivative to extrapolate the behaviour of a function and predict for which value of x it will reach a given value.

- Use the rate of change to extrapolate the behaviour of a function given only a series of input-output pairs.
- Find critical points in a function.
- Identify critical points as maximum, minimum or terrace points.
- Distinguish between terrace point and inflection point.
- Finding the absolute maximum and the absolute minimum of a function in a given interval.
- Obtaining a function that expresses the behaviour of a problem as function of one single variable.
- Optimizing a function to find for what value of x the output of the function is maximum or minimum.

Integration

Knowledge

- Of the concept of primitive of a function.
- Of the relationship between differentiation and antidifferentiation.
- Of the basic rules for calculating the primitive of a function.
- Of the meaning of the integral of a function between two points.
- Of the relationship between the integral of a function and the area below its curve.
- Of the differential relationships that naturally exist between functions such as (but not restricted to) force and work, flow and volume, power and energy.

Skills

- Obtain the primitives of a polynomial function.
- Obtain the primitive of any function in which x is exponentialized to any rational number.
- Obtain the primitive of basic trigonometric and exponential functions.
- Use the integral of a function to calculate the area below its curve and the x -axis.
- Use the integral of a function to calculate the area between said function and the x -axis in a given interval of x even when the function crosses said axis.
- Using integrals to calculate the area between two functions in a given interval of x .
- Using integrals to calculate the area trapped between two functions.
- Using integrals to calculate the overall sum of a property that is expressed as a function of x (such as “number of trees per kilometer”)

Trigonometry

Knowledge

- Of the definitions of sine, cosine and tangent both for any right angle triangle and in the unit circle.
- Of the sines and cosines of the basic simple angles (multipliers of 30 and 45).
- Of the limitations of the sine rule and in which cases it may be ambiguous to use it.
- Of the cases in which the cosine rule must be used.
- Of the nature of the equation of a circle and where it comes from.

Skills

- Representing approximately any angle in the unit circle and indicating its sine and cosine.
- Use the unit circle to deduce the sine and cosine of angles.
- Solve exercises in which the sine and cosine rule need to be used at least once.
- Write the equation of a circle given the coordinates of the center and the radius.
- Sketching a circle given its equation.
- Using equations and inequalities to refer to the circumference of a circle, the interior and the

exterior of said circle.

- Using the calculator to obtain which angle has a given sine, cosine or tangent.
- Solving any coherent triangle given enough information (three sides, two sides and one angle or two angles and one side)