

Integration test

Maths 3c

Non-calculator section.

Name: _____

1. Say whether these sentences are true or false and state why.

a) If I have a function $a(x)$ and I derive it and get $b(x)$ and then calculate the primitive of $b(x)$, then I'll end up with the same function $a(x)$. (1 0 0)

b) If I have a function $a(x)$ and calculate its primitive, $c(x)$ and then I derive $c(x)$ I'll end up with the same function $a(x)$. (1 0 0)

c) A function has only one primitive. (1 0 0)

2. Bonus question: find the mistake in this sentence: "The function $x^3 + 2x + 3$ is the primitive of the function $3x^2 + 2$ ". (1 0 0)

3. Find the primitives of these functions.

a) (1 0 0)

$$f(x) = x^2 + 3x + 11$$

b) (0.5 0 0)

$$g(x) = \frac{1}{x}$$

c) (0 0.5 0)

$$h(x) = \frac{1}{x^2}$$

d) (0.5 0 0)

$$i(x) = -\cos x$$

e) (0 0.5 0)

$$j(x) = \sqrt{x}$$

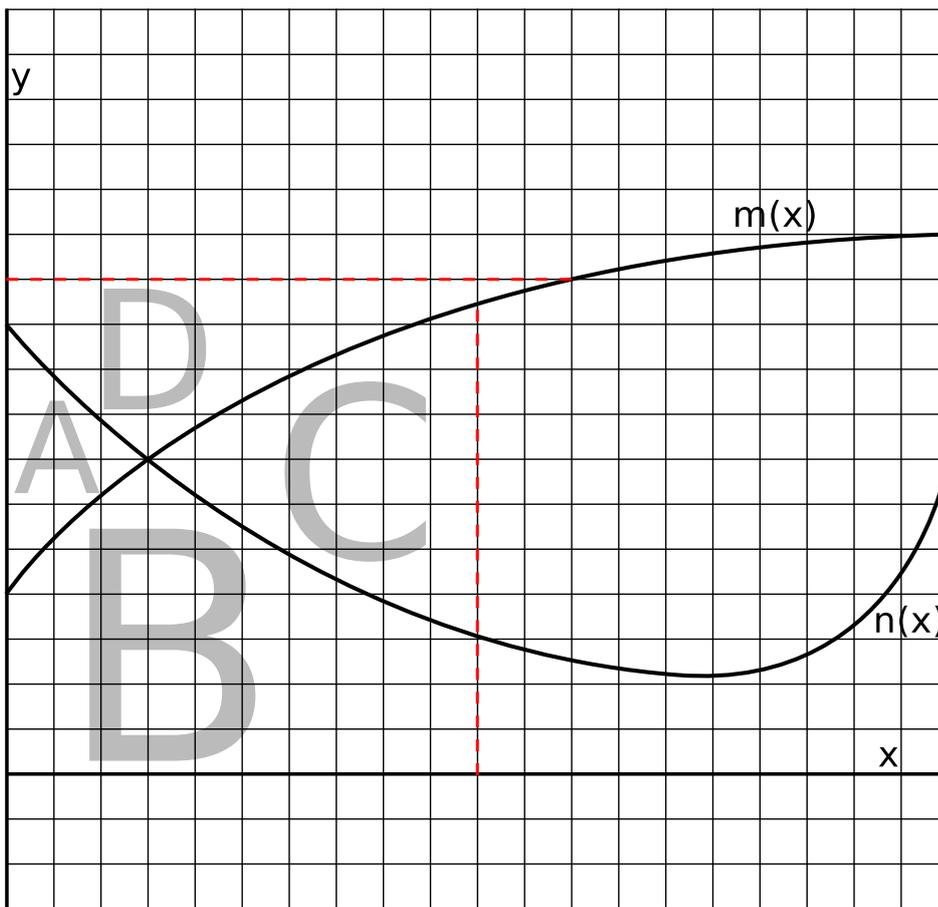
f) (0 0.5 0)

$$k(x) = \frac{1}{\sqrt{x}}$$

g) (0 0.5 0)

$$l(x) = \frac{2x + 5}{4}$$

4. Given these graphs of the functions $m(x)$ and $n(x)$:



Write the integrals that you must use to calculate the areas A, B and C. (0 1 0 each)

5. Bonus question: write the integral that calculates the area D. (0 1 0)

6. Write the expression that calculates the area trapped between the function $o(x) = x^3 - 8x^2 + 19x - 12$ and the x-axis between $x=0$ and $x=5$.(0 1 0)

7. Calculate (0 1 0)

$$\int_1^3 \left(2x^2 + \frac{2}{3}x + 4 \right) dx$$

8. Calculate the area between the function $p(x) = x^2 - 2x$ and the x-axis between $x=0$ and $x=3$.(0 0 1)

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9. Calculate the area trapped between the functions(0 0 2)

$$q(x) = x^2 - 3x - 10$$

and

$$r(x) = x^3 - 4x^2 - x - 2$$

10. The hydropower plant in Floda is producing electricity thanks to the water that flows out of a lake. Unfortunately there hasn't been enough water to turn it on until today. When the turbines are turned on, they don't operate at full power directly, it takes some time to reach the maximum power of 6000000 Watts. From the moment they are turned on, the power changes over time according to the function

$$s(t) = 2000000 + 2t + 0,1t^2$$

and when the power reaches 6000000 then it remains constant until the plant is turned off again (s is expressed in watts and t is expressed in seconds).

- a) How long does it take for the plant to reach the maximum power?(1 0 0)

b) How much energy does it generate during that time?(0 0 1)

c) Draw an approximate graph of how the power changes over time during the first three hours.(0 1 0)

11. The spaceship Nicky Minaj is entering the atmosphere of planet [insert humorous name here] and, as it so happens in these cases, the hull starts to heat up.

In order to control the landing, the ship is entering the planet rear first and will use the thrusters to stop the fall, but the thrusters won't work if they get hotter than 1700°C.

The science officer, Mr. Flock, has determined that only the first 150 Km will be dangerous, and the ship won't heat up much after that. The atmosphere is thinner at higher altitudes, so the ship will heat up slower first and then faster, and Mr. Flock has come up with a function that can tell how many degrees the temperature will increase per kilometer.

$$Ti(x) = 0,5 + 0,001x + 0,001x^2$$

Where T_i is the temperature increase in degrees celsius per kilometer and x is the distance in kilometers. Remember: T_i is not the temperature depending on the position, but the **temperature increase** in degrees per kilometer.

a) Calculate how many degrees the hull will heat up.(0 0 1)

b) Knowing that the initial temperature of the hull is -120°C , calculate the final temperature of the Nicky Minaj's rear.(1 0 0)

c) Bonus question: Mr. Flock is born in the USA (United Sectors of Asteroids) and has accidentally given the function $T_i(x)$ for x expressed in miles instead of Km. What is the final temperature, knowing that a mile is approximately 1.6 km?(0 0 1)